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Description	

US external debt sustainability revisited: Bayesian analysis of extended Markov switching unit root test

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Abstract

The sustainability of US external debt, which has been an issue of global concern, is analyzed using a Markov switching (MS) unit root test applied to the flow of debt, i.e., the current account. The first to apply the MS unit root test to the issue of US external debt in order to examine local stationarity and global stationarity were Raybaudi et al. (2004). This paper introduces an extended MS unit root test where the transition probability is time-varying rather than fixed, as is usually the case, and the change of probability is explained by the real exchange rate, which theory suggests has a close relationship with the external balance. The extended MS unit root test calculated by the Markov Chain Monte Carlo (MCMC) method provides us with new insights on the issue of US external debt in recent years, suggesting that even though the debt/current account-GDP ratio remains relatively high, the probability of stationarity (sustainability) is unexpectedly high when recent US dollar depreciation is taken into account.

JEL classification : C11; C22; F21; F32

Key words: US external debt sustainability; Markov switching unit root test

1 Introduction

At the end of 2008, the ratio of the stock of US external net debt to GDP was 24.0 percent while the current account deficit in the forth quarter of 2008 stood at 4.4 percent of GDP (Figures 1 and 2).[1] The two deficits started to balloon in the late 1990s, and although recently this has come to

[1] External debt includes "the valuation effect" which includes capital gains and losses owing to exchange rate and other asset price changes. This paper does not take this effect into account when assessing the sustainability of US external debt. That is, it is not the first difference of the external debt including the valuation effect but the current

a halt, skepticism concerning the sustainability of US external debt remains, given that the deficit has not been really corrected.

[Figure 1 and Figure 2]

Against this background, the aim of this paper is to analyze the sustainability of the growth in US external debt, following the methodology of Trehan and Walsh (1988,1991) and Ahmed and Rogers (1995) by applying the unit root test to the current account deficit. If the current account deficit is stationary, the expected external debt will grow linearly, so that, as discussed in the next section, the present value of the future debt converges to zero because the discount rate grows explosively.

Sustainability of the external balance can be analyzed by two different approaches. One is the aforementioned time series analysis, while the other is the use of some open economy model where the real external balance and the theoretical balance derived from the model simulation are compared. Most of the preceding studies belonging to the first group are non-switching unit root and cointegration tests testing for global stationarity only, while only a few (e.g., Perron 1997) take structural breaks into account.[2]

However, there are also other studies, such as the one by Raybaudi et al. (2004), who adopted a Markov switching (MS) unit root test and analyzed the current accounts of five countries, including the United States. This unit root test calculates the probabilities of being in stationary and non-stationary regimes for each time point of an estimation period. Other studies apart from the one by Raybaudi et al. (2004) that adopted this test include, for example, Hall et al. (1999) and Nelson et al. (2001), but these applied the test not to external debt sustainability but other issues. The former study sought to detect periodically collapsing bubbles, while the latter, by using Monte Carlo experiments, investigated the performance of the test when the true process undergoes various types of MS regime change. There are not many empirical studies that have adopted the MS version of the unit root test, and the MS model has been more widely used in business cycle analyses where the two regimes are assumed to be the expansionary and contractionary phases of the business cycle.

Compared to a non-switching unit root test, which can only judge global stationarity, i.e., stationarity for the estimation period as a whole, the MS unit root test has the advantage that one

account without the effect that the unit root test is applied to. This is due to constraints with regard to external debt data, which are available only on an annual basis. For more on the valuation effect, see Corsetti and Konstantinou (2005) and IMF (2005).

[2]The sustainability of US external debt was analyzed by non-switching unit root test and cointegration test in Trehan and Walsh (1991), Ahmed and Rogers (1995), Ogawa and Kudo (2004) and by a combination of these two tests with the Perron test in Matsubayashi (2005).

can derive a "realistic" estimation result in that the probabilities of the stationarity for phases of deficit expansion can be different from those for phases of deficit contraction. Moreover, in general, any economic variable can indicate global stationarity and local nonstationarity simultaneously under the MS unit root test, and Raybaudi et al. (2004) called such local nonstationary phases “red signals” by which one can recognize the economy departing from the global stationary condition. Also in this sense, the MS unit root test can be used as a realistic analytical method because in practice nations never go easily bankrupt, although they sometimes fall into critical situations.

However, one drawback of the MS test is that, because it requires more parameters to be estimated, it is difficult to obtain stable and robust results.

To overcome this problem, the present study employs the extended MS unit root test. “Extended” here refers to the following two points: (1) the transition probability is time-varying rather than fixed, as is usually the case, and the probability change is explained by the real exchange rate which theoretical studies show to have a close relationship with the external balance; and (2) the MS unit root test with time-varying transition probabilities is calculated using the Markov Chain Monte Carlo (MCMC) method. This approach differs from the one adopted by Raybaudi et al. (2004), who used a fixed transition probability and maximum likelihood estimation.

With regard to (1), that is, the relationship between the real exchange rate and the external balance, the present study follows standard two-country models of trade assuming that a country’s trade balance depends on the real exchange rate as well as the real domestic and foreign incomes. Figure 2, depicting the moving average of the quarter-to-quarter change of the real exchange rate as well as the US current account-GDP ratio, clearly shows that there is a close relationship between the two. In this context, there are a considerable number of studies that have argued that a reduction in the US current account deficit would require a large decline in the dollar (e.g., Obstfeld and Rogoff 2004).

Compared with the fixed transition probability model, the extended model which incorporates the real exchange rate as a variable explaining the transition probability with respect to the external balance will be expected to have the advantage that it distinguishes the stationary and non-stationary phases more clearly. It is because the two phases differ in the way the external balance and the real exchange rate change. As is shown in the Figure 3, the exchange rate varies more widely when it is depreciated (usually in the stationary phase of external balance) than appreciated (in the non-stationary phase) and accordingly the movement of external balance is more volatile when it is depreciated than appreciated.[3] So, the time-varying probability model including

[3] Figure 3 shows that for the phase when the current account deficit declined in the first half of the 1970s and the early 1980s, the absolute values of the rate of change of the exchange rate were larger than those values in the

the two variables - the change of the real exchange rate and the volatility of the external balance (current account-GDP ratio) will be able to trace the asymmetricity observed between stationary and non-stationary phases. This point will be discussed again in the following sections.

[Figure 3]

The remainder of this paper is organized as follows. Section 2 discusses the sustainability condition for external debt. Next, Section 3 introduces the extended MS unit root test, while the Bayesian estimation method (MCMC, i.e. the Gibbs sampling algorithm) is presented in Section 4. Section 5 then discusses the empirical results of the extended MS unit root test and compares them with those of the MS unit root test with a fixed transition probability. Section 6 concludes the paper.

2 The sustainability condition

Trehan and Walsh (1988, 1991) presented the sustainability condition of external debt as follows.

Consider an economy with net external debt S_t, and current account $S_t - S_{t-1}$. If the current account is stationary, it has a moving average representation,

$$S_t - S_{t-1} = \delta + \Psi(L)\epsilon_t \tag{1}$$

where L is the lag operator, ϵ_t is white noise, δ is a constant, and $\Psi(L) = \Psi_0 + \Psi_1 L + \Psi_2 L^2 + ...$ Using the Beveridge and Nelson decomposition, S_{t+j} can be written as the sum of a linear time trend, a stochastic trend, a stationary process, and an initial condition:

$$S_{t+j} = \delta(j+1) + \Psi(1)(\epsilon_1 + \epsilon_2 + \epsilon_3 + ...\epsilon_{t+j}) + \eta_{t+j} + (S_{t-1} - \eta_{t-1}) \tag{2}$$

where $\eta_t = \alpha(L)\epsilon_t = \sum_{j=0}^{\infty} \alpha_j L^j \epsilon_t$, $\alpha_j = -(\Psi_{j+1} + \Psi_{j+2} + ...)$.

A sufficient condition for external debt sustainability is that the present value of the future debt converges to zero under the no-Ponzi condition. We can say that if the current account $S_t - S_{t-1}$ is stationary and (2) holds, the present value of the expected future debt converges to zero because

subsequent middle of the 1980s when the exchange rates were in turn appreciated and accordingly the current account deficit expanded. In the period after the latter half of the 1990s, the same asymmetricity could be observed. The variances of the external balance were larger in the depreciation phases than the appreciation phases. It is possible that the asymmetric movement of the real exchange rate depends on the nominal exchange rate pass-through, which measures the elasticity of domestic currency export or import prices with respect to changes in the nominal exchange rate. Webber (2000) argues that the nominal exchange rate depreciation causes import prices to rise by more than the same magnitude appreciation which will cause them to fall in the case of eight countries across the Asia and Pacific.

the expected future debt grows linearly while the discount rate (${}_t\rho_{t+j}$) grows exponentially as ${}_t\rho_{t+j} = \prod_{v=0}^{j}(1 + r_{t+v})$ (Eq. (3)). r_{t+v} denotes the nominal interest rate.

$$\lim_{j\to\infty} E({}_t\rho_{t+j}^{-1} S_{t+j} \mid I_{t-1}) = 0 \tag{3}$$

The stationary condition of external debt is usually analyzed in terms of the debt-GDP ratio and in this case, the discount rate should be modified to $t\rho'_{t+j} = \prod_{v=0}^{j} \frac{1+r_{t+v}}{1+g_{t+v}}$, where g_t denotes the GDP growth rate. If the economy is not dynamically inefficient, i.e. $r_t > g_t$, then the above-mentioned sustainability condition also holds and we can make use of a unit root test of the current account-GDP ratio to check the sustainability of the external debt-GDP ratio.

One thing to be mentioned before we use the above analytical framework is that currently, the US income balance is in surplus even though the United States has a net external debt. Assume the identity $S_t = S_{t-1}r_t + d_t$, where d_t is the current account deficit. The interest rate should be negative for the income balance to be positive. If this is the case, then $1 + r_t < 1$, and this usually leads to $\frac{1+r_t}{1+g_t} < 1$, so that the analytical framework becomes inappropriate.

The coexistence of a surplus in the income balance with external debt means that the rate of interest earned on US-owned assets abroad is higher than that on foreign-owned assets in the United States. However, it can be said that, over time, such differentials should disappear and the continued large current current deficits are likely to push the U.S. income balance below zero in the not too distant future (Higgins et al. 2005).[4] For this reason, the analytical framework presented here seems appropriate.

3 The extended Markov switching unit root test

This section introduces the extended Markov switching unit root test. The basic model underlying this test is as follows:

$$\Delta x_t = \phi_1 s_t + \phi_2(1 - s_t) + \phi_3(1 - s_t)x_{t-1} + \sigma\eta_t \tag{4}$$

where x_t is the current account-GDP ratio, Δ is the first difference operator, η_t is $\eta_t \sim N(0, 1)$, and s_t indicates the state that the regime is in at time t. Thus, the time series satisfies a model which

[4] In fact, looking at the long-term trend in the interest rate differential, this is indeed declining until 2000. From 2000 to 2004, U.S. net income receipts and the interest differential edged up with departing from the long-term trend and Higgins et al. (2005) pointed that some fortunate and temporary factors including the sharp falling of U.S. interest rates contributed to that.

allows the dynamic behavior of the series to be governed by either a stationary (sustainable) regime when $s_t = 0$, or a non-stationary (unsustainable) regime when $s_t = 1$. ϕ_1 is a parameter for the unsustainable regime and ϕ_2 and ϕ_3 are parameters for the sustainable regime where $-2 < \phi_3 < 0$.

This paper adopts not only Eq. (4), but also the following Eq. (4') which incorporates the time-varying variance. Raybaudi et al. (2004) did not adopt this modified version of the MS unit root test and in this paper it will be shown later that this time-varying variance model is most appropriate to assess the probability of the sustainability of US external debt. This modification is made by taking into account the fact that US external balance moves asymmetrically and is more volatile when the exchange rate is depreciated (in the stationary phase) than appreciated (in the non-stationary phase) as mentioned in the section 1.

$$\Delta x_t = \phi_1 s_t + \phi_2(1 - s_t) + \phi_3(1 - s_t)x_{t-1} + (1 + \omega s_t)^{\frac{1}{2}}\sigma\eta_t \tag{4'}$$

The variance has two different values: $(1 + \omega)\sigma^2$ if $s_t = 1$, and σ^2 if $s_t = 0$.

Following the research about business cycle durations by Filardo and Gordon (1998), the time-varying transition probabilities are defined as

$$\Pr(S_t = s_t \mid S_{t-1} = s_{t-1}, z_t) = \begin{pmatrix} q(z_t) & 1 - p(z_t) \\ 1 - q(z_t) & p(z_t) \end{pmatrix} \tag{5}$$

where $S_t \in \{1, 0\}$, $p(z_t) = \Pr(S_t = 1 \mid S_{t-1} = 1, z_t)$, and $q(z_t) = \Pr(S_t = 0 \mid S_{t-1} = 0, z_t)$.

A univariate probit model is estimated to measure the transition probability matrix at each time t. Using latent variable S_t^*, the probit model is set as

$$\Pr(S_t = 1) = P(S_t^* \geqslant 0) \tag{6}$$

$$S_t^* = \gamma_0 + \gamma_z z_t + \gamma_s s_{t-1} + u_t \tag{7}$$

where z_t is an information variable that affects the transition probabilities between stationary and non-stationary regimes. This specification is quite general and can incorporate various kinds of parameters. The candidate variable for z_t in this study is the moving-averaged quarter-to-quarter change of the US real exchange rate. Equations (6) and (7) can be said to be equations for measuring transition probabilities because the variable s_{t-1} is included in Eq. (7).

The random variable u_t is drawn from a process of independently distributed standard normal variables, $u_t \sim N(0, 1)$. The calculation of transition probabilities at each time t is performed by evaluating a conditional cumulative distribution function (CDF) for u_t. Let $\Phi_{u|z}$ represent a $N(0,1)$ conditional CDF, while transition probabilities $p(z_t)$ and $q(z_t)$ are derived as follows:

$$\begin{aligned} p_t &\equiv \Pr(S_t = 1 \mid S_{t-1} = 1) = \Pr(u_t \geqslant -\gamma_0 - \gamma_z z_t - \gamma_s \mid z_t) \\ &\equiv 1 - \Phi_{u|z}(-\gamma_0 - \gamma_z z_t - \gamma_s) \\ q_t &\equiv \Pr(S_t = 0 \mid S_{t-1} = 0) = \Pr(u_t < -\gamma_0 - \gamma_z z_t \mid z_t) \equiv \Phi_{u|z}(-\gamma_0 - \gamma_z z_t) \end{aligned} \tag{8}$$

The necessary and sufficient condition for global stationarity in a Markov switching ARMA process has been examined by Francq and Zakoïan (2001).[5] Whether the US current account GDP ratio meets this condition and how local stationary probabilities do change at each time are discussed below.

4 A Bayesian method for estimation

The model described in the previous section is estimated by the Markov Chain Monte Carlo (MCMC) method, not by maximum likelihood.[6] The Gibbs Sampler, a powerful MCMC computation method, is adopted.

The parameters of interest are:

$$\theta = \left\{ \phi_1, \phi_2, \phi_3, \omega, \sigma, \gamma_0, \gamma_z, \gamma_s, \{s_t\}_1^T, \{s_t^*\}_2^T, \{p_t\}_2^T, \{q_t\}_2^T \right\} \tag{9}$$

The application of the MCMC algorithm involves the following steps.

4.1 Step1 $(\phi_1, \phi_2, \phi_3, \omega, \sigma)$

First, the prior beliefs about the parameters of Eq. (4) are represented by conjugate priors. Define $Y = \{\Delta x_2, ...\Delta x_T\}'$; $X = (X_2, ...X_T)'$; and $X_i = (s_i, (1 - s_i), (1 - s_i)x_{i-1})$. The prior distribution for the variance has the inverse-gamma form $IG(\frac{\nu_0}{2}, \frac{\delta_0}{2})$, and the posterior distribution of σ^2 then has the form

$$IG(\frac{\nu_0 + n}{2}, \frac{\delta_0 + \|Y - \Phi X\|^2}{2}) \tag{10}$$

[5] For the time series $\{x_t\}$ that evolves according to Eq. (4), a necessary and sufficient condition for global stationarity is $q(1+\phi_3)^2 + p + (1-q-p) * (1+\phi_3)^2 < 1$, $q(1+\phi_3)^2 + p < 2$.

[6] The following analytical work was undertaken using Gauss, version 6.

where $\Phi = (\phi_1, \phi_2, \phi_3)$ and $\|\cdot\|$ denotes the Euclidean norm.

Given drawn σ^2 from Eq. (10) and the multivariate normal distribution as a prior of Φ, $N(\widehat{\Phi}, \widehat{A_\Phi}\sigma^2)$, the posterior for Φ is

$$\begin{aligned} &N(\overline{\Phi}, \overline{A_\Phi}\sigma^2) \\ &\overline{A_\Phi} = (\widehat{A_\Phi}^{-1} + X^{'}X)^{-1} \\ &\overline{\Phi} = \overline{A_\Phi}(\widehat{A_\Phi}^{-1}\widehat{\Phi} + X^{'}Y) \end{aligned} \tag{11}$$

In the next section, the model with the variance varying across time (Eq.(4')) will be examined along with the fixed variance model shown as Eq. (4) since the Bayes factor analysis assigns greater probability to the time-varying variance model than the fixed variance model. In this case, by additively defining $(1+s\omega)^{-\frac{1}{2}} = [(1+s_2\omega)^{-\frac{1}{2}}, ..., (1+s_T\omega)^{-\frac{1}{2}}]$ and transforming Y to $Y^* = Y*(1+s\omega)^{-\frac{1}{2}}$ and X to $X^* = X*(1+s\omega)^{-\frac{1}{2}}$, with the same inverse-gamma prior distribution of σ^2 as in the fixed variance model, the posterior distribution of the variance is

$$IG(\frac{\nu_0+n}{2}, \frac{\delta_0 + \|Y^* - \Phi X^*\|^2}{2}) \tag{10'}$$

Following Albert and Chib (1993), we next derive the posterior distribution for the parameters $\Phi = (\phi_1, \phi_2, \phi_3)$ in Eq. (4'). Given $\{s_t\}_1^T$, omega depends only on the observations for which $s_t = 1$. Letting $T = \{t : s_t = 1\}$, transforming ω to $\overline{\omega} = \omega + 1$, and choosing the prior for $\overline{\omega}$, $IG(\frac{\omega_0}{2}, \frac{\eta_0}{2})I_{\overline{w}>0}$, one can show that the complete conditional distribution of $\overline{\omega}$ is

$$IG(\frac{\omega_0+n^{'}}{2}, \frac{\eta_0 + \sum\{(\widetilde{Y} - \Phi\widetilde{X})/\sigma\}^2}{2})I_{\overline{w}>0} \tag{10''}$$

where $\widetilde{Y}$ and $\widetilde{X}$ are obtained by selecting data for Y ; X belongs to regime $s_t = 1$, and $n^{'}$ is the cardinality of T.

Given drawn ω and σ^2, and the multivariate normal distribution as a conjugate prior of Φ, $N(\widehat{\Phi}, \widehat{A_\Phi}\sigma^2)$, the posterior for Φ is

$$\begin{aligned} &N(\overline{\Phi}, \overline{A_\Phi}\sigma^2) \\ &\overline{A_\Phi} = (\widehat{A_\Phi}^{-1} + X^{*'}X^*)^{-1} \\ &\overline{\Phi} = \overline{A_\Phi}(\widehat{A_\Phi}^{-1}\widehat{\Phi} + X^{*'}Y^*) \end{aligned} \tag{11'}$$

4.2 Step2 ($\{s_t\}_1^T$)

4.2.1 The case of $t \geqq 2$

Applying Bayes theorem, the probability of being s_t, or $\Pr(s_t|Y_T, X_T, S_{-t}, \theta)$ can be introduced as follows:

$$\Pr(s_t|Y_T, S_{-t}) = \frac{\Pr(s_t|Y_t, S_{-t}) f(\Delta x_{t+1}, ..., \Delta x_T|Y_t, S_{-t}, s_t)}{f(\Delta x_{t+1}, ..., \Delta x_T|Y_t, S_{-t})} = \Pr(s_t|Y_t, S_{-t}) \tag{12}$$

where $Y_T = (\Delta x_2, ...\Delta x_T)$; $X_T = (X_2, ...X_T)$; and , $\theta = (\phi_1, \phi_2, \phi_3, \omega, \sigma)$ with the conditioning θ and X suppressed for simplicity.

Eq. (12) is derived because the second term in the numerator, $f(\Delta x_{t+1}, ..., \Delta x_T|Y_t, S_{-t}, s_t)$ and $f(\Delta x_{t+1}, ..., \Delta x_T|Y_t, S_{-t})$ in the denominator cancel each other out. That is, if $(\Delta x_{t+1}, ..., \Delta x_T)$ is independent of s_t, given S_{-t}, as shown in Eq. (4). Using Bayes theorem, we can rewrite $\Pr(s_t|Y_t, S_{-t})$ as

$$\Pr(s_t|Y_t, S_{-t}) = \Pr(s_t|s_{t-1}) \Pr(s_{t+1}|s_t) f(\Delta x_t|Y_{t-1}, S_t) \tag{13}$$

Working backwards from t=T, values for s_t can be simulated from a series of Bernoulli distributions using the probabilities generated by (13).

4.2.2 The case of $t = 1$

Using $\Pr(s_2|p_2, q_2, s_1)$ provided by the Markov process and using Bayes rule with $\widehat{\pi}$ as prior probability $\Pr(s_1 = 1)$, we can derive the posterior probability $\overline{\pi}$:

$$\Pr(\overline{\pi}|s_2 = 1) = \frac{p_2\widehat{\pi}}{p_2\widehat{\pi} + (1 - q_2)(1 - \widehat{\pi})}$$

$$\Pr(\overline{\pi}|s_2 = 0) = \frac{(1 - p_2)\widehat{\pi}}{(1 - p_2)\widehat{\pi} + q_2(1 - \widehat{\pi})} \tag{14}$$

Given a value of $\Pr(\overline{\pi}|s_2 = 1)$ or $\Pr(\overline{\pi}|s_2 = 0)$, the value of s_1 can be simulated from a Bernoulli distribution.

4.3 Step3 ($\{s_t^*\}_2^T, \gamma_0, \gamma_z, \gamma_s, \{p_t\}_2^T, \{q_t\}_2^T$)

With the $\{s_t\}_1^T$ drawn in the previous step, the probit model from Equations (6) and (7) can be calculated. Based on the inequality constraint in Eq. (6), $\{s_t^*\}_2^T$ can be simulated from appropriate truncated standard normal distributions. Given $\{s_t^*\}_2^T$, Eq. (7) becomes a linear regression model with unit variance. Define W to be the matrix right-hand side variables of Eq. (7) and $N(\widehat{\gamma}, \widehat{A_\gamma})$ to be the conjugate prior for $\gamma = (\gamma_0, \gamma_z, \gamma_s)$, and denote the vector of $\{s_t^*\}_2^T$ by s^*. Then the posterior distribution is

$$N(\overline{\gamma}, \overline{A_\gamma}) \tag{15}$$

where

$$\overline{A_\gamma} = (\widehat{A_\gamma}^{-1} + W'W)^{-1}$$
$$\overline{\gamma} = \overline{A_\gamma}(\widehat{A_\gamma}^{-1}\widehat{\gamma} + W's^*)$$

Given values for γ, the transitional probabilities $\{p_t\}_2^T$ and $\{q_t\}_2^T$ are obtained from the normal cumulative distribution function, as described in Eq. (8).

5 Empirical results

5.1 Data description and priors

The quarterly US current account and nominal GDP series are from the Bureau of Economic Analysis and are seasonally adjusted. The quarterly real exchange rate series is calculated as the export price index divided by the import price index and is converted to a four-quarter lagged twelve-quarter backward moving average of the quarter-to-quarter change. These price indexes (with 2000=100) are taken from the IMF's International Financial Statistics. The estimation period is from the first quarter of 1961 to the forth quarter of 2008.

The application of the Gibbs Sampler algorithm demands prior information on some elements of θ. The priors are derived from the maximum likelihood (ML) estimation applied to data running from the first quarter of 1961 to the third quarter of 2005.[7] This estimation period is shorter than

[7] The ML-based analysis was executed according to Hamilton (1994). The calculation procedure is as follows. Define the fixed transition probability as

$\Pr(S_t = s_t \mid S_{t-1} = s_{t-1}) = \begin{pmatrix} q & 1-p \\ 1-q & p \end{pmatrix}$; $p = \Pr(S_t = 1 \mid S_{t-1} = 1)$, and $q = \Pr(S_t = 0 \mid S_{t-1} = 0)$.

Then $P\{s_t = 1\} = \frac{1-q}{2-p-q}$ and $P\{s_t = 0\} = \frac{1-p}{2-p-q}$ are derived. Next, let $\xi_{t|t}$ be the 2×1 matrix containing $P\{s_t = 1\}$ and $P\{s_t = 0\}$, and η_t be 2×1 matrix containing each regimes' probability densities of Δx_t, and set up the following simultaneous equations:

$\xi_{t|t} = \frac{\xi_{t|t-1} \otimes \eta_t}{1(\xi_{t|t-1} \otimes \eta_t)}$ and $\xi_{t+1|t} = P\xi_{t|t}$

where P is the transition probability matrix as just defined, $1 = (1, 1)$ and $\otimes$ denotes element-by-element multiplication. Given a starting value $\xi_{1|0}$, and an assumed value for the estimation error and variance of Eq. (4), one can that of the MCMC method since the ML method fails to get convergent results for cases with a longer estimation period. The prior parameters $\widehat{\Phi_{fv}} = (\phi_1, \phi_2, \phi_3, \sigma_{fv})$ of the fixed variance model (Eq. (4)) and those for the time-varying variance model (Eq. (4')), $\widehat{\Phi_{vv}} = (\phi_1, \phi_2, \phi_3, \sigma_{vv}, \omega)$, come from this ML estimation.

The prior parameters of Eq.(7), $\gamma = (\gamma_0, \gamma_z, \gamma_s)$ are specified by OLS estimation of Eq.(7) using the $\{s_t\}_1^T$ obtained by the ML estimation of Eq. (4) as an independent variable.[8] Two priors for γ are prepared: $\widehat{\gamma_{tvtp}}$ for the time-varying transition probability model and $\widehat{\gamma_{ftp}}$ for the fixed transition probability model, which omits variable z_t.

Priors specified as mentioned above are as follows.

$$\widehat{\Phi_{fv}} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} -0.1501 \\ -0.0588 \\ -0.1651 \end{pmatrix} \quad \widehat{\Phi_{vv}} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} -0.0882 \\ -0.0290 \\ -0.1619 \end{pmatrix}$$

$$\widehat{A_{\Phi_{fv}}} = \widehat{A_{\Phi_{vv}}} = diag(10^3, 10^3, 10^3)$$

$$\widehat{\gamma_{tvtp}} = \begin{pmatrix} \gamma_0 \\ \gamma_z \\ \gamma_s \end{pmatrix} = \begin{pmatrix} -0.2349 \\ 0.0518 \\ 0.5820 \end{pmatrix} \quad \widehat{\gamma_{ftp}} = \begin{pmatrix} \gamma_0 \\ \gamma_s \end{pmatrix} = \begin{pmatrix} -0.2632 \\ 0.6149 \end{pmatrix}$$

$$\widehat{A_{\gamma_{tvtp}}} == diag(10^3, 10^3, 10^3) \qquad \widehat{A_{\gamma_{ftp}}} = diag(10^3, 10^3)$$

$$\sigma_{fv} = 0.0977 \qquad \sigma_{vv} = 0.3000 \qquad \omega = -0.3703 \qquad \Pr(s_1 = 1) = 0.5747$$

5.2 Parameter estimates

The estimated posterior means and standard deviations for θ of the time-varying transition probability (TVTP) model with fixed variance are reported in Table 1(a), while those of the fixed transition probability (FTP) model with fixed variance are reported in Table 1(b).[9] As for the parameter means, the two models do not show large differences and the signs are estimated as predicted: $-2 < \phi_3 < 0$, and $\gamma_z > 0$. The column labeled C.D. shows the convergence diagnostic

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iterate the above simultaneous equation for $t = 1, 2, ...T$ to calculate the values of $\widehat{\xi}_{t|t}$ and $\widehat{\xi}_{t+1|t}$ for each time t in the sample.

[8] The dependent variable S^* in Eq. (7) is calculated as $S^* = \Pr(s_t = 1) - 0.5$.

[9] In Tables 1(a), 1(b), and 2, $x = (1 + \phi_3)^2 + p + (1 - q - p) * (1 + \phi_3)^2$ and $y = q(1 + \phi_3)^2 + p$. $x < 1$ and $y < 2$ is a necessary and sufficient condition for global stationarity of the current account-GDP ratio. See footnote 5. The results show that US external debt is stationary in the global sense i.e., stationary for the estimation period as a whole.

statistic proposed by Geweke (1992), which can be used to assess the convergence of MCMC sampling. This statistic compares those values occurring early in the sequence with those occurring later in the sequence and if the difference between the two is insignificant, this indicates convergence.

Table1(a) Simulated Data From the Time-Varying Transition Probability Model With Fixed Variance

	Prior		Posterior						Sampling
Parameter	Mean	S.D.[a]	Mean	S.D.[a]	Lower 5%	Lower 10%	Upper 5%	C.D.[b]	Form
ϕ_1	-0.150	9.884	-0.119	0.011	-0.139	-0.137	-0.102	0.154	Normal
ϕ_2	-0.059	9.884	-0.012	0.007	-0.022	-0.021	0.001	0.897	Normal
ϕ_3	-0.165	9.884	-0.094	0.003	-0.103	-0.100	-0.090	0.744	Normal
σ^2	0.098	1.120	0.094	0.011	0.078	0.081	0.114	-0.078	Inverse-gamma
γ_0	-0.235	31.623	0.931	0.128	0.755	0.771	1.136	-1.499	Normal
γ_z	0.052	31.623	0.552	0.144	0.320	0.345	0.738	1.488	Normal
γ_s	0.582	31.623	1.153	0.121	0.967	1.000	1.352	0.018	Normal
$x = 0.949;\ y = 1.537$									

[a] S.D. denotes standard deviation.

[b] C.D. denotes convergence diagnostic.

Table1(b) Simulated Data From the Fixed Transition Probability Model With Fixed Variance

	Prior		Posterior						Sampling
Parameter	Mean	S.D.[a]	Mean	S.D.[a]	Lower 5%	Lower 10%	Upper 5%	C.D.[b]	Form
ϕ_1	-0.150	9.884	-0.080	0.007	-0.093	-0.092	-0.070	-0.090	Normal
ϕ_2	-0.059	9.884	-0.060	0.008	-0.072	-0.071	-0.047	0.506	Normal
ϕ_3	-0.165	9.884	-0.092	0.002	-0.097	-0.095	-0.090	-0.193	Normal
σ^2	0.098	1.120	0.102	0.011	0.084	0.088	0.122	-0.023	Inverse-gamma
γ_0	-0.263	31.623	-0.278	0.202	-0.580	-0.536	0.077	-0.730	Normal
γ_s	0.614	31.623	4.732	0.262	4.351	4.414	5.173	-1.443	Normal
$x = 0.999;\ y = 1.501$									

[a] S.D. denotes standard deviation.

[b] C.D. denotes convergence diagnostic.

Letting $\overline{g_1}$ be the mean of the early part of the sequence of sampled data and $\overline{g_2}$ that of the later part of the sequence of sampled data, the C.D. statistic can be written as: [10]

[10] In Eq. (16), $\widehat{var}(\overline{g}_M) = \frac{\widehat{\sigma}^2_{g_M}}{n_M}\left(1 + 2\sum_{i=1}^{B_M} K\left(\frac{i}{B_M}\right)\rho_{g_M}(i)\right)$

$$Z = \frac{\overline{g_1} - \overline{g_2}}{\sqrt{\widehat{var}(\overline{g_1}) + \widehat{var}(\overline{g_2})}} \tag{16}$$

Z can be compared to critical values from a standard normal statistical table. If the null hypothesis $H_0 : \overline{g_1} = \overline{g_2}$ is rejected, this indicates that we have not used enough replications.

In order to attenuate the effect of the choice of starting values, the first 30,000 draws are discarded, leaving the next 30,000 draws to calculate the posterior moments.

The estimated probabilities of US external debt being in a sustainable regime based on the TVTP and FTP models are graphed in Figure 4 and Figure 5. For the phase when the current account deficit expanded in the middle of the 1980s, and for the period after the latter half of the 1990s when the deficit began to soar again, the probabilities of these models are relatively low.

In the period after 2005, these models show more volatile probability figures and this makes it impossible to discern a clear pattern of the phase shift from low to high probabilities.

[Figure 4 and Figure 5]

On the other hand, Figure 6 which shows the probability calculated by the modified TVTP model incorporating time-varying variance as in Eq. (4') paints a quite different picture: for the phase when the current account deficit declined in the first half of the 1970s and the early 1980s, and for the period after 2005 when the deficit began to shrink again, the probabilities of the modified TVTP model are higher than the previous two models.

[Figure 6]

What is the reason for the contrast? The key point is the real exchange rate explaining the time-varying transition probabilities in the TVTP model (Eq. (7)). In periods when the contrast can be observed, i.e., the first half of the 1970s, the early 1980s, and the period after 2005, the exchange rate is in a phase of depreciation.

In Table 2, the estimated coefficient γ_z is 1.882 which is larger than the same coefficient in the unmodified TVTP model ($\gamma_z = 0.552$ in Table 1(a)). This positive figure indicates that the

where the Parzen window $K(\cdot)$ is

$$\begin{aligned} K(z) &= 1 - 6z^2 + 6z^3; z \in [0, \frac{1}{2}] \\ &= 2(1-z)^3; z \in [\frac{1}{2}, 1] \\ &= 0; elsewhere \end{aligned}$$

The suffix M takes value 1 (in the early part of the sequence) or 2 (in the later part of the sequence) and $n_1 = 0.1n$, $n_2 = 0.5n$, (n is the number of total drawings) as proposed by Geweke (1992). $\rho_{g_M}(i)$ denotes the autocorrelation between drawing set $\{\theta^1, \theta^2, ..., \theta^{n_M - i}\}$ and its i-lagged drawing set $\{\theta^i, \theta^2, ..., \theta^{n_M}\}$. B_M is the bandwidth. A bandwidth of 100 is selected for the early part of the sequence and 500 for the later part of the sequence.

depreciation of the dollar ($z_t < 0$) leads to a increasing probability of being in a stationary regime (Eq. (6)(7)). Conversely, the estimated probability of stationarity is lower in the modified TVTP model during the phases of dollar appreciation in the 1960s and middle of the 1980s. Compared with the unmodified time-varying transition probability model, it can be concluded that this significant positive coefficient γ_z could be obtained by the simultaneous estimation of ω that switches the variance of the current account movements between the two phases.

The probability of being in a stationary regime in Figure 6 is relatively high for the period after the latter half of the 1990s when the current account deficit expanded more than ever, and it should be stressed again that the modified TVTP model shows the probability shifts to a level above 50 percent after 2005. The former result is quite different from the one obtained by Raybaudi et al. (2004) where the probability of being in a stationary regime plunged below 10 percent around 1998 and the probability level almost unchanged even in the 2000s.[11] This long-run quite low probability of stationarity (sustainability) looks inconsistent with the continuing expansion of external capital inflow into U.S. from the late of 1990s.

Table2 Simulated Data From the Time-Varying Transition Probability Model With Time-Varying Variance

	Prior		Posterior						Sampling
Parameter	Mean	S.D.[a]	Mean	S.D.[a]	Lower 5%	Lower 10%	Upper 5%	C.D.[b]	Form
ϕ_1	-0.088	17.321	-0.088	0.004	-0.096	-0.095	-0.080	1.201	Normal
ϕ_2	-0.290	17.321	-0.139	0.007	-0.156	-0.148	-0.130	0.851	Normal
ϕ_3	-0.161	17.321	-0.091	0.001	-0.095	-0.094	-0.090	0.829	Normal
σ^2	0.300	0.119	0.163	0.019	0.133	0.139	0.197	-0.055	Inverse-gamma
ω	-0.370	0.164	-0.709	0.081	-0.828	-0.807	-0.563	0.063	Inverse-gamma
γ_0	-0.234	31.623	0.010	0.084	-0.117	-0.089	0.157	-0.646	Normal
γ_z	0.051	31.623	1.882	0.055	1.765	1.799	1.962	1.393	Normal
γ_s	0.582	31.623	0.479	0.048	0.408	0.422	0.563	0.704	Normal
$x = 0.926;\ y = 1.022$									

[a] S.D. denotes standard deviation.

[b] C.D. denotes convergence diagnostic.

The next question is which is the most appropriate among the aforementioned three types of models. The first comparison is between the two base models: TVTP with fixed variance (M1) and

[11] In Raybaudi (2004), the U.S. current account data set used for the MS unit root test was over the period from the first quarter of 1970 to the forth quarter of 2002.

FTP with fixed variance (M2). For the model comparison, the Bayes factor (BF) is introduced.[12] The BF_{ij} comparing model i and model j is defined as

$$BF_{ij} = \frac{p(Mi|Y)}{p(Mj|Y)} = \frac{p(Y|Mi)p(Mi)}{p(Y|Mj)p(Mj)} = \frac{p(Y|Mi)}{p(Y|Mj)} \tag{17}$$

where Y denotes the dependent variable in Eq. (4) and (4').

BF_{ij} is derived when equal prior weight is attached to each model ($p(Mi) = p(Mj)$). The result for BF_{12}, which compares M1 and M2 is $2.0600 * 10^9$ and $p(M1|Y) = \frac{BF_{12}}{1+BF_{12}} = 1.0000$ which indicates that M1 should be selected rather than M2. The next comparison is between M1 and M3 (the TVTP with time-varying variance model). The result for BF_{13} is $1.1195 * 10^{-13}$ and $p(M3|Y) = \frac{1}{1+BF_{13}} = 1.0000$. M3 is superior to M1. Based on this two-step comparison of models, we can conclude that the TVTP with time-varying variance model is the most appropriate model to assess the probability of the sustainability of US external debt. The analysis has shown that the assessment of US external debt especially for the period after 2005 changes if a TVTP with fixed variance model or an FTP model is used.

6 Conclusion

In this paper, it was proposed to use the extended MS unit root test to investigate the sustainability of US external debt. The unit root test with the time-varying transition probability and the regime switching variance calculated by the Markov Chain Monte Carlo (MCMC) methodology provides us with new insights into the sustainability of US external debt. For the period when the deficit soared during the late 1990s and early 2000s, we found that the probabilities of stationarity (sustainability) estimated by the modified model are higher than those obtained based on the traditional FTP model, and it is especially notable that the probability shifts to a level above 50 percent after 2005.

This result is quite different from the one obtained by Raybaudi et al. (2004) and is due to the varying transition probability linked to the real exchange rate. The specification introduced here is very general in that it can incorporate state-dependent, duration-dependent and other various kinds of parameters. Based on this general analytical framework, the MS model will contribute to the more efficient analysis of unit root tests, just as the model has contributed to the analyses of business cycles.

[12] The marginal likelihood for each regime $p(Y|Mi)$; $i = 1, 2, 3$ in the Bayes factor is calculated following Koop (2003). In the time-varying variance model $M3$, $p(Y|M3) = p(Y^1|M3) * p(Y^2|M3)$ where Y^1 denotes samples of Y in a stationary regime and Y^2 denotes samples of Y in a non-stationary regime.

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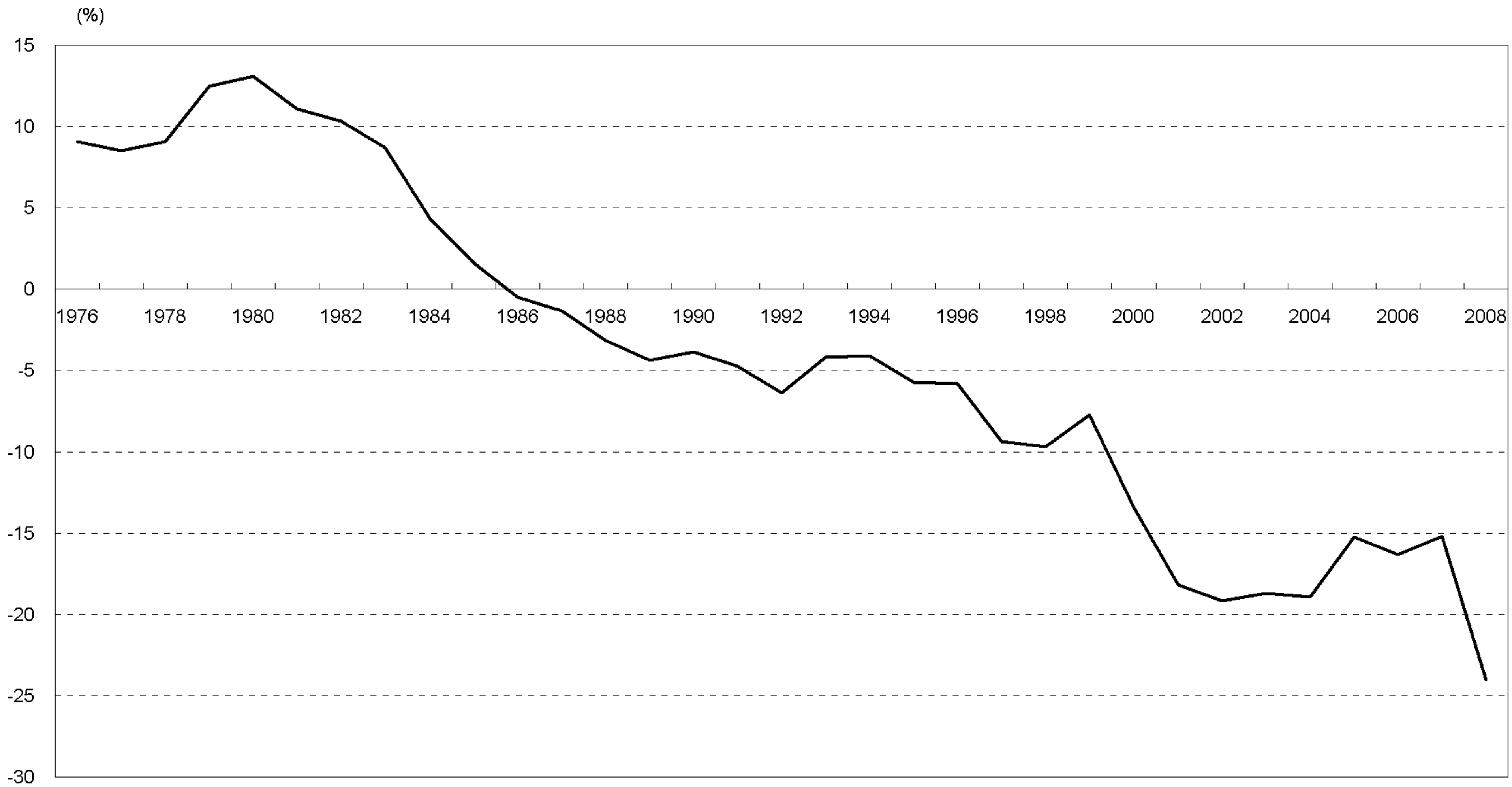
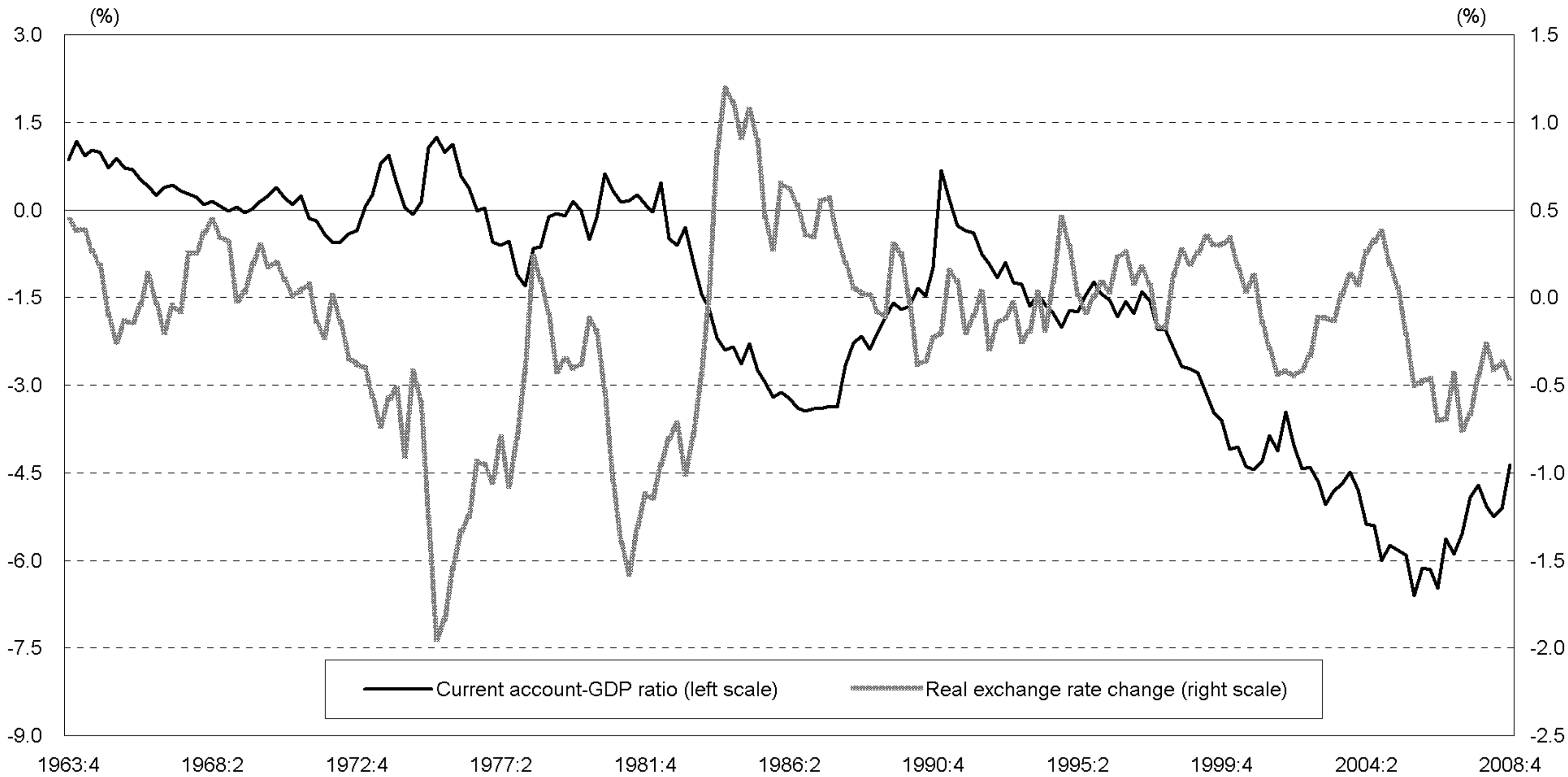


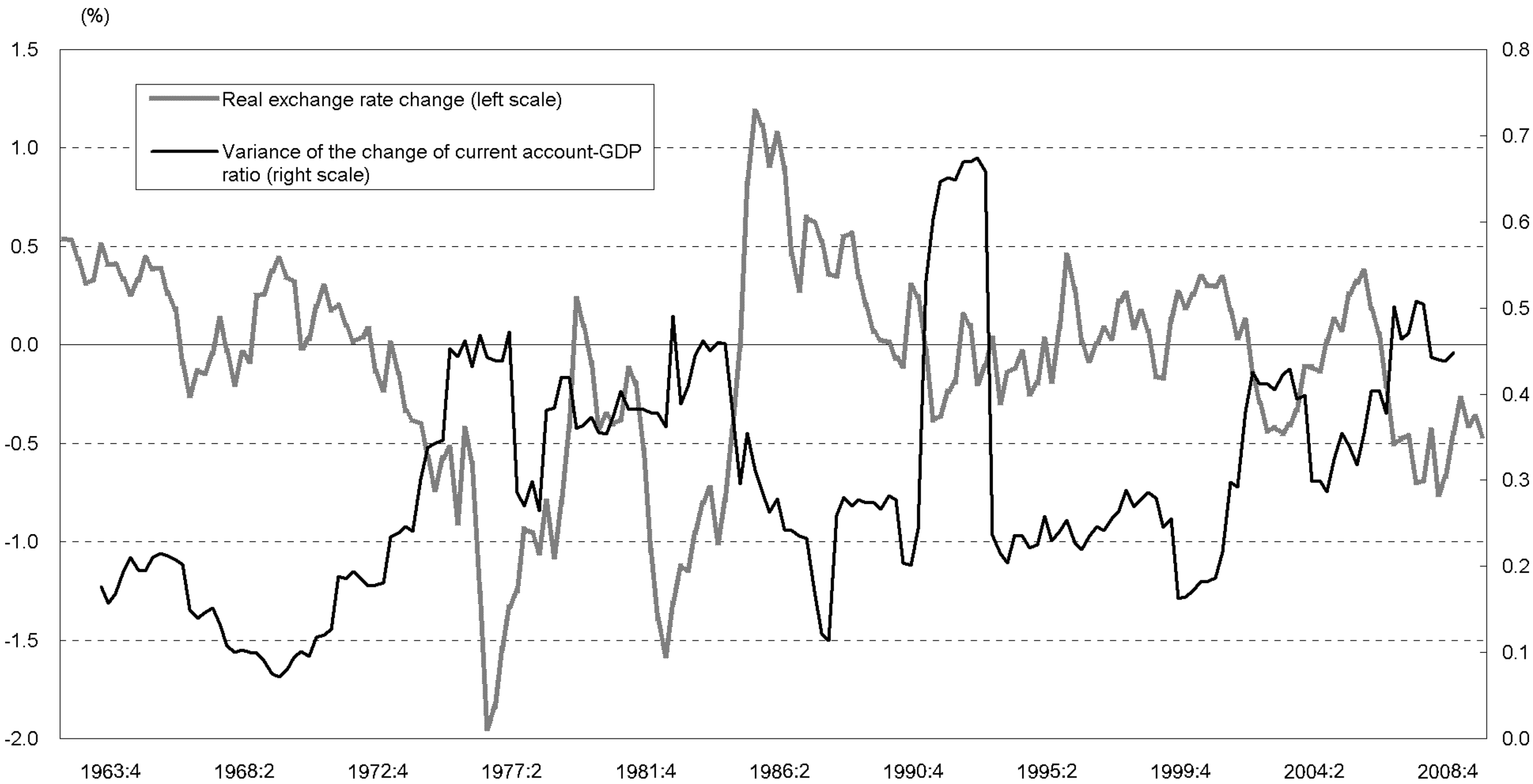
The data in 2008 is preliminary

Fig. 1. US Net International Investment Position At Year End Relative to GDP



The real exchange rate is the export price index divided by the import price index and converted to a 4-quarter lagged 12-quarter backward moving average of the quarter-to-quarter change. A positive value means appreciation and a negative value means depreciation.

Fig. 2. US Current Account GDP-Ratio And Real Exchange Rate



The time-varying variance of the quarter-to-quarter change of current account-GDP ratio is calculated for the rolling window of nine quarters.The plotted data corresponds to the center of the rolling window. The values of real exchange rate change is the same as in Fig. 2. A positive value means appreciation and a negative value means depreciation.

Fig. 3. External Balance Volatility and Real Exchange Rate Change in US

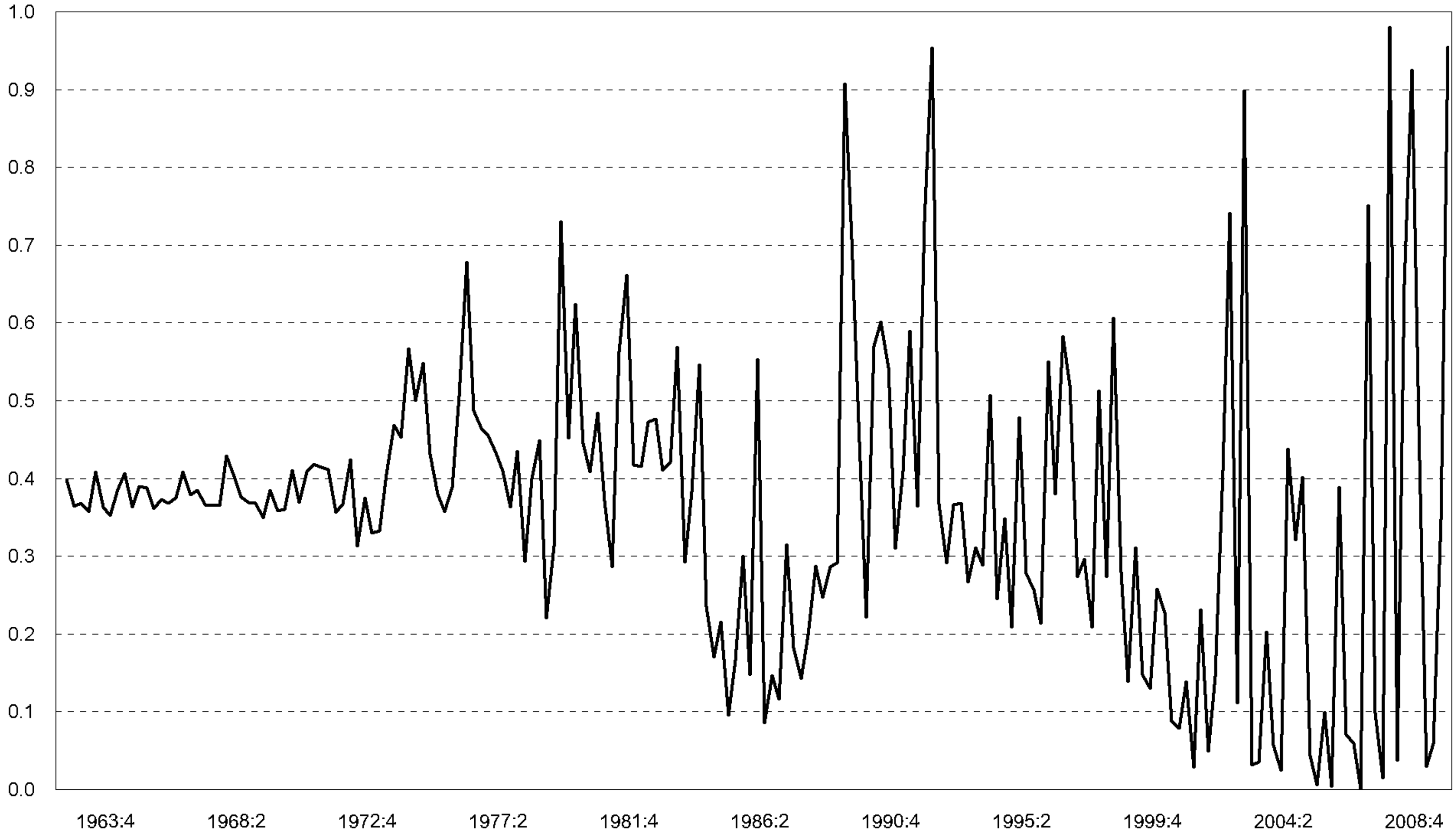


Fig. 4. The Probability of US External Debt Being Stationary (Sustainable), Results of theTVTP (Fixed variance) Model

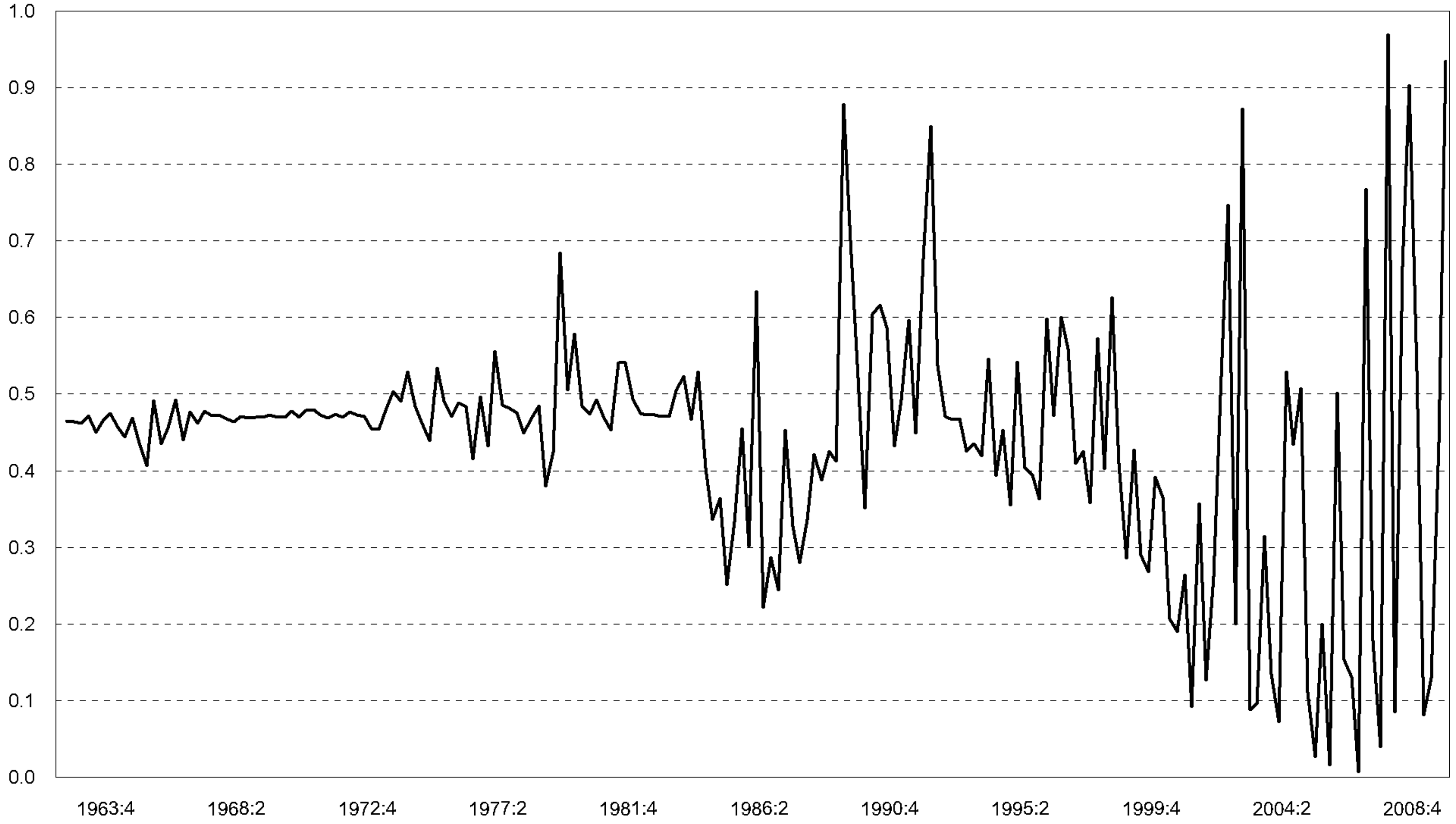


Fig. 5. The Probability of US External Debt Being Stationary (Sustainable), Results of the FTP Model

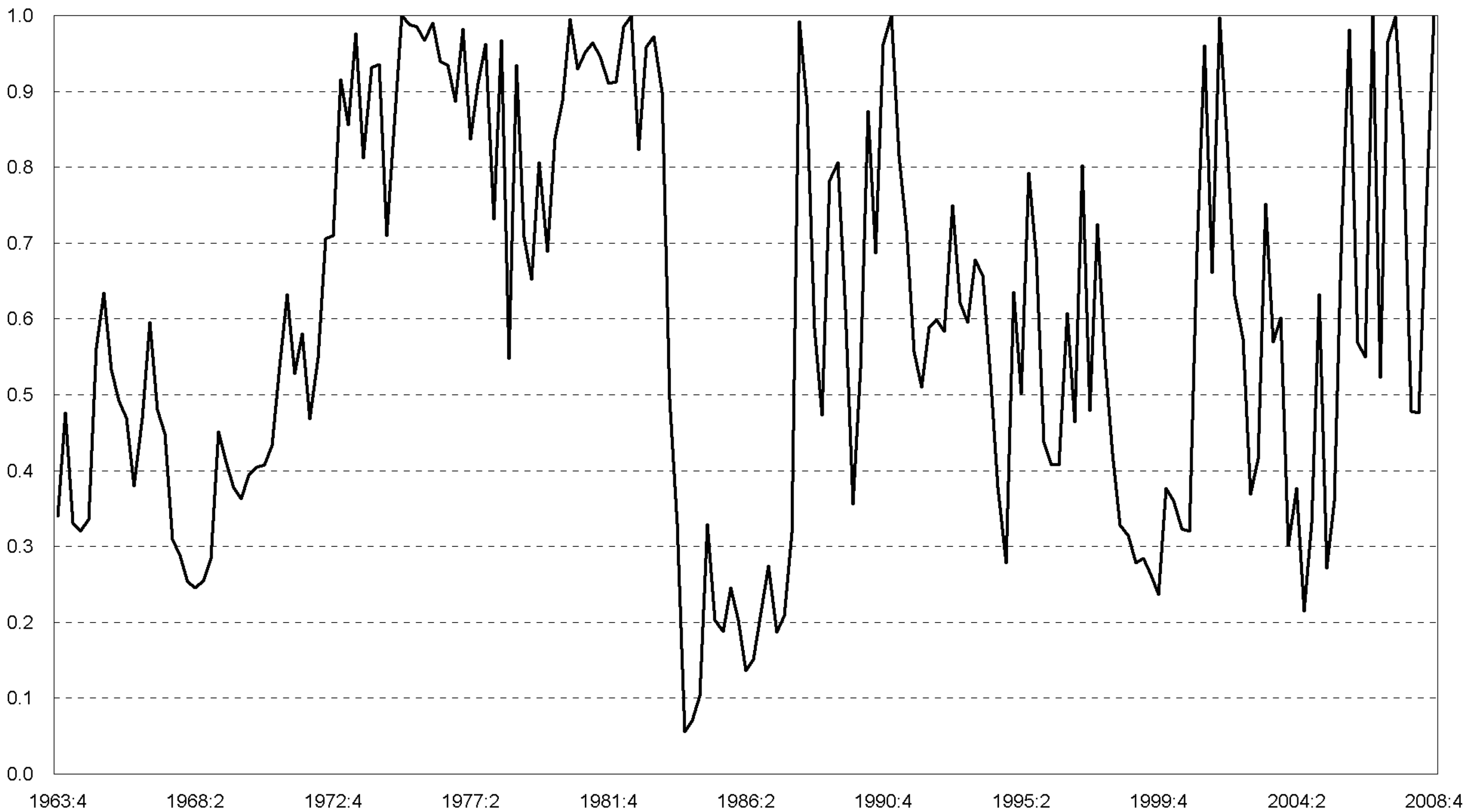


Fig. 6. The Probability of US External Debt Being Stationary (Sustainable), Results of the TVTP (Time-Varying Variance) Model