Title	Equational Reasoning and Completion
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Citation	
Issue Date	2012-06
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/10560
Rights	
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Abstract

Equations are a versatile and natural tool for various reasoning tasks. This thesis investigates how to fully automate equational reasoning. Central to automation is Knuth and Bendix' ground-breaking completion procedure, which searches for a complete term rewriting system (TRS) for a given system of equations. It was originally formulated as an algorithm, and requires a user-provided reduction order. The choice of the reduction order is non-trivial in general and heavily affects the success of the procedure.

During the early 2000's, significant progress has been made in automated constraint solving. Very often it turned out that encoding a problem to boolean constraints and using a SAT solver is more effective than developing a dedicated algorithm. However since the search space for a completion problem is not known in advance, it is not possible to directly encode it to constraints and benefit from the success of SAT solvers.

We show that completion can be characterized as an optimization problem. Using this formulation, called *Maximal Completion*, algorithmic aspects can be abstracted away by encoding the optimization problem to boolean constraints, and employing a MaxSAT solver to find a solution. Full automation of the completion procedure can then be achieved by constructing a suitable order from the solution of the constraint problem.

The success of Knuth and Bendix' procedure has led to several adaptations to related reasoning tasks, such as Inductionless Induction or Rewriting Induction. We propose a new uniform framework based on *constrained equalities*, which allows to easily express several different joinability requirements that are needed by the different procedures. Soundness proofs are vastly simplified, since all necessary conditions are encoded in the constraint part. Similar to Maximal Completion, an encoding as an optimization problem over boolean constraints can be used to automatically find a solution, i.e. a suitable order, for the joinability requirements. Experiments show that this approach is especially effective for the above mentioned methods for inductive reasoning.

Lastly we investigate confluence of TRSs. Testing confluence automatically is a central property not only for completion, but in other applications as well, such as narrowing or type theory. While confluence for terminating TRSs is decidable, finding confluence criteria for non-terminating and non-left-linear TRSs is very challenging and few criteria exist. Moreover very often these criteria face the issue of computability. We present a *new confluence criterion* for non-left-linear and non-terminating TRSs. Since it requires joinability of uncomputable extended critical pairs, we provide a new result on unification, which enables full automation of our criterion.

All results have been implemented and experimentally evaluated to show their effectiveness. Moreover, we provide the free completion tool MAXCOMP which is based on Maximal Completion.