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Petri Nets as Models of Linear Logic

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In the years 1960-1962, Carl Adam Petri defined Petri nets which is a general purpose mathematical model for describing relations existing between conditions and events. Petri nets consist of two types of elements, places and transitions. Each place models a process in terms of types of resources, and can hold arbitrary nonnegative multiplicity. Each transition represents a state transition rule, i.e., how those resources are consumed or produced by actions. They are described using the notion of multisets. A multiset over a set P is a function, $m : P \rightarrow \mathcal{N}$ [7], [10].

Linear Logic was discovered by J. Y. Girard in 1987 [3], [4], [13], [14]. Linear logic (intuitionistic, classical and predicate) are obtained by deleting the contraction and the weakening rules from standard sequent calculus formulations of corresponding logics. In the Gentzen sequent calculus for intuitionistic logic, a sequent $A_1, \cdots, A_n \rightarrow A$ is written to mean that the formula A is deducible from the assumption formulas $A_1, \cdots, A_n$ (we shall use capital Greek letters as an abbreviation for a sequence of formulas). The calculus has the two structural rules for adding a vacant assumption and removing of a duplicate of assumption.

$$\frac{\Gamma \rightarrow B}{\Gamma, A \rightarrow B}\ (\text{weakening}),$$

$$\frac{\Gamma, A, A \rightarrow B}{\Gamma, A \rightarrow B}\ (\text{contraction}).$$

In the presence of these rules the following two right introduction rules for conjunction

$$\frac{\Gamma \rightarrow A \quad \Delta \rightarrow B}{\Gamma, \Delta \rightarrow A \wedge B}\ (1),$$

$$\frac{\Gamma \rightarrow A \quad \Gamma \rightarrow B}{\Gamma \rightarrow A \wedge B}\ (2)$$

become interderivable in the sense that the first rule can be derived from the second by weakening, and the second from the first by contraction. In linear intuitionistic logic these rules (weakening and contraction) are deleted and the rule of (1) and (2) are no longer interderivable. Without them, propositions cannot be introduced arbitrarily into a list of assumption and a duplication in the list cannot be removed. It is in this sense that linear logic is a *resource conscious* logic.

The connection between linear logic and Petri nets has recently been the subject of great interest [2], [5], [6]. Girard's linear logic has a great deal of interest in how might be useful in the theory of parallelism. The places are like atomic propositions in linear logic and transitions like provability relation. Girard's phase semantics for linear logic in [3] uses quantales [1], [9], [11], [12], [15], and Engberg and Winskel [2] showed a straightforward way in which a Petri net induces a quantale and so becomes a model for intuitionistic linear logic. But they did not prove a completeness theorem for models induced by Petri nets.

In this thesis, we prove completeness for quantales generated by Petri nets. To prove completeness the quantales used in [2] do not work. Although the following proof shows that

$$(A \wedge B) \vee (A \wedge C) \to A \wedge (B \vee C)$$

is derivable in intuitionistic linear logic,

$$\dfrac{\dfrac{\dfrac{A \to A}{A \wedge C \to A}(\wedge \to) \quad \dfrac{\dfrac{B \to B}{B \to B \vee C}(\to \vee)}{A \wedge B \to B \vee C}(\wedge \to)}{A \wedge B \to A \wedge (B \vee C)}(\to \wedge) \quad \dfrac{\dfrac{A \to A}{A \wedge C \to A}(\wedge \to) \quad \dfrac{\dfrac{C \to C}{C \to B \vee C}(\to \vee)}{A \wedge C \to B \vee C}(\wedge \to)}{A \wedge C \to A \wedge (B \vee C)}(\to \wedge)}{(A \wedge B) \vee (A \wedge C) \to A \wedge (B \vee C)}(\vee \to) \quad ,$$

we cannot prove the sequent

$$A \wedge (B \vee C) \to (A \wedge B) \vee (A \wedge C).$$

That is, the *distributivity* of $\wedge$ and $\vee$ does not hold in intuitionistic linear logic. In the quantale given in [2], the distributivity is always valid. Therefore, if we want to prove completeness using the quantales of [2], then we have to add the distributivity to intuitionistic linear logic. However this is not what we intend to do. We construct quantales in which the distributivity is not always valid, and prove completeness. We can also prove completeness of intuitionistic linear logic with a modal operator for quantales by using similar construction.

In Chapter 2, we overview Petri nets and algebraic structures [8]. We introduce algebras including quantales, and closure operations on the algebras which play a crucial role in the proof of completeness.

In Chapter 3, we discuss intuitionistic linear logic (its syntax and semantics) and then prove soundness theorem for quantales generated by Petri nets. Next we show why we cannot prove completeness in the quantales used in [2], and then prove completeness using the quantale based on our construction.

In Chapter 4, we discuss a modal operator of course !. The absence of the rules for weakening and contraction is compensated, to some extent, by the addition of the modal operator !. We consider a semantics with the modal operator using similar construction, and then prove completeness of intuitionistic linear logic with the modal operator for quantales generated by Petri nets.

In Chapter 5, we consider classical quantales for classical linear logic generated by Petri nets.

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