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Description	

Accumulator-assisted Distributed Turbo Codes for Relay Systems Exploiting Source-Relay Correlation

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***Abstract*—In relay systems, the probability of errors occurring in the source-relay (S-R) link can be viewed as representing correlation between the source and the relay. This letter proposes a very simple iterative decoding technique, accumulator-assisted distributed turbo code (ACC-DTC) using 2-state (memory-1) convolutional codes, where the correlation knowledge between the source and the relay is estimated and exploited at the destination.**

***Index Terms*—relay system, iterative decoding, decode-and-forward, distributed Turbo codes, doped accumulator**

I. Introduction

In this letter, we consider a new relay system, where it is assumed that the relay can not always decode correctly the message from the source. This issue has actually been a core in the research community of cooperative communications.

The motivation of this contribution is based on our previous work on the turbo equalization technique for multiple-input multiple-output (MIMO) systems [1]. The major finding of [1] is that the *vertical iterations* (*VI*) between the two convolutional decoders at the destination (see Fig. 1) can convert the shape of the decoder's extrinsic information transfer (EXIT) curve of the *horizontal iterations* (*HI*) into that very similar to a Turbo code. We propose a very simple iterative decoding technique for relay systems, accumulator-assisted distributed turbo code (ACC-DTC) that exploit the error probability in the source-relay (S-R) link as the source-relay correlation. The proposed ACC-DTC utilizes four very simple CCs, all with memory-1. The relay only *extracts*[1] the source information bits, interleaves, re-encodes and forwards it to the destination.

At the destination, the *VI*s are performed between the source and relay's decoders, where the LLR is updated by the function $f_c(\cdot)$ that compensates the LLR according to the correlation; the error probability p of the S-R link is estimated by a modified version of the algorithm presented in [2], and the estimate $\hat{p}$ of p is used in $f_c(\cdot)$. Doped-accumulator (DACC) [3] is used to help the convergence tunnel in the HI's EXIT chart open until a point very close to the (1,1) mutual information (MI) point.[2]

The results of the computer simulations confirm that the proposed ACC-DTC significantly outperforms the conventional DTC [4] and super Turbo codes (SuTC) [2] in static additive white Gaussian noise (AWGN) and in frequency-flat block-Rayleigh fading channels, even though the decoding complexity of the proposed ACC-DTC is very low.

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[1]The decoder $D_{da,s}$ of DACC performs the BCJR algorithm to provide decoder D_s of C_s with soft input. In this letter, D_s performs Viterbi algorithm to produce hard decision of the uncoded bits (No iteration takes place between the $D_{da,s}$ and D_s). Full iterative decoding at the relay with the expense of power consumption due to computation may achieve better performances. However, eliminating error in the S-R link is out of the scope of this letter.

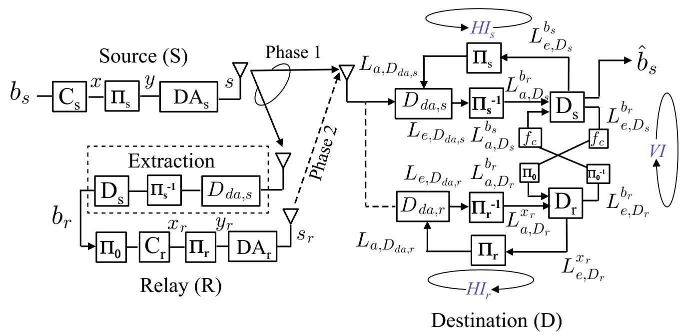

Fig. 1. Relay R with the proposed accumulator-assisted distributed turbo code with LLR updating function $f_c(\cdot)$ at the destination

II. System Model

This letter assumes a single-relay single-source system.[3] The relay is assumed to operate in half-duplex mode, in which the relay receives and transmits signals during the Phase-1 and Phase-2, respectively.

Given that the distance between the source and the destination is $d_{\mathrm{sd}} = d$, two relay location scenarios are considered; location A where the distances of relay from the source is $d_{sr} = d$, and to the destination $d_{\mathrm{rd}} = d$; location B where $d_{\mathrm{sr}} = d/4$ and $d_{\mathrm{rd}} = 3d/4$. As a consequence, the signal-to-noise power ratio (SNR) of the three channels between three terminals are approximated by [5] $\gamma_{\mathrm{sr}} = g_{\mathrm{sr}}\gamma_{\mathrm{sd}}, \gamma_{\mathrm{rd}} = g_{\mathrm{rd}}\gamma_{\mathrm{sd}}$, where $\gamma_{\mathrm{sd}}, \gamma_{\mathrm{sr}}, \gamma_{\mathrm{rd}}$ are the SNRs of source-destination (S-D), S-R, and relay-destination (R-D) links, respectively. g_{sr} and g_{rd} are the gains of the S-R and R-D links, given by $g_{\mathrm{sr}} = (d_{\mathrm{sd}}/d_{\mathrm{sr}})^n$ and $g_{\mathrm{rd}} = (d_{\mathrm{sd}}/d_{\mathrm{rd}})^n$, respectively. n denotes the path-loss exponent with $2 \leq n \leq 6$. In this letter, we assume $n = 3.52$ [5] that translates into (a) $\gamma_{\mathrm{sd}} = \gamma_{\mathrm{sr}} = \gamma_{\mathrm{rd}}$, and (b) $\gamma_{\mathrm{sr}} = \gamma_{\mathrm{sd}} + 21.19$ dB, $\gamma_{\mathrm{rd}} = \gamma_{\mathrm{sd}} + 4.4$ dB. [4]

[2]Due to the space limitation, the convergence analysis using EXIT chart is not presented in this letter.

[3]An extension to multiple-relay systems is straightforward by changing the structure so that LLRs are exchanged over more than two decoders via *VI*s.

[4]$g_{\mathrm{sr}} = 4^{3.25} = 21.19$ dB and $g_{\mathrm{rd}} = (4/3)^{3.25} = 4.4$ dB.

The block-wise expression of the received signals at the relay and at the destination are expressed as

$$\mathbf{y}_{\mathrm{sd}} = \sqrt{\gamma_{\mathrm{sd}}} \cdot h_{\mathrm{sd}} \cdot \mathbf{s} + \mathbf{n}_{\mathrm{sd}}, \quad (1)$$

$$\mathbf{y}_{\mathrm{sr}} = \sqrt{\gamma_{\mathrm{sr}}} \cdot h_{\mathrm{sr}} \cdot \mathbf{s} + \mathbf{n}_{\mathrm{sr}}, \quad (2)$$

$$\mathbf{y}_{\mathrm{rd}} = \sqrt{\gamma_{\mathrm{rd}}} \cdot h_{\mathrm{rd}} \cdot \mathbf{s}_r + \mathbf{n}_{\mathrm{rd}}, \quad (3)$$

where $\mathbf{s} \in \mathbb{R}^{K\times 1}$ and $\mathbf{s}_r \in \mathbb{R}^{K\times 1}$, representing the symbol vectors transmitted from the source and relay, respectively, are assumed to be modulated by binary-phase shift keying (BPSK). $\mathbf{n}_{\mathrm{sd}}$, $\mathbf{n}_{\mathrm{sd}}$, and $\mathbf{n}_{\mathrm{rd}}$ are the zero-mean additive white Gaussian noise (AWGN) vectors with variance of σ^2_{sd}, σ^2_{sr} and σ^2_{rd}, respectively. $h_{\mathrm{sd}}, h_{\mathrm{sr}}$ and h_{rd} are the frequency-flat block-Rayleigh fading channel gains, which are constant in one block.

III. Proposed Decoding Structure

The proposed system is shown in Fig. 1, where all constituent codes are memory-1 CC. At the source transmitter, the binary information b_s is encoded by C_s, interleaved by Π_s, doped-accumulated (DACC) with a doping rate P_s and BPSK-modulated. In Phase-1 the source broadcasts signal to the relay and destination. The DACC uses a memory-1 systematic recursive CC (SRCC), where every P_s-th input systematic bit is replaced by its accumulated-coded bit. Its decoder, $D_{da,s}$, uses the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm.

At the relay, to avoid the heavy decoding complexity, only uncoded information bits b_s are *extracted* (no iteration). Hard decision is then performed on the output of D_s to obtain the binary source information b_r. Because D_s is very weak, the error between the source and relay may occur with probability $\Pr(b_s \neq b_r) = p$. The correlation between the source and relay is expressed by the error probability p as $\rho = 1 - 2p$.

Fig. 2 shows the error probability p at the relay when S-R link is AWGN channel.[5] With the conventional DTC and SuTC, $p = 0.082$ and $p = 0.077$, respectively, at $\gamma_{\mathrm{sr}} = -2.75$ dB, while with only *extraction* in our proposed ACC-DTC, $p = 0.191$ representing the worst performance at the relay among those techniques. The binary sequence b_r is then interleaved by Π_0, re-encoded by C_r, re-interleaved by Π_r, doped-accumulated with a doping rate P_r, BPSK-modulated, and forwarded to the destination in Phase-2.

The destination performs *HI* for the signal reception from the source during the Phase-1, denoted as HI_s, and another *HI* for the signal received from the relay during the Phase-2, denoted as HI_r. The extrinsic LLRs of the source information obtained by HI_s and HI_r are then exchanged via the *VI* between decoders D_s and D_r where the LLR is updated by the updating function $f_c(\cdot)$. This process is repeated. Finally, binary decision is made on the *a posteriori* LLR from D_s to obtain the estimate $\hat{b}_s$ of the source information b_s.

1) Estimation of p: At the destination, p is estimated using the *a posteriori* LLRs $L^{b_s}_{p,D_s}$ and $L^{b_r}_{p,D_r}$, of the uncoded bits b_s and b_r output from the decoders D_s and D_r, respectively,

[5] Due to space limitation, the value of p when S-R link is Rayleigh fading channel, is not shown in this letter.

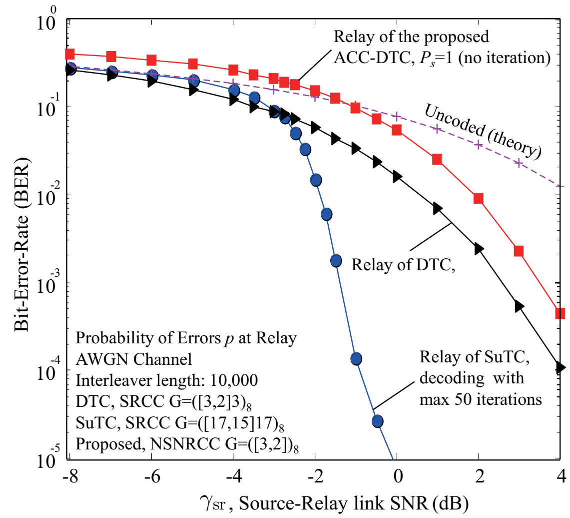

Fig. 2. Probability of errors p at the relay with channel coding rate $R = 1/2$

as

$$\hat{p} = \frac{1}{N}\sum_{n=1}^{N} \frac{\exp\{L^{b_s}_{p,D_s}\} + \exp\{L^{b_r}_{p,D_r}\}}{(1+\exp\{L^{b_s}_{p,D_s}\})(1+\exp\{L^{b_r}_{p,D_r}\})}, \quad (4)$$

where N is the number of *reliable a posteriori* LLRs used[6].

2) LLR Updating Function: As in [2], we use the following updated probability of b_r based on the estimated error $\hat{p}$ obtained by (4), as

$$\Pr(b_r = 0) = (1-\hat{p})\Pr(b_s = 0) + \hat{p}\Pr(b_s = 1), \quad (5)$$

$$\Pr(b_r = 1) = (1-\hat{p})\Pr(b_s = 1) + \hat{p}\Pr(b_s = 0), \quad (6)$$

which leads to the LLR updating function $f_c(\cdot)$ for b_r, to obtain the updated extrinsic LLR of $L^{b_s}_{e,D_s}$ as

$$L^{b_s}_{e,D_s,updated} = f_c(\hat{p}, L^{b_s}_{e,D_s}), \quad (7)$$

$$= \ln \frac{(1-\hat{p})\exp\{L^{b_s}_{e,D_s}\} + \hat{p}}{(1-\hat{p}) + \hat{p}\,\exp\{L^{b_s}_{e,D_s}\}}. \quad (8)$$

Similarly, the update of $L^{b_r}_{e,D_r}$, $L^{b_r}_{e,D_r,updated}$, is obtained in the same way as (8) using $f_c(\hat{p}, L^{b_r}_{e,D_r})$.

IV. Performances Evaluation

This section provides simulation results with the parameter settings as shown in Figs. 3 and 4. All results are based on the estimate $\hat{p}$ of p with channel coding rate $\mathcal{R} = 1/2$.[7] For each single HI_s and HI_r for D_s and D_r, respectively, 5 *VI*s took place, and the whole process was repeated 50 times, resulting in 50×2 *HI*s plus 50×5 *VI*s in total. The performance of SuTC was evaluated with 50 external iterations plus $50\times 2\times 1$ internal turbo iterations, while with the conventional DTC 50 turbo iterations in total[8] between D_s and D_r, because in both

[6] In the calculation of $\hat{p}$ using (4), LLRs having larger absolute values than a given threshold T are used. The authors of [2] uses $T = 3$ to estimate $\hat{p}$ based on the extrinsic LLR. In this letter, we use different threshold T because the code used in the proposed ACC-DTC, memory-1 CC, is very weak. It should be noted here that unlike [2], we use *a posteriori* LLR instead of extrinsic LLR for higher accuracy.

[7] Except for the conventional DTC, because the relay keeps silent when error is detected. Therefore, estimation of p is not needed.

[8] There is no internal iterations in the conventional DTC.

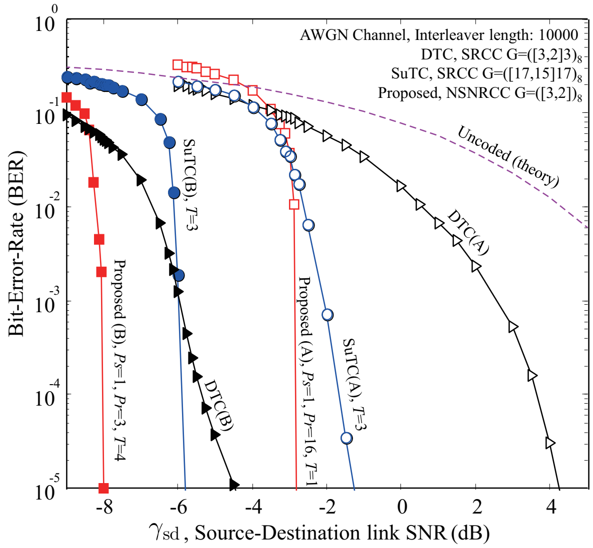

Fig. 3. BER performances over all S-D, S-R, R-D links being AWGN channels

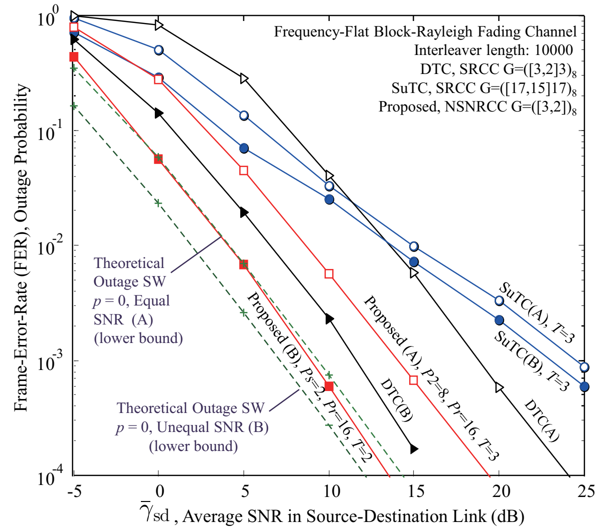

Fig. 4. FER performances and outage probability over all S-D, S-R, R-D links being frequency-flat block-Rayleigh fading channels

cases no more gain was observed by increasing iteration times. For SuTC, estimate $\hat{p}$ was also used in $f_c(\cdot)$. However, for the conventional DTC as in [4], the knowledge about error probability p is not utilized at the destination.

Fig. 3 shows BER performances of the proposed ACC-DTC structure in AWGN channels (for all S-D, S-R, R-D links) with the relay locations A and B. At the relay location A, the proposed ACC-DTC, denoted as ACC-DTC(A) in the figure, has a clear turbo-cliff at $\gamma_{\rm sd} = -2.75$ dB, which outperforms the SuTC and the conventional DTC (SuTC(A) and DTC(A)) by 1.24 dB and 6.69 dB, respectively, at BER of 10^{-4}. At the relay location B, the ACC-DTC(B) has clear turbo-cliff at $\gamma_{\rm sd} = -8.125$ dB, which also outperforms SuTC(B) and DTC(B) by 2.18 dB and 2.76 dB, respectively.

The frame-error-rate (FER) performance results in frequency-flat block-Rayleigh fading channel (for all S-D, S-R, R-D links) are shown by Fig. 4. The theoretical outage curves at the locations A and B for BPSK with coding rate $R = 1/2$ are also shown for comparison, where extrinsic LLR exchange of the information b_s and b_r between the two decoders with $p = 0$ is assumed. Therefore, the theoretical curves are lower bounds of the outage probability [6]. The X-axis indicates average SNR of the channel between the source and the destination, denoted as $\bar{\gamma}_{\rm sd}$, and the Y-axis shows FER. At FER of 10^{-3} the performance gains over SuTC are 10.42 dB to 13.97 dB, and the gains over the the conventional DTC are 4.65 dB and 2.71 dB when the relay location changes from A to B.

The performance gains in AWGN and frequency-flat block-Rayleigh fading channels can be achieved by the combined use of the DACC and memory-1 CC with the selected P_s and P_r values shown in the figures, based on in-depth observation on EXIT chart, yielding better matching of the two EXIT curves. Another reason is that our proposed technique uses the *a posteriori* LLR in estimating p resulting in more accurate estimate $\hat{p}$. The serial concatenation of DACC and memory-1 CC for the signal transmissions in Phase-1 and Phase-2 is followed by an LLR updating function $f_c(\cdot)$ in *VI*, which can adaptively "adjust" the LLR value based on the estimate $\hat{p}$ of the S-R link error probability p.

V. Conclusions

This letter has focused on the relaying system in the presence of error in the S-R link. We have proposed a novel distributed turbo coding technique, ACC-DTC, to solve the problem. The probability of error occurring in the S-R link is treated as representing correlations between the source and the relay, which is then estimated at the destination. The correlation is exploited by the LLR updating function in the *VI* loop. With help of the rate-1 memory-1 DACC, the convergence tunnel opens until a point very close to the (1,1) MI point in the *HI* EXIT chart. Although with a help of a relay, the 2nd order diversity is unachievable by SuTC, but it can be achieved by the conventional DTC. The proposed ACC-DTC can also achieve the 2nd order diversity and outperforms the conventional DTC at both the relay locations A and B. This indicates that the proposed ACC-DTC can achieve the best performance among those three techniques, so far as $d/4 \leq d_{sr} \leq d$.

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