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A New Approach of Ordering for AC-TRS

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Term rewriting system (TRS) is regarded as computational model that reduces terms by applying directed equations, called rewrite rules. TRS is widely used as a model of functional programming languages and as basis of automated theorem proving, specification and verification.

The termination and confluent properties are fundamental notions of TRS as computational models. A rewriting system is said to be terminating, if there are no infinite reducton sequences of reductions. If TRS is terminating, any strategy can always reduce a term into the its normal form. Moreover, we can easily check whether a terminating TRS is confluent. Thus, the termination property is very important in the theory of TRS.

In general, it is undecidable whether a given TRS is terminating or not. Thus several sufficient conditions have been successfully developed in particular cases. Generally speaking, those techniques can be clasified into two approaches: semantic ordering method and syntactic orderinfg one. In the former, terms are interpreted compositionally in some (well-founded monotone) algebra in order to prove termination. The semantionc ordering methoed is difficult to be used in automated proof. The other while, the syntactic ordering is based on the syntactical structure of terms. The precedence is an order on function symbols. Then, Syntactic ordering extends precedece to order on terms. This paper adopts syntactic approach, since syntactic orderings can be implemented easily for the termination of TRS. The simplification ordering plays an important roll in syntactic ordering method. The simplification ordering is defined as a partial order that has monotonic and subterm properties, and the simplification ordering is well-founded. Then it is well known if $>$is simplification ordering and all reduction rules $l \to r$ in a TRS R satisfy $l > r$, then R is terminating. Many kinds of simplifiation orderings have been proposed by many researches. For example, D. Plaisted introduced the Path of Subterm Ordering (PSO), N. Dershowitz introduced the Recursive Path Ordering (RPO), and P. Lescanne introduced the Recursive Decomposition Ordering (RDO), and so on.

Associative-Commutative-TRSs (AC-TRSs) is equational term rewriting system, and equations in E are either an Associativity or Commutativity axiom. In AC-TRS, the equations can be reagrded as unoriented rewrite rules. Thus the equation make infinitly reduction sequences, because equations make non-terminating sequences no matter how they are used as a rewrite rule. In termination proof of AC-TRS, we must consider AC-equivalence classes of terms instead of simple terms. Indeed, this method has been widly used in automatic termination proofs of rewrite systems with AC equations.

The simplification ordering can not use directly for termination proof of AC-TRS. To overcome this difficulty, the transformation by flatting have been introduced. Let f be AC-operator, and X, Y be multisets of terms. Then flatting transformation is defined by $f(X, f(Y)) \rightarrow_{Fl} f(X, Y)$.

The flattened terms have no nesting occurrences of AC operators and the order of subterms occuring as arguments does not matter. We use a multiset notation to denote such subterms. Through flatting transformation, AC-equivalence class is represented by a single term. The simplification ordering that satisfies AC-compatibility can be applied for proving termination of AC-TRS. The ordering of RPO with flatting have AC-compatibility. Unfortunatly, this ordering is not simplification ordering. Therefore, for being simplification ordering, a number of attempts have been tried to overcome this diffculty.

Bachmair introduced Associative Path Ordering (APO), but this require a limitation on precedence. Kapur introduced AC-ordering with pseudo-copying transformation and elevating transformation. It supposed precedence is total order. This ordering does not require limitations on precedence, but it uses non-deterministic procedure that is hard to inplement. Rubio introduced Interpretation rule that contains selecting maximal ordering term. This method does not have non-deterministic procedure. These orderings use flatting transformations for satisfying simplification ordering and AC-compatibility. In this paper, we introduces another method without flatting.

In chapter 4, instead of flatting, we give the stacking that is a new transformation. Let f be AC-operator and X, Y are multisets of terms. The stacking is $f(X, f(Y)) \rightarrow_{St} f(f(X, Y))$, where $\underbrace{f(\cdots(f}_{n}(T)\cdots)$ represents $f^n(T)$ $(n \geq 1)$ and n is degree. The flatting compress coherents AC-operators to one. The stacking keeps the infomation of a number of continuous AC-operators in degree. We first consider RPO with stacking. Unfortunately, this ordering is not simplification ordering. It does not have monotonicity. Because stacking assembles AC-operators to the root symbol, it disturbs relation between subterms and AC-operators. We must treat transformation carefully for satisfying simplification ordering and AC-compatibility.

The vanishing AC-operators causes that RPO with flatting is not simplification ordering and AC-compatibility. The stacking does not have vanishing of AC-operator. However, the AC-operators in subterms moves to the root symbol. Hence the stacking crashes the relation between AC-operators and subterms. We think the degree of AC-operators is the inclination of subterms, the stacking changes the inclination of subterms, and the ordering that RPO with stacking does not have monotonicity.

We introduced new transformations for the inclination. We treat the inclination of AC-operators carefully. The first transformation is called Reconstruction. The inclination of AC-operators for subterms is caused by assembling AC-operator to the root symbol. Since the Reconstruction distribute AC-operators for subterms, it may vanish the inclination of AC-operators. Reconstruction Ordering (RCO) is based on Reconstuction and RPO. We apply Reconstruction for the stacking term, and compare them by RPO.

We prove that RCO satisfy AC-compatibility, irreducibility, subterm property, and monotonicity. Unfortunatly, transivity have never proven, because Reconstuction has a lot of case of degree of AC-operators and subterms. Thus, it is too diffcult to prove the transivity.

The second approach is called Department transformation and Lifting transformation. We examine the condition that disturb realtion between AC-operators and subterms. There are conditions that stacking crashs relaton and, it is clasifyed into two case. The first condition is called inclination, the second one called take-in. We can search terms that have possibility to disturb monotonicity for two condition.

If a term satisfies inclinatoin condition, we apply Department transformation. Department transformation extracts subterms from original term. If the term satisfy take-in condition, we apply Lifting transformation. The Lifting transformation changes the term to multiset of terms. These transformations are applied to terms that has AC-operators as the root symbol. SDLO is based on RPO, and terms are transfomed by stacking, Department, and Lifting transformations. However, SDLO does not saitisfy the monotonicity. and it has a problem in trsnaformation.

Therefor, from these observatoins we can conclude that a new ordering method based on stacking transformation has several esential difficulties for guaranteeing AC-compatibility.