Title	Soft-feedback MMSE Equalization for Non-orthogonal Frequency Division Multiplexing (n-OFDM) Signal Detection
Author(s)	Ahmad, Norulhusna; S-Yusof, S. Kamilah; Fisal, Norsheila; Anwar, Khoirul; Matsumoto, Tad
Citation	2012 International ITG Workshop on Smart Antennas (WSA): 248-255
Issue Date	2012-03-07
Type	Conference Paper
Text version	author
URL	http://hdl.handle.net/10119/10532
Rights	This is the author's version of the work. Copyright (C) 2012 IEEE. 2012 International ITG Workshop on Smart Antennas (WSA), 2012, 248-255. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
Description	

Soft-feedback MMSE Equalization for Non-Orthogonal Frequency Division Multiplexing (n-OFDM) Signal Detection

Norulhusna Ahmad*, S. Kamilah S-Yusof*, Norsheila Fisal*, Khoirul Anwar** and Tad Matsumoto**§
*UTM-MIMOS Center of Excellence in Telecommunication Technology, Universiti Teknologi Malaysia,
UTM Skudai, 81310 Johor, MALAYSIA.
**School of Information Science, Japan Advanced Institute of Science and Technology (JAIST),
1-1 Asahidai, Nomi, Ishikawa, JAPAN 923-1292
§Center for Wireless Communication, FI-90014, University of Oulu, Finland
Email: *{nhusna@ic,kamilah@fke,sheila@fke}.utm.my,
**{anwar-k,matumoto}@jaist.ac.jp, **§tadashi.matsumoto@ee.oulu.fi

***Abstract*—A frequency division multiplexing technique, non-orthogonal frequency division multiplexing (n-OFDM), is proposed in [1]- [2] to enhance the efficiency of bandwidth utilization. This paper reveals that the smaller the frequency separation, the larger sum capacity can be achieved compared with the conventional OFDM technique. However, n-OFDM system introduces inter-carrier interference (ICI) at the transmitter because the orthogonality between the subcarriers no longer holds. Moreover, since the channel covariance matrix of n-OFDM has high condition number when the overlapping factor, $1-\alpha$, is large, conventional linear detectors suffers from severe noise enhancement. To solve this problem, this paper proposes the use of soft cancellation- minimum mean-squared error (SC-MMSE) turbo equalization. Binary constellation constrained mutual information (CCMI) is calculated by utilizing the area property for the EXtrinsic Information Transfer (EXIT) chart of the SC-MMSE equalizer. Results of the EXIT chart analysis and bit-error-rate (BER) simulations in additive white Gaussian noise (AWGN) channel are presented.**

I. Introduction

Nowadays, improving spectrum efficiency is one of the most crucial issues when developing future generation wireless communication systems, given the fact of rapid growth of broadband service demands. One of the techniques that can enhance the bandwidth utilization of multi-carrier systems is orthogonal frequency division multiplexing (OFDM) in which the complexity of the OFDM signal detection is considered practical due to orthogonality among the subcarriers. However, by allowing the overlapping of the subcarriers and hence violating the orthogonality rule, higher spectrum efficiency can be realized. This concept is known as non-orthogonal frequency division multiplexing (n-OFDM).

Several techniques of realizing n-OFDM concept have been proposed, e.g., [1]–[5]. Nevertheless, n-OFDM leads to two major problems which are; 1) severe inter-carrier interference (ICI) due to the close overlapping between the subcarriers, and 2) the transmitter needs a bank of analogue waveform generator to generate n-OFDM signals [4].

ICI imposes significant distortion in the received signal; even though it can be solved by using the Maximum Likelihood sequence estimation (MLSE) at the expense of complexity. This is because the larger the overlapping factor, the more interference components from the farther subcarriers due to the overlapping have to be taken into account. Additionally, with a large overlapping factor, conventional linear detection techniques such as zero forcing (ZF) and/or minimum mean-squared error (MMSE) signal detection schemes suffer from severe noise enhancement, due to large condition number of the equivalent channel covariance matrix. Recently, Generalized Sphere Decoding (GSD) technique has been proposed to reduce the complexity for MLSE [1], but only half of the bandwidth is utilized in the technique presented.

The problem inherent within the generation of the FDM signal having uniform sub-carrier spacing can be solved by the techniques presented in [3] and [4] where generalized inverse discrete Fourier transform (GIDFT) is used. However, the n-OFDM signal is generated by appending multiple zeros at the input of the GIDFT matrix which makes the system inefficient in terms of frequency utilization.

This paper aims to provide information theoretical analysis of n-OFDM signal based on capacity analysis using singular-value-decomposition (SVD), where the GIDFT matrix, used to generate the n-OFDM signal, is regarded as a part of the channel. We propose soft cancellation- MMSE (SC-MMSE) turbo equalization to solve the problem due to the large noise enhancement in the n-OFDM system. The soft ICI components within one n-OFDM symbol, is obtained by utilizing the feedback from the maximum a posteriori probability (MAP) decoder, which is then used as apriori information to the equalizer. The concept of ICI elimination is very similar to turbo equalization for single carrier signalling [6]. For n-OFDM signal generation, we propose the use of ”fat” GIDFT matrix, thus higher frequency utilization efficiency can be achieved than the technique in [5]. Moreover, with the fat GIDFT matrix, the number of the input symbols is larger than the output, by which more data can be transmitted compared to the technique in [3] and [4].

It is obvious that even though the n-OFDM may outperform OFDM in terms of spectrum efficiency, it is not sensible to use it in frequency-flat fading channels, because even single carrier signaling utilizing the entire bandwidth does not need equalization. In fact, the channel capacity is determined only by the channel property, and hence is independent of the signaling schemes. However, when considering practical applications of n-OFDM in frequency-selective channels as an alternative solution to the single carrier signaling with equalization, deriving computationally efficient algorithms for n-OFDM signal detection in frequency-flat channels is still very important. Hence, the goal of this paper is to reveal the theoretical limit of n-OFDM signaling with the proposed turbo equalization in frequency-flat channel in terms of bandwidth efficiency and noise enhancement introduced by the overlapping factor in the system.

The paper is organized as follows: the model of the n-OFDM system investigated in this paper is described in Section II. Bandwidth efficiency with GIDFT matrix and information theoretic consideration are discussed. In Section III, the proposed time domain SC-MMSE equalizer is explained together with binary constellation constraint mutual information (CCMI) analysis. Results of the simulations conducted to evaluate the performance of a coded n-OFDM system is presented in Section IV in terms of EXIT analysis and BER performance. Lastly, the paper is concluded in Section V with some concluding statement.

II. System Model

A stream of modulated signal is multiplexed by a GIDFT matrix to produce n-OFDM subcarriers in one n-OFDM block. The n-OFDM signal is given by

$$x[t] = \frac{1}{\sqrt{T}} \sum_{m=\infty}^{\infty} \sum_{n=0}^{N_b-1} s_{n,m} \exp\left(j2\pi n \Delta f (t-mT)\right) \quad (1)$$

where $s_{n,m}$ is the n^{th} modulated subcarriers in the m^{th} n-OFDM block, $j=\sqrt{-1}$ and Δf is frequency separation of modulated subcarriers which depends on the ratio between a scalar α and symbol duration T, as defined in [2]

$$\Delta f = \frac{\alpha}{T} \quad (2)$$

From (2), OFDM signal is generated when $\alpha = 1$ but when $\alpha < 1$, the orthogonality between the subcarriers no longer holds. Consider one n-OFDM block which is sampled at T/N_c $(N_c < N_b)$, the equation is expressed as (3) where $1/\sqrt{N_c}$ is the normalization factor.

$$X[k] = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} s_n \exp\left(\frac{j2\pi k n \alpha}{N_c}\right) \quad (3)$$

Fig.1 details the proposed n-OFDM system structure where there are N_b input subcarriers and N_c output samples to and from the GIDFT block, respectively. As described in Section I, we consider that GIDFT is part of the channel, which

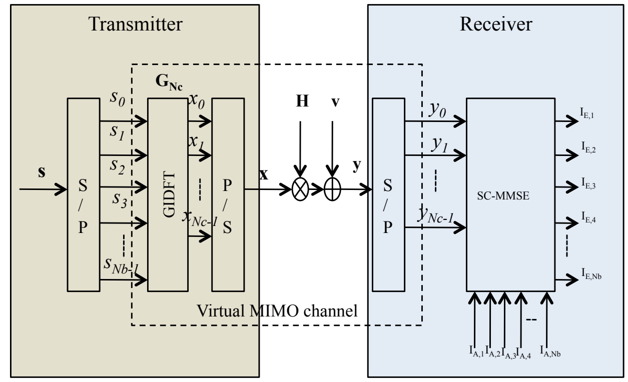


Fig. 1. n-OFDM system model with SC-MMSE detector

further simplifies the system model by using multiple input multiple output (MIMO) concept. Hence, the received signal is simplified to

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (4)$$

and

$$\mathbf{x} = \mathbf{G}_{N_c}\mathbf{s} \quad (5)$$

where $\mathbf{y} = [y_0, y_1, \ldots, y_{N_c-1}]^T$ is the received signal, $\mathbf{s} = [s_0, s_1, \ldots, s_{N_b-1}]^T$ is the subcarriers input, $\mathbf{v} = [v_0, v_1, \ldots, v_{N_c-1}]^T$ is zero-mean complex Gaussian noise with variance σ_0^2, $\mathcal{CN} = (0, \sigma_0^2)$ and $[.]^T$ is the transpose operation. Notes that, we assume $H = I_{N_c}$ where I_{N_c} is an identity matrix with size of $N_c \times N_c$, with which by properly setting the variance of the noise components, we can evaluate the performance of our proposed technique at arbitrary values of signal-to-noise power ratio (SNR). The sampled of n-OFDM signal is given by $\mathbf{G}_{N_c} \in \mathbb{C}^{N_c \times N_b}$ where the matrix elements with $k = 0, 1, \ldots, N_c-1$ and $n = 0, 1, \ldots, N_b-1$ is given as

$$g_{k,n} = \frac{1}{\sqrt{N_c}} \exp\left(\frac{j2\pi k n \alpha}{N_c}\right) \quad (6)$$

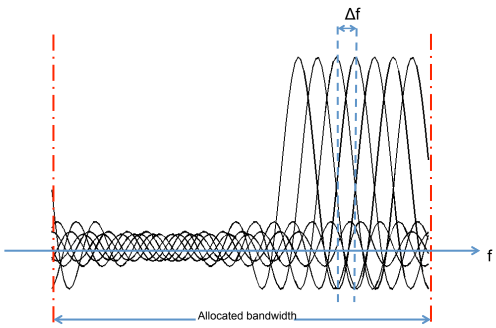
In other words, $\mathbf{G}_{N_c}$ is actually a part of a full Discrete Fourier Transform (DFT) matrix of which the size depends on input substream number N_b and output subcarriers number N_c. The input output (I/O) of GIDFT is connected by α, as

$$\alpha = \frac{N_c}{N_b} \qquad N_c < N_b \quad (7)$$

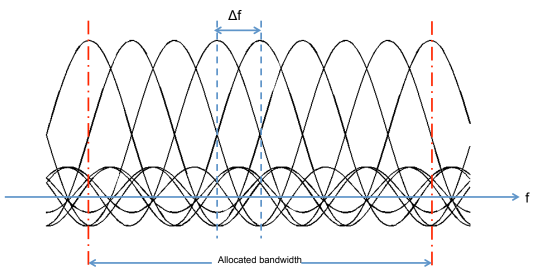
The overlapping factor can be defined as $1-\alpha$. Thus, when α is small, the overlapping factor is high. Since $N_c < N_b$, the matrix of $\mathbf{G}_{N_c}$ becomes fat and hence higher bandwidth efficiency can be achieved, of which the details are described in the next section.

A. Bandwidth Efficiency with GIDFT Matrix

In OFDM system, the full matrix of Inverse Discrete Fourier Transform (IDFT) as in (6) is used with $\alpha = 1$. This means that, $N_b = N_c$ and the orthogonality of the subcarriers can be preserved.



(a) n-OFDM spectrum when $N_b = N_c$



(b) n-OFDM spectrum when $N_b > N_c$

Fig. 2. n-OFDM spectrum in the same allocated bandwidth with $1-\alpha = 0.6$

For n-OFDM systems, the subcarrier spacing is reduced by a factor of α, which increases the ICI. In terms of bandwidth efficiency, n-OFDM system can transmit more data compared to OFDM system by using a "fat" $\mathbf{G}_{N_c}$. If the input subcarriers is the same as output samples ($N_b = N_c$) and $\alpha < 1$, this means the input subcarriers are compressed but the bandwidth is not fully utilized as shown in [2]. The spectral efficiency comparison is depicted in Fig. 2 for $1 - \alpha = 0.6$. Given the same bandwidth allocation, Fig. 2(a) shows that almost 40% of the bandwidth is used with $N_b = 7$ and sampled at $N_c = 7$. Whereas in Fig. 2(b), the bandwidth is fully utilized with $N_b = 7$ and $N_c = 3$.

B. Information Theoretic Consideration

Given the channel SNR, the communication channel is associated with a parameter that expresses the channel's signal transmission capability. In 1948 [7], Shannon derived the limit, which is, since then, referred to as the channel capacity, C; if the information transmission rate, R per channel use is less than or equal to C, i.e., $R \leq C$, bit error rate can be reduced arbitrarily. In order to achieve that, with the given SNR, the channel capacity,

$$R \leq C = \log_2(1 + \text{SNR}) \tag{8}$$

has to hold.

Since we assume $H = I_{N_c}$, (4) is equivalent to

$$\mathbf{y} = \mathbf{G}_{N_c}\mathbf{s} + \mathbf{v}. \tag{9}$$

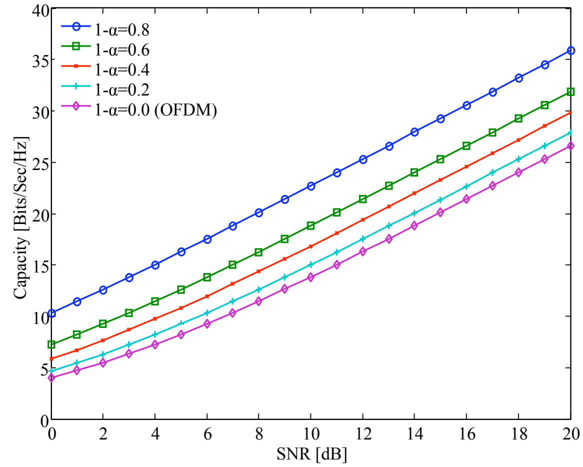


Fig. 3. Capacity analysis for n-OFDM system with different overlapping factor, $1 - \alpha$ when $N_c = 4$.

The matrix of $\mathbf{G}_{N_c}$ can be singular-value-decomposed (SVD) as

$$\mathbf{G}_{N_c} = \mathbf{U}\mathbf{D}\mathbf{V}^H, \tag{10}$$

where $[.]^H$ is the Hermitian transpose, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices ($\mathbf{U}\mathbf{U}^H = \mathbf{V}\mathbf{V}^H = 1$). The elements of $\mathbf{D}$ are the singular values of $\mathbf{G}_{N_c}$ with its component, d_n, expressed as

$$\mathbf{D} = \text{diag}[d_0, d_1, \ldots, d_{N_b-1}]. \tag{11}$$

Since

$$\mathbf{G}_{N_c}\mathbf{G}_{N_c}^H = \mathbf{U}\mathbf{D}\mathbf{D}^T\mathbf{V}^H \tag{12}$$

the squared singular values of d_n are the eigenvalues of the matrix $\mathbf{G}_{N_c}\mathbf{G}_{N_c}^H$.

Assuming each subcarrier's SNR is the same for the whole bandwidth (i.e., the channel is frequency-flat), sum capacity in n-OFDM system over all the subcarriers can then be calculated using the singular values obtained from (11), as

$$C = \sum_{n=1}^{N_b} \log_2(1 + |d_n|^2\text{SNR}) \tag{13}$$

Fig. 3 shows the sum capacity of all subcarriers, versus SNR with the overlapping factor, $1 - \alpha$, as a parameter. It is found that the bigger the $1-\alpha$, the greater the sum capacity. For e.g., when $1 - \alpha = 0.5$ and $N_c = 4$, n-OFDM system ($N_b = 8$) can transmit almost 50% more data for the allocated bandwidth compared to OFDM system ($N_b = 4$). However, as explained in the Section I, channel capacity is determined by the channel property only. Hence, the results presented in this paper shows that the theoretical sum capacity limit of n-OFDM overlapping signaling in Gaussian channel is greater than OFDM thus enhances the bandwidth efficiency considerably.

C. Condition Number

The efficiency of detection scheme for n-OFDM signal can be identified by calculating the condition number of the n-OFDM covariance matrix. The condition number is defined

TABLE I
THE CONDITION NUMBER FROM THE COVARIANCE MATRIX, $\mathbf{G}_{N_c}\mathbf{G}_{N_c}^H$ WITH DIFFERENT OVERLAPPING FACTOR, $1-\alpha$ FOR $N_c = 4$

$1-\alpha$	Condition Number
0.8	9.64×10^{16}
0.6	3.28×10^{16}
0.4	3.18×10^{16}
0.2	1.76×10^{16}
0	1

as the ratio of the largest to the smallest eigenvalues of the covariance matrix, $\mathbf{G}_{N_c}\mathbf{G}_{N_c}^H$. Table I shows that the noise enhancement factor is very large even with a small overlapping factor, $1-\alpha = 0.2$.

BER performance of the MMSE equalizer is shown in Fig. 4(a) and 4(b) versus number of iterations for different SNR values, which is equivalent to the BER performance with the SC-MMSE turbo equalization for the first iteration. Figs. 4(a) and 4(b) are for high and low overlapping factors, respectively. It is found from the figure that with $1-\alpha$ is 0.8, which corresponds to the condition number = 9.64×10^{16}, MMSE equalizer cannot detect the n-OFDM signal, resulting in unsatisfactory error performance. However, when the overlapping factor is small $(= 0.2)$, the condition number = 1.76×10^{16} which yields to better BER performance, as shown in Fig. 4(b).

As noted before, since n-OFDM system imposes to large noise enhancement, therefore, we propose SC-MMSE turbo equalization because SC-MMSE can work in high condition number and provides better BER performance [8].

III. SC-MMSE TURBO DETECTOR IN N-OFDM SYSTEM

SC-MMSE turbo detector has been proven to mitigate the interference efficiently, and achieve excellent performance [6]. The received data is fed to the SC-MMSE equalizer which is composed of soft interference cancellation, and MMSE filtering, both efficiently suppress the remaining interference. A priori log likelihood ratio (LLR), $\mathrm{L}_{\mathrm{A,EQ}}$ of the symbols delivered by the MAP decoder from the previous iteration is given as

$$\mathrm{L}_{\mathrm{A,EQ}}[s_n] = \ln \frac{\Pr[y|s(n) = +1]}{\Pr[y|s(n) = -1]} \tag{14}$$

where $\Pr[\cdot]$ indicates the probability of its argument. The first moment of the soft symbol is then generated by $\tilde{s} = \tanh\left(\mathrm{L}_{\mathrm{A,EQ}}/2\right)$ for a vector $\tilde{\mathbf{s}}$ as

$$\tilde{\mathbf{s}} = [\tilde{s}(0), \tilde{s}(1), \tilde{s}(2), \ldots, \tilde{s}(N_b - 1)] \tag{15}$$

$$\tilde{\mathbf{s}}(n) = [\tilde{s}(0), \ldots, \tilde{s}(n-1), 0, \tilde{s}(n+1), \ldots, \tilde{s}(N_b - 1)] \tag{16}$$

where for the desired symbol, $s(n)$, $n = 0, 1, \ldots, N_b - 1$ to be detected, we set $\tilde{s}(n) = 0$. The soft symbol, $\tilde{s}(n)$ is used to cancel the interference components by multiplying with the estimated channel, $\mathbf{G}_{N_c}$. The soft canceler output is then expressed as

$$\tilde{\mathbf{y}}(n,k) = \mathbf{y}(k) - \mathbf{G}_{N_c}\tilde{\mathbf{s}}(n) \tag{17}$$

$$= \mathbf{y}(k) - \mathbf{c}(n,k) \tag{18}$$

for $k = 0, 1, \ldots, N_c - 1$ where

$$\mathbf{c}(n,k) = \mathbf{G}_{N_c}\tilde{\mathbf{s}}(n). \tag{19}$$

MMSE filtering is used to further reduce the interference components by multiplying $\tilde{\mathbf{y}}(n,k)$ by the vector $\mathbf{m}(n,k)$ to obtain

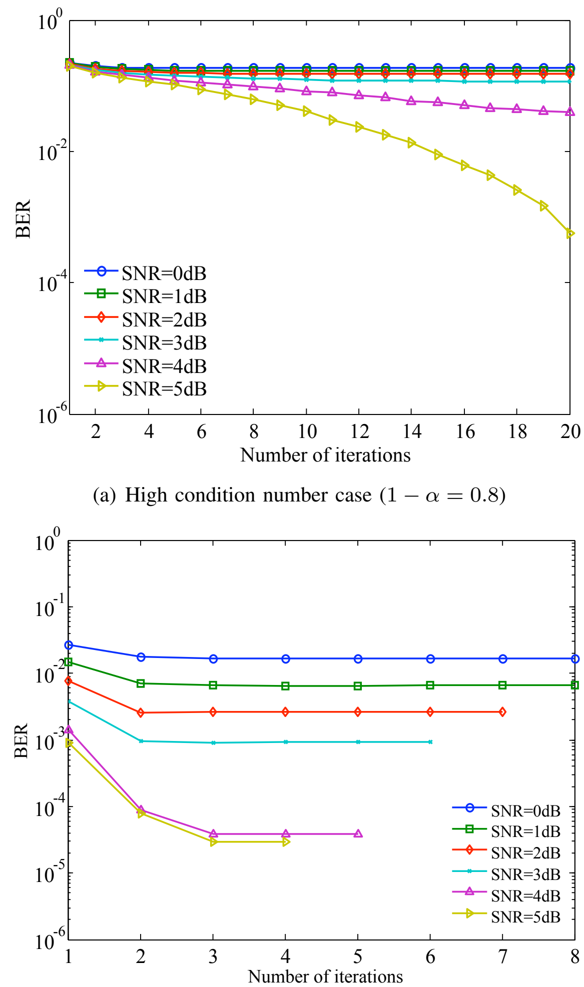
$$z(n) = \mathbf{m}^H(n,k)\tilde{\mathbf{y}}(n,k) \tag{20}$$

for which the criterion is

$$\mathbf{m}(n,k) = \arg\min_{\mathbf{m}} \mathrm{E}\left\{\left\|s(n) - \mathbf{m}^H\tilde{\mathbf{y}}(n,k)\right\|^2\right\} \tag{21}$$

$$= \arg\min_{\mathbf{m}} \mathbf{m}^H \mathrm{E}\left\{\tilde{\mathbf{y}}(n,k)\tilde{\mathbf{y}}(n,k)^H\right\}\mathbf{m} - 2\mathbf{m}^H\mathrm{E}\left\{s(n)\tilde{\mathbf{y}}(n,k)\right\} + 1 \tag{22}$$

$$\approx \left[\mathbf{G}_{N_c}\Delta(n)\mathbf{G}_{N_c}^H + \sigma^2\mathbf{I}\right]^{-1}\mathbf{G}_{N_c}\mathbf{e}_{N_b}, \tag{23}$$



(a) High condition number case $(1-\alpha = 0.8)$

(b) Low condition number case $(1-\alpha = 0.2)$

Fig. 4. BER vs number of iteration for different SNR when $N_c = 4$

where

$$\mathrm{E}\{\tilde{\mathbf{y}}(n,k)\tilde{\mathbf{y}}(n,k)^H\} = \mathbf{G}_{N_c}\Delta(n)\mathbf{G}_{N_c}^H + \sigma^2\mathbf{I} \quad (24)$$

$$\mathrm{E}\{s(n)\tilde{\mathbf{y}}(n,k)\} = \mathbf{G}_{N_c}\mathbf{e}_{N_b} \quad (25)$$

with

$$\begin{aligned}\Delta(n) &= \mathrm{cov}\{\mathbf{s}(n) - \tilde{\mathbf{s}}(n)\}\\ &= \mathrm{diag}\{1-\tilde{s}(0)^2, \ldots, 1-\tilde{s}(n-1)^2, 1,\\ &\quad 1-\tilde{s}(n+1)^2, \ldots, \tilde{s}(N_b-1)^2\}\end{aligned} \quad (26)$$

and $\mathbf{e}_{N_b}$ is a N_b vector length, of which the n^{th} element is equal to 1 and the other elements are zero.

Assuming that the MMSE filter output can be approximated as a Gaussian channel output having input $s(n)$,

$$z(n) = \mu(n)s(n) + \eta(n), \quad (27)$$

where

$$\mu(n) = \mathrm{E}\{z(n)s(n)\} = \mathbf{e}_{N_b}^T\mathbf{G}_{N_c}^H\mathbf{m}(n,k) \quad (28)$$

and $\eta(n)$ is the equivalent noise sample. The extrinsic information of the coded bit can be defined as

$$\begin{aligned}\mathrm{I_E}[s_n] &= \ln\frac{\Pr[z(n)|s(n)=+1]}{\Pr[z(n)|s(n)=-1]}\\ &= \frac{4\mathrm{Re}\{z(n)\}}{1-\mu(n)}.\end{aligned} \quad (29)$$

Figure 5 shows the BER versus a priori mutual information, IA with the parameter of $1-\alpha$ for SNR = 5 dB and Nc = 4. It is clearly found that the BER with $1-\alpha = 0.5$ is better than with $1-\alpha = 0.8$ because of the high interference when overlapping factor, $1-\alpha$ is large.

The binary constellation constraint mutual information (CCMI) can be evaluated using (30) by measuring the areas under the EXIT curves of the equalizer for each sub-carrier, which are then summed up to calculate the total CCMI, as

$$\mathrm{CCMI}_T = \sum_{0}^{N_b-1}\int_0^1 \mathrm{I_{A,EQ}}\,\mathrm{d}i \quad (30)$$

The result of the total CCMI versus SNR is shown in Fig. 6 with the $1-\alpha$, as a parameter. It is found that, the total CCMI at $\alpha = 0.9$ is the largest among those overlapping factor values tested, even though highest interference conditions occurs. This is mainly because of the SC-MMSE's ICI cancellation capability.

IV. Simulation Result

A block diagram of the chained simulation of the proposed SC-MMSE turbo detector for n-OFDM system is depicted in Fig. 7. We assumed a 1/2 rate memory length, recursive systematic convolutional (RSC) code with generator polynomial $(3,2)_8$ and binary phase shift keying (BPSK) modulation for performance evaluation. The interleaved encoded bit sequence is sampled at every bit timing, and the samples are forwarded to the GIDFT matrix. At the receiver, the extrinsic information is forwarded via a random interleaver to the log-MAP BCJR decoder as a priori information where the extrinsic LLR is calculated using (29). The performance is evaluated for different number of GIDFT subcarriers output, $N_c = 4, 16, 32, 64$ with different values of $1-\alpha$, as a parameter. The process is repeated until no more iteration gain in BER performance is achieved. The simulation parameters are also summarized in Table II.

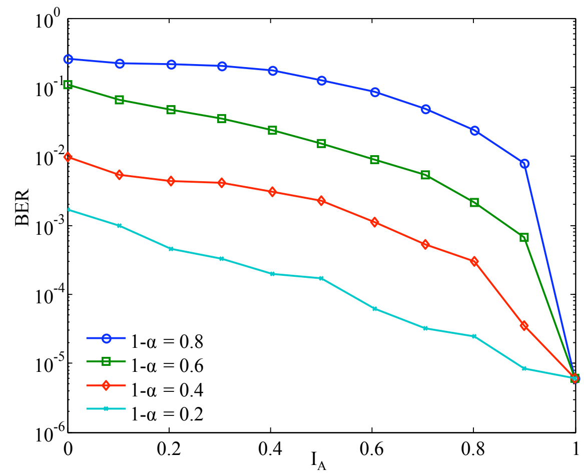


Fig. 5. BER vs IA at SNR = 5dB for Nc = 4.

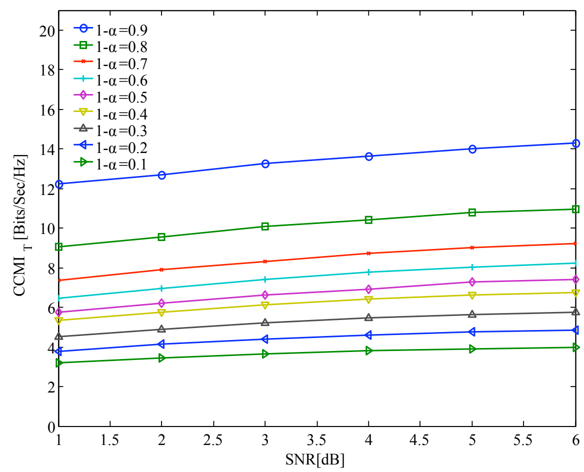


Fig. 6. Total CCMI vs SNR for n-OFDM system with SC/MMSE turbo detector.

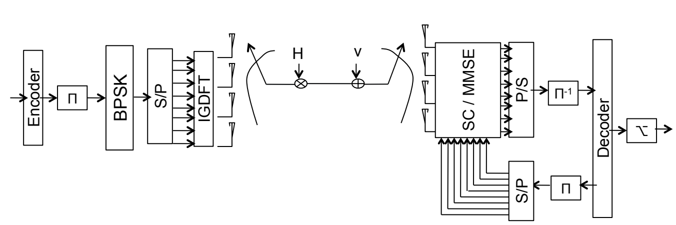


Fig. 7. Simulation model for chained simulation for n-OFDM system with SC/MMSE turbo detector.

TABLE II
SIMULATION PARAMETERS FOR CHAINED SIMULATION IN N-OFDM SYSTEM

Type	Simulation parameter
Encoder	Convolutional code $(3,2)_8$
Coding rate	$1/2$
Decoder	Log-MAP BCJR
Channel	AWGN
Modulation	BPSK
GIDFT output	$N_c = 4, 8, 16, 64$
Interleaver	Random interleaver

A. EXIT analysis

EXtrinsic Information Transfer (EXIT) chart is used to analyze the matching between the decoder and equalizer. Assume that LLRs have Gaussian distribution and the transmitted bits $s = \pm 1$, the conditional probability function (pdf) can be define as $\Pr(\xi|S)$. The mutual information (MI), $I(S;\xi)$ is then, expressed as

$$I(S;\xi) = \frac{1}{2} \sum_{s=+1,-1} \int_{\infty}^{\infty} \Pr(\xi|S=s) \log_2 \left[\frac{2\Pr(\xi|S=s)}{\Pr(\xi|s=-1) + \Pr(\xi|s=+1)} \right] \mathrm{d}\xi, \quad (31)$$

where (31) can be evaluated via histogram measurement followed by numerical integration. However, this complexity can be reduced by using the J-function approximation, that connects mutual information (I) and the root squared variance (σ) of LLR, as

$$I(S;\xi) = J(\sigma) \approx \left(1 - 2^{-H_1 \sigma^2 H_2}\right)^{H_3} \quad (32)$$

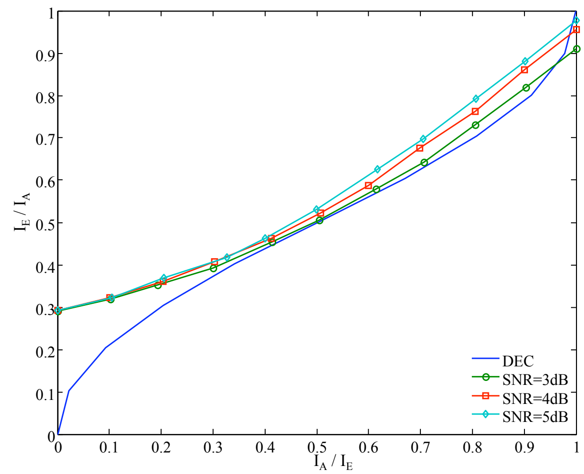
and the inverse as

$$\sigma = J^{-1}(I) = \left(\frac{-1}{H_1} \log_2 \left(1 - I^{\frac{1}{H_3}}\right)\right)^{\frac{1}{2H_2}}, \quad (33)$$

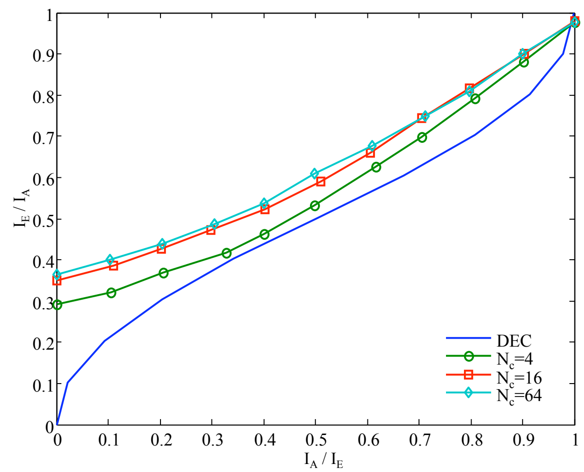
where $H_1 = 0.3073, H_2 = 0.8935, H_3 = 1.1064$ [9].

The EXIT curves with the SC-MMSE equalizer and the BCJR decoder in the n-OFDM system is shown in Figure 8(a) for different values of SNR when $N_c = 4$ and $1 - \alpha = 0.78$. It founds that the EXIT curve is stuck at SNR = 3dB but the tunnel opens until a point close to the (1.0, 1.0) MI point at SNR = 4dB, which means that a turbo cliff is expected to happen at SNR = 4dB. However, since the ending point still does not reach exactly (1.0, 1.0) point, error floor is still unavoidable with higher SNR. In fact, there is a technique that brings the ending point of the equalizer to the really very close to the (1.0, 1.0) MI point, but it is out of the scope of this paper.

The expectation of bit-error- rate (BER) can also be calculated based on the intersection between the EXIT function of decoder and the EXIT function of equalizer. From Fig. 8(a), the intersection when SNR = 4dB happened at a priori MI of equalizer, $I_{A,Eq} = 0.997$. Noted that, a priori MI of equalizer is equal to extrinsic MI of decoder, $I_{E,Dec}$.



(a) EXIT chart for different value of SNR when $N_c = 4$.



(b) EXIT chart for variety of N_c with SNR = 5dB

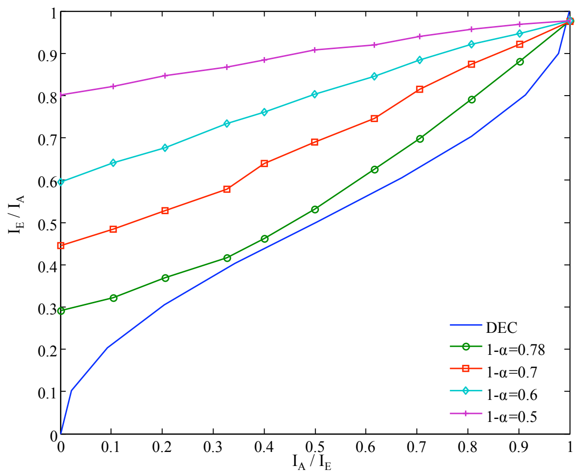
Fig. 8. The impact on the EXIT chart for SC-MMSE equalizer and decoder when $1 - \alpha = 0.78$.

Therefore, a posteriori MI of decoder, $I_{p,Dec}$ approximately equals to extrinsic MI in the case of high MI which can lead to the expectation of BER [10] as

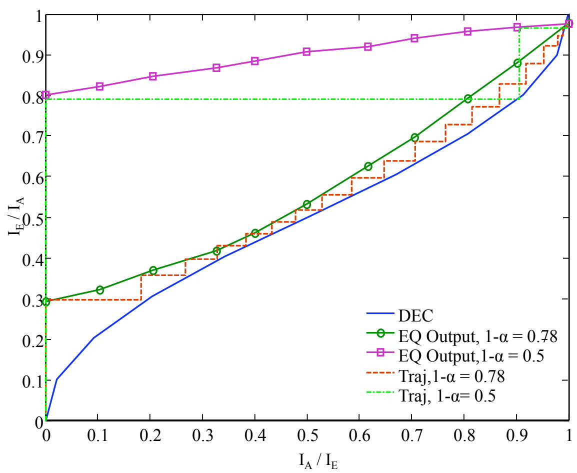
$$\begin{aligned} \mathrm{P_b} &\approx \frac{1}{2}\mathrm{erfc}\left(\frac{J^{-1}(I_{p,Dec})}{2\sqrt{2}}\right) \\ &\approx 6.623 \times 10^{-4} \end{aligned} \quad (34)$$

Figure 8(b) depicts the impact of different N_c in the EXIT chart for $1-\alpha = 0.78$. It shows that the tunnel is opened more widely as N_c is increased, especially when $N_c = 4$ compared to $N_c = 64$. This means that the number of iterations can be reduced as N_c gets larger and more data can be transmitted with lower complexity.

In Fig. 9(a), the impact on the EXIT chart for different values of overlapping factor, $1 - \alpha$ when SNR = 5dB and $N_c = 4$ are evaluated. We can see that, as $1 - \alpha$ decreases, the tunnel is widely opened because the distortion due to ICI becomes smaller, and the number of the iterations for the turbo equalization can also be reduced. To determine the required



(a) EXIT chart for different overlapping factor, $1-\alpha$.



(b) Trajectory on the EXIT chart with $1-\alpha = 0.78$ and $1-\alpha = 0.5$.

Fig. 9. EXIT chart between SC-MMSE equalizer and decoder when SNR = 5dB and $N_c = 4$

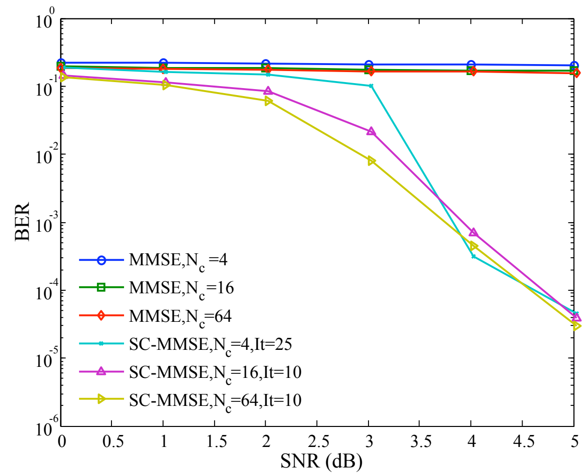


Fig. 10. BER performance for variety of N_c with $1-\alpha = 0.78$.

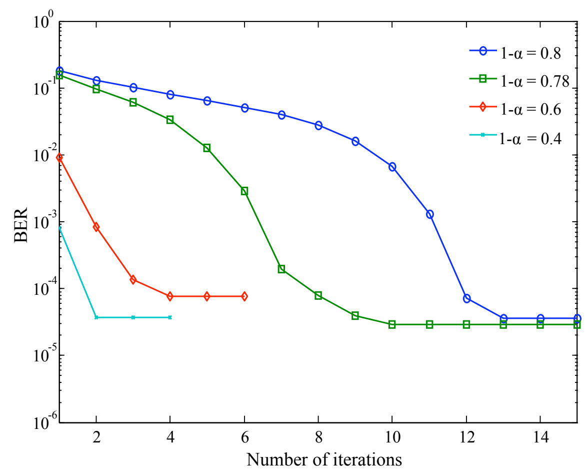


Fig. 11. BER performance vs Number of Iterations for different overlapping factor, $1-\alpha$ when $N_c = 64$.

number of iterations, trajectory analysis is performed for $1-\alpha = 0.78$ and $1-\alpha = 0.5$, of which the results are illustrated in Fig. 9(b). It is clearly shown that, with $1-\alpha = 0.78$, 18 iterations are needed due to heavy ICI whereas with $1-\alpha = 0.5$, only 2 iterations are needed.

B. BER performance

The BER performances are evaluated based on the parameters shown in Table II which are the same as in EXIT analysis in Section IV-A. BER performance with n-OFDM SC-MMSE is shown in Fig. 10 for different values of GIDFT subcarriers input number, N_c with $1-\alpha = 0.78$. The BER performance is well matched to the EXIT chart in Fig. 8(b) and hence a sharper BER cliff occurred at SNR = 3dB for $N_c = 4$. The error floor starts to happen as expected in (34) when SNR = 4dB at BER= 3.1×10^{-4} with slightly different around 3.523×10^{-4}.

The BER performance against number of iterations for different overlapping factor when $N_c = 64$ is shown in Figure 11. The number of iterations or turbo loops between equalizer and decoder is reduced as $1-\alpha$ becomes smaller. This means that, we can reduced the complexity of the system and at the same time, we can transmit more data by controlling the parameter of N_c and $1-\alpha$.

V. Conclusion

In this paper, we first provide information theoretic analysis of n-OFDM signal in terms of sum Gaussian capacity and sum CCMI. It is shown that n-OFDM system can utilize the bandwidth better than OFDM and hence increase sum capacity, totaling over the subcarriers. It is obvious that the capacity loss in OFDM system can be recovered by introducing the overlapping factor. However, the channel capacity is independent of the signaling schemes. Thus, in this paper we provide the theoretical limit in terms of bandwidth efficiency for the n-OFDM system in flat- fading channel.

To avoid the large noise enhancement resulted from high condition number, SC-MMSE turbo equalization is proposed. The EXIT chart analysis shows a good matching between

SC-MMSE equalizer and log-MAP decoder. As a result, it has been shown that the proposed technique provides excellent BER performance even with large overlapping factor. Although equalization is not needed in the flat fading channel for OFDM system, equalization is used in n-OFDM system to eliminate the ICI and the noise enhancement introduces at the transmitter. The findings of the n-OFDM system in this paper so far provides a strong basis for extending the work in frequency selective fading channel.

ACKNOWLEDGMENT

The authors would like to thanks the MOHE and Universiti Teknologi Malaysia for GUP grant that have been supported this research

REFERENCES

[1] I. Kanaras, A. Chorti, M. Rodrigues, and I. Darwazeh, "A combined MMSE-ML detection for a spectrally efficient non orthogonal FDM signal," in *Broadband Communications, Networks and Systems, 2008. BROADNETS 2008. 5th International Conference on*, Sept. 2008, pp. 421 –425.

[2] ——, "Spectrally efficient FDM signals: Bandwidth gain at the expense of receiver complexity," in *IEEE International Conference on Communications, 2009. ICC '09.*, June 2009, pp. 1 –6.

[3] M. Hamamura and S. Tachikawa, "Bandwidth efficiency improvement for multi-carrier systems," in *15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 2004. PIMRC 2004.*, vol. 1, Sept. 2004, pp. 48 – 52.

[4] S. Isam and I. Darwazeh, "Simple DSP-IDFT techniques for generating spectrally efficient FDM signals," in *7th International Symposium on Communication System Networks and Digital Signal Processing (CSNDSP)*, 2010, pp. 20–24.

[5] K. Anwar, "Peak power reduction for multicarrier systems in satellite and radio communications," Ph.D. dissertation, Nara Institute of Science and Technology, March 2008.

[6] K. Kansanen and T. Matsumoto, "An analytical method for MMSE MIMO turbo equalizer EXIT chart computation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 1, pp. 59–63, Jan. 2007.

[7] C. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, July, October 1948.

[8] F. Vogelbruch and S. Haar, "Low complexity turbo equalization based on soft feedback interference cancellation," *IEEE Communications Letters*, vol. 9, no. 7, pp. 586 – 588, July 2005.

[9] F. Brannstrom, "Convergence analysis and design of multiple concatenated codes," Ph.D. dissertation, Chalmers University of Technology, Sweeden, 2004.

[10] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. on Communications,*, vol. 49, no. 10, pp. 1727 –1737, Oct. 2001.