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Equational Axioms for Process Algebra with Iteration

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1 Introduction

The purpose of this papar is to introduce a new iteration operator in BPP(Basic Parallel Process). In 1994, Bergstra, Bethke and Ponse introduced iteration in process algebra by means of Kleene's star operation and proposed an axiomatization for BPA(Basic Process Algebra) extended by binary iteration. Then Fokkink and Zantema proved that this axiomatization is complete with respect to bisimulation equivalence. In 1993, BPP(Basic Parallel Process) were introduced by Christensen. In this paper, we introduce a system for BPP extended by binary iteration, called $\mathrm{BPP}^{\star\|}$. We will prove the axiomatization for $\mathrm{BPP}^{\star\|}$ is sound with respect to bisimulation equivalence. But we don't know how $\mathrm{BPP}^{\star\|}$ should be extended in order to axiomatize bisimilarity.

Process algebra is a mathematical model for concurrent communicating processes. It is presented syntactically as a formal system and semantically as an algebra. We have some different views of processes, and there are some systems called process algebra in the wide sense, e.g. CCS(Calculus of Communicating Systems), CSP(Communicating Sequential Processes) and ACP(Algebra of Communicating Processes) etc. We define here process algebraically as follows. We will take a set of constants and a set of operators, and define a set of terms which are constructed from constants and operators inductively. Its semantics is presented as transition systems. There are some equivalence relations between processes. The concept of bisimilarity is two processes have same behavior is important role in equivalence relations. So it is an interesting question whether sound and complete equational theories exist or not.

2 BPP

BPP consists of an alpahbet A of atomic actions, together with two binary operators, i.e. alternative composition $+$ and parallel composition $\|$.

BPP terms are terms that can be constructed the atomic actions A and the two binary operators. Process terms can be defined in the BNF notation as follows

$$p \ ::= \ a \ | \ ap \ | \ p+p \ | \ p \| p$$

where $a \in A$ In the following, we assume that $\|$ binds stronger than $+$.

Table 1 presents action relations for BPP. The special symbol $\surd$ in this table represents successful termination.

$$a \in A \qquad a \xrightarrow{a} \surd$$

$$+ \qquad \frac{x \xrightarrow{a} x'}{x+y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x+y \xrightarrow{a} y'} \qquad \frac{x \xrightarrow{a} \surd}{x+y \xrightarrow{a} \surd} \qquad \frac{y \xrightarrow{a} \surd}{x+y \xrightarrow{a} \surd}$$

$$\| \qquad \frac{x \xrightarrow{a} x'}{x \| y \xrightarrow{a} x' \| y} \qquad \frac{y \xrightarrow{a} y'}{x \| y \xrightarrow{a} x \| y'} \qquad \frac{x \xrightarrow{a} \surd}{x \| y \xrightarrow{a} y} \qquad \frac{y \xrightarrow{a} \surd}{x \| y \xrightarrow{a} x}$$

Table 1: Action relation for BPP

The equivalence between BPP terms which we are interested in here is bisimilarity.The bisimilarity defined as follows.

Definition A binary relation R is a *bisimulation* if whenever $R(p,q)$ then for each $a \in A$

1. if $p \xrightarrow{a} p'$then $q \xrightarrow{a} q'$ for some q' with $R(p',q')$
2. if $q \xrightarrow{a} q'$ then $p \xrightarrow{a} p'$ for some p' with $R(p',q')$
3. if $p \xrightarrow{a} \surd$ and only if $q \xrightarrow{a} \surd$

Two processes p and q are called *bisimilar*, denoted by $p \leftrightarroweq q$, if there exists a relation R between processes.

We now give an equational theory for BPP process. Table 2 contains an axiom system for BPP. It contains standard inference rules for the equality.

A1	$x + y$	$=$	$y + x$
A2	$(x + y) + z$	$=$	$x + (y + z)$
A3	$x + x$	$=$	x
A6	$x \parallel y$	$=$	$y \parallel x$
A7	$x \parallel (y \parallel z)$	$=$	$(x \parallel y) \parallel z$

Table 2: axiom for BPP

3 Iteration

Systems of recursion equations play a fundamental role in process algebra as a means for specifying or analyzing infinite behaviour. So we introduce a new operation that give sufficient expressive power to study infinite behaiour in process algebra without such systems.

In 1956, Kleene introduced the binary operation * for describing regular events . He defined regular expressions, and gave algebraic transformation rules for these,

$$E^*F = F \vee E(E^*F)$$

(E^*F being the iterate of E on F) Kleene also noted the correspondence with the conventions of algebra, treating $E \vee F$ as analogous to $E + F$,and EF as the product of E and F. Following Kleene, Bergstra, Betheke and Ponse introduced x^*y in process algebra with defining equation

$$x^*y = x \cdot (x^*y) + y$$

So x^*y is the process that chooses between x and y, and upon termination of x has this choice again.

The definition of * contains sequential composition $\cdot$, but BPP doesn't contain it. So we will define a new iterate operation. We introduce a operation $^\bigstar \parallel$ which we call the BPS(Binary Parallel Star). This operation is defined by

$$x^\bigstar \parallel y = x \parallel (x^\bigstar \parallel y) + y$$

So $x^\bigstar \parallel y$ is the process that executes processes x and y in parallel, and x can execute many times in parallel.

This operator has two meanings.

- a point of execute y the number of execute x is fixed
- a point of execute y the number of execute x isn't fixed

We investigated both cases to define axioms and action relations. As a result the former we can get an axiom and action relations, but the latter we cann't get them.

In the case that a point of execute y the number of execute x is fixed, we will think of $x^\bigstar \parallel y$ as

$$y + x \parallel (y + x \parallel (y + x \parallel (y + x \parallel (y + x \parallel (y + x \parallel (\cdots)\cdots))))).$$

Table 3 presents action relations for BPS of this case. We shall write p^n to represent the term $p \parallel \cdots \parallel p$ consisting of n copies of p combined in parallel.

Note that BPS's action relations contain infinite action relations, e.g. $\dfrac{x \xrightarrow{a} x'}{x^\bigstar \parallel y \xrightarrow{a} x^n \parallel x' \parallel (x^\bigstar \parallel y)}$ says that if x can execute a and there by turn into x' then $x^\bigstar \parallel y$ can execute a and there by turn into $x^n \parallel x' \parallel (x^\bigstar \parallel y)$. So it contains a variable n, and it presents infinite action relations.

$$\text{BPS} \quad \begin{array}{cc} \dfrac{x \xrightarrow{a} x'}{x^\bigstar \parallel y \xrightarrow{a} x^n \parallel x' \parallel (x^\bigstar \parallel y)} & \dfrac{y \xrightarrow{a} y'}{x^\bigstar \parallel y \xrightarrow{a} x^n \parallel y'} \\ \dfrac{x \xrightarrow{a} \surd}{x^\bigstar \parallel y \xrightarrow{a} x^n \parallel (x^\bigstar \parallel y)} & \dfrac{y \xrightarrow{a} \surd}{x^\bigstar \parallel y \xrightarrow{a} x^n} \end{array}$$

Table 3: Action relation for BPS

In Table 4 we introduce the axiom system BPS.

$$\text{BPS1} \quad x \parallel (x^\bigstar \parallel y) + y \quad = \quad x^\bigstar \parallel y$$

Table 4: Axiom for BPS

The axiomatization for BPS is sound with respect to bisimulation equivalence.

Theorem(Soundness) if $p = q$ then $p \leftrightarrow q$

4 Conclusion

In this paper we gave a axiom and action relations for BPS, and we proved soundness with respect to bisimulation equivalence. Here are some left issues. Our axiom is constructed by only one rule. To prove completeness with respect to bisimulation equivalence will need more axiom. In the case of BPA* there are three axioms. How should BPS be extended in order to axiomatize bisimilarity?