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Description	

Iterative Spatial Demapping for Two Correlated Sources over Fading Multiple Access Channel

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***Abstract*—This paper investigates in frequency-flat Rayleigh fading multiple access channel (MAC) the performances of Slepian-Wolf (SW)-based iterative spatial demapping (SW-ISM) for two correlated sources. The correlation between the sources is exploited in log-likelihood ratio (LLR) exchange via the *vertical iteration* (*VI*) loop between the two outer decoders. The results via computer simulations confirm that the proposed SW-ISM structure achieves excellent performances over frequency-flat Rayleigh fading MAC. Results of the SW and MAC rate regions analysis are also presented to demonstrate the effectiveness of the technique. Potential applications of the technique investigated in this paper are sensor and/or relay networks requiring high throughput.**

I. Introduction

The correlation in information transmitted from multiple nodes can be utilized to reduce the energy consumption [1] for reliable transmission to a common destination/receiver. A goal of this paper is to make efficient use of the source correlation in the framework of multiple access channel (MAC) with single antenna receiver, where achieving higher spectrum efficiency is aimed at, by reducing transmission phases.

Slepian-Wolf (SW) coding theorem identifies a region of achievable rates $\mathcal{R}_1$ and $\mathcal{R}_2$ when considering the lossless compression of two correlated sources b_1 and b_2 sending data to a common destination [2]. The contribution of SW coding theorem is the discovery that the compression can be performed even if both sources are encoded separately so far as the destination has the knowledge of the source correlation. However, to the authors' best knowledge, only a few techniques have been known with the aim of its practical utilization to wireless sensor or relaying networks.

Inspired by [3], we proposed in [4] the SW-based iterative spatial demapping (SW-ISM) for two correlated sources over instantaneous power-controlled Rayleigh fading MAC.[1] By assuming that the correlation between source b_1 and b_2, as shown in Fig. 1, can be expressed by $\rho = 1 - 2p_e$, with $p_e = \Pr(b_1 \neq b_2)$ being the bit-flipping probability, the SW rate region is given by three inequalities,

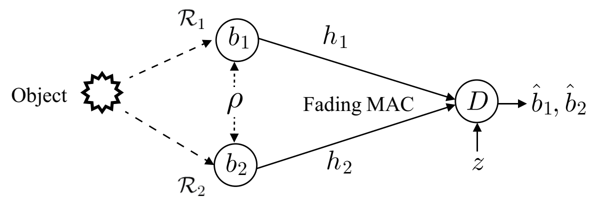

Fig. 1. Communication between sensors with source correlation ρ to the common destination D over fading MAC

$$\begin{aligned} \mathcal{R}_1 &\geq \mathcal{H}(p_e), \\ \mathcal{R}_2 &\geq \mathcal{H}(p_e), \\ \mathcal{R}_1 + \mathcal{R}_2 &\geq 1 + \mathcal{H}(p_e), \end{aligned} \tag{1}$$

where $\mathcal{H}(\cdot)$ is the entropy of the source [5].

In this paper, we extend our proposed SW-ISM structure presented in [4] to more generic fading MAC where the channel state information (CSI) is only known to the receiver.[2] The proposed SW-ISM, combined with doped accumulator, achieves excellent performance even with the randomness of the received composite signal points composed of the two sources at the destination. Furthermore, it results in a better matching to the decoder extrinsic information transfer (EXIT) curves that provide performance very close to the Slepian-Wolf/Shannon theoretical performance limit.[3] The receiver performs ISM demapping according to the turbo principle where the log-likelihood ratios (LLR) are exchanged between the ISM demapper and outer decoder separated by interleavers. Detection of the sources b_1 and b_2 are performed via *horizontal iteration* (*HI*) loop, while the correlation between the two sources is exploited in *vertical iteration* (*VI*) loop.

The main objectives of this paper are: (1) to extend the performance evaluation of SW-ISM structure for two correlated sources to fading MAC (without instantaneous power control), and (2) to analyze theoretically the separation of source and channel coding by evaluating both SW and MAC rate regions. The results presented in this paper are useful in

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[1]The channel gains h_1 and h_2, as shown in Fig. 1, are set to $|h_1| = h_2 = 1$, but the phase difference $\theta = \angle(h_1, h_2)$ is unknown to the transmitter.

[2]The both power and θ are unknown to the transmitter.

[3]EXIT analysis is presented in [4] but not in this paper.

the applications of relay systems and/or sensor networks over MAC channels.

With frequency-flat (1-path) Rayleigh fading MAC, the phase difference θ between the complex channel coefficient h_1 and h_2 changes randomly. The aim of evaluation in frequency-flat Rayleigh fading channel is to clarify its applicability to the sensor networks and/or relay systems allowing intra-link errors, where the source b_2 works as the relay.

II. Transceiver Structure

Single carrier signaling is considered.[4] Fig. 2 shows a block-diagram of the SW-ISM structure [4], where the binary streams b_1 and b_2 are encoded separately. Puncturing for each coded source may be performed to adjust the total compression and channel coding rates, denoted as $\mathcal{R}_1$ and $\mathcal{R}_2$.

A. Correlated Sources Model

The correlation between the two sources can be expressed using bit-flipping [6] error probability p_e as

$$\rho = 1 - 2p_e. \tag{2}$$

The k-th bit of b_2 is defined as

$$b_2^k = b_1^k \oplus e^k, \tag{3}$$

where $\oplus$ indicates a modulo 2 addition and $e^k \in \{0, 1\}$ is a random variable with probabilities $\Pr(e^k = 1) = p_e$ and $\Pr(e^k = 0) = 1 - p_e$, which is independent of b_1^k. The conditional probabilities are then given by

$$\Pr(b_2^k = 0 | b_1^k = 1) = \Pr(b_2^k = 1 | b_1^k = 0) = p_e \tag{4}$$
$$\Pr(b_2^k = 0 | b_1^k = 0) = \Pr(b_2^k = 1 | b_1^k = 1) = 1 - p_e, \tag{5}$$

where parameter p_e can take a value in the range of $0 \leqslant p_e \leqslant 0.5$ [6].

By assuming that the appearances of 0 and 1 in b_1 and b_2 are equiprobable, the corresponding entropy $\mathcal{H}(b_1) = \mathcal{H}(b_2) = 1$. If the information block length K is long enough, the conditional entropy $\mathcal{H}(b_1|b_2)$ is given by

$$\begin{aligned} \mathcal{H}(b_1|b_2) &= \lim_{K\to\infty} \frac{1}{K} \mathcal{H}(b_2^1, \cdots, b_2^K | b_1^1, \cdots, b_1^K) \\ &= \mathcal{H}(p_e), \end{aligned} \tag{6}$$

where $\mathcal{H}(p_e) = -p_e \log_2(p_e) - (1-p_e)\log_2(1-p_e)$ is a binary entropy of the random sequence e. Finally, the achievable SW region $(\mathcal{R}_1, \mathcal{R}_2)$ with this model is given by (1).

B. Transmitters

The bitstream b_1 is convolutionally encoded using C_1, interleaved by Π_1, doped accumulated, and modulated to binary phase shift keying (BPSK)[5] symbol s_1, with $\mathbb{E}[s_1] = 1$. The bitstream b_2 is first Π_0-interleaved, convolutionally encoded using C_2, interleaved by Π_2, doped accumulated and modulated to BPSK symbol s_2 with $\mathbb{E}[s_2] = 1$. It should be noted here that the interleaver Π_0 is introduced to exploit the correlation knowledge via the *VI* loop at the receiver. Interleavers Π_1 and Π_2, which are longer than Π_0, are used to interleave the coded bits x and y to obtain sequences x' and y'.[6]

The rate-1 doped accumulator (D-ACC) [7], [8] (after the interleaver) is utilized to flexibly adjust the shape of the EXIT curve of the ISM demapper depending on the correlation between the sources. The structure of D-ACC is very simple since it is composed of a memory-1 systematic recursive convolutional codes (SRCC) with octal code generator of $[3, 2]_8$ followed by *heavy puncturing* of the coded bits such that its coding rate $R_{dacc} = 1$. With a doping rate P, the D-ACC replaces every P-th systematic bits with the accumulated coded bit.

C. Channel

The BPSK symbols s_1 and s_2 are transmitted simultaneously over the frequency-flat Rayleigh fading channels with their coefficients h_1 and h_2,[7] respectively. Since the two source symbols are transmitted simultaneously from their transmit antennas to a single receive antenna, the received signal r is a result of superposition of the two receive signals, as

$$r = h_1 s_1 + h_2 s_2 + z, \tag{7}$$

where z is a noise component modeled by complex Gaussian random variable with zero mean and variance $\sigma_n^2/2$ per dimension, with which the noise power is $N = \sigma_n^2$. h_1 and h_2 are assumed to be known only to the receiver. The instantaneous received signal-to-noise power ratio (SNR) for the sources 1, 2, and for the total received symbols are defined, respectively, as

$$\gamma_1 = \frac{|h_1|^2}{N}, \quad \gamma_2 = \frac{|h_2|^2}{N}, \quad \gamma_{total} = \frac{|h_1|^2 + |h_2|^2}{N}. \tag{8}$$

The superposition causes different constellations of the receive symbols which depends on $|h_1|$ and $|h_2|$ and their phase difference $\theta = \angle(h_1, h_2)$. Fig. 3 plots some possible constellations at the receiver.[8]

D. Receiver

The receiver consists of a common ISM demapper, two *HI* loops to detect the two sources, and one *VI* loop to exploit the advantage due to the correlation between the sources. The ISM demapper performs demapping of spatial constellation from r into s_1 with the help of *a priori* information about s_2, and vices versa, in the form of LLR provided by the decoders. The demapper output is the extrinsic LLR which is then fed into the decoder of D-ACC (referred to as D_{dacc}), deinterleaved, and then decoded by D_1 or D_2.

[4] An extension to other signaling schemes, such as multicarrier systems as well as their mixture systems, is straightforward.

[5] There is no restriction to other higher order modulations.

[6] A delay τ is added to antenna 1 as a compensation of the additional interleaver Π_0 at source b_2.

[7] h_1 and h_2 is i.i.d complex Gaussian random variables with zero mean, unit variance and remains constant during a block of symbol interval.

[8] It should be emphasized here that for Fig. 3(a) $1/\sqrt{2}$ is not needed because it is a composite signal (not a constellation of a quadrature phase shift keying (QPSK) symbol).

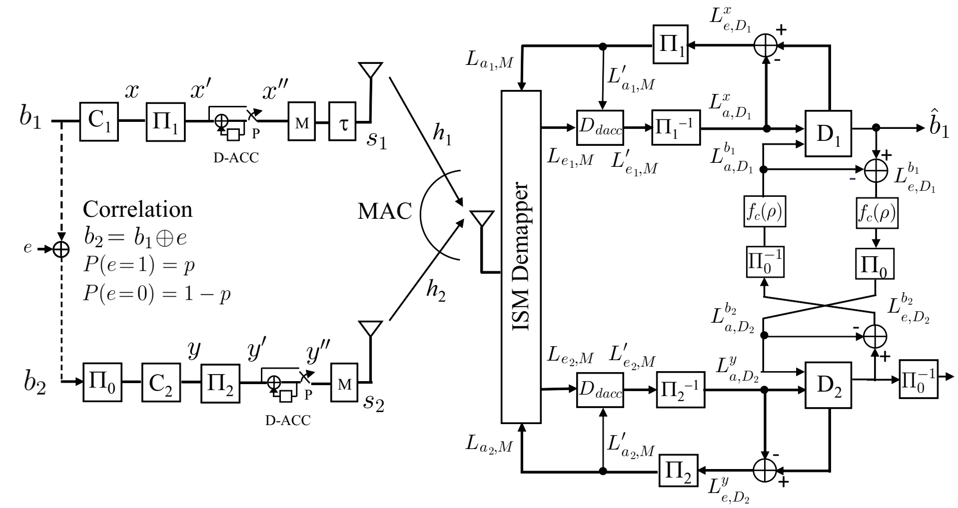

Fig. 2. Single antenna receiver for two correlated sources over fading MAC

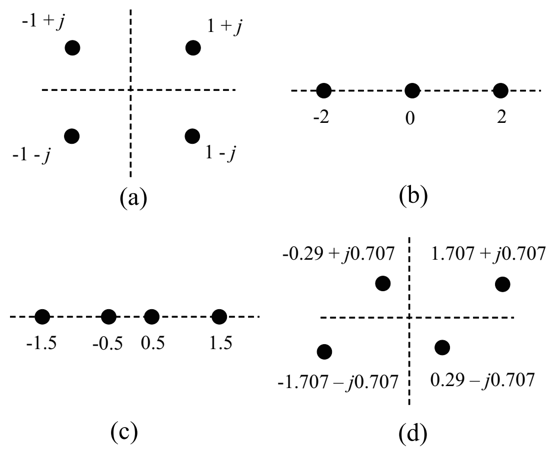

Fig. 3. Possible constellations of spatially separated two BPSK symbols when: (a) $|h_1| = |h_2| = j, \theta = 90°$, (b) $|h_1| = |h_2| = 1, \theta = 0°$, (c) $|h_1| = 1/2, |h_2| = 1, \theta = 0°$, and (d) $|h_1| = |h_2| = 1, \theta = 45°$

At the receiver D_{dacc} is performed using the Bahl-Cokce-Jelinek-Raviv (BCJR) algorithm [9] immediately after the ISM demapping. It should be noted here that interleaver between D_{dacc} and the equalizer is not needed because the extrinsic LLR is not exchanged between them.

Decoders D_1 and D_2 provide extrinsic LLRs of the uncoded bits $L^{b_1}_{e,D_1}$ and $L^{b_2}_{e,D_2}$, respectively, to performs the *VI* loop where the correlation knowledge is utilized. Decoders D_1 and D_2 also provide extrinsic LLRs of the coded bits L^{x}_{e,D_1} and L^{y}_{e,D_2}, respectively, via the *HI* loop to improve the ISM demapper performance by providing the updated LLRs.

III. ISM Demapper for Correlated Sources

A. ISM Demapper

As shown in Fig. 2, with the help of *a priori* information about s_2 provided by the decoder D_2 in the form of $L_{a_2,M}$, the demapper calculates the extrinsic LLR $L_{e_1,M}$ of the symbol s_1 from the received signal r by [4]

$$L_{e_1,M} = \ln \frac{\Pr(s_1 = +1|r)}{\Pr(s_1 = -1|r)} = \ln \frac{\sum_{\mathcal{S}\in\mathcal{S}_{+1}} \exp\left\{-\frac{|r - h_1 s_1 - h_2 s_2|^2}{\sigma_n^2} + b_2 L_{a_2,M}\right\}}{\sum_{\mathcal{S}\in\mathcal{S}_{-1}} \exp\left\{-\frac{|r - h_1 s_1 - h_2 s_2|^2}{\sigma_n^2} + b_2 L_{a_2,M}\right\}}, \quad (9)$$

where $\mathcal{S}_{+1}$, $\mathcal{S}_{-1}$ are the sets of superposition symbols in (7) having symbol s_1 being $+1$ and -1, respectively, with $s_1 = 1 - 2b_1$. Similarly, with the help of the *a priori* information about symbol s_1 provided by decoder D_1 in the form of $L_{a_1,M}$, the extrinsic LLR $L_{e_2,M}$ of symbol s_2 is updated.

The extrinsic LLRs, $L_{e_1,M}$ and $L_{e_2,M}$, are then de-doped-accumulated by D_{dacc}, de-interleaved by Π_1^{-1} and Π_2^{-1}, respectively, to provide *a priori* LLR, L^{x}_{a,D_1} and L^{y}_{a,D_2}, of the coded bits to the decoders D_1 and D_2.

B. Vertical Iterations and LLR Updates

Due to the correlation between the sources, the extrinsic LLR of the uncoded information bits, obtained as the result of the BCJR algorithm, has to be adjusted in the *VI* loop. As in [6], we use the following probability update for b_2:

$$\Pr(b_2 = 0) = (1 - p_e)\Pr(b_1 = 0) + p_e \Pr(b_1 = 1),$$
$$\Pr(b_2 = 1) = (1 - p_e)\Pr(b_1 = 1) + p_e \Pr(b_1 = 0). \quad (10)$$

The probability update for b_1 is performed in the same way as (10). The LLR updating function $\rho(\cdot)$ corresponding to (10) for b_2 is equivalent to

$$L^{b_2} = f_c(\rho, L^{b_1}) = \ln \frac{(1-\rho) + (1+\rho)e^{L^{b_1}}}{(1+\rho) + (1-\rho)e^{L^{b_1}}} \quad (11)$$

in the log-domain, where L^{b_1} and L^{b_2} are the extrinsic LLRs of b_1 and b_2, respectively. Similarly, the updating function for b_1, $L^{b_1} = f_c(p, L^{b_2})$, is obtained from (11) by replacing $L^{b_2}(L^{b_1})$ with $L^{b_1}(L^{b_2})$. In this paper, we assume that p_e is perfectly known to the receiver.[9]

IV. MAC and Slepian-Wolf Rate Region Analysis

With the definitions $P_1 = |h_1|^2$, $P_2 = |h_2|^2$, the power of the interference due to P_1 on to P_2 and P_2 on to P_1 can be calculated, respectively, by

$$I_{12} = P_1 \cos^2\theta \quad \text{and} \quad I_{21} = P_2 \cos^2\theta. \tag{12}$$

Finally, with the Gaussian code-book approximation, the average MAC rate region, given θ, P_1, and P_2 can be approximated by

$$\begin{aligned} \mathcal{R}_1 &\leq \int_0^{2\pi} \log_2\left(1 + \frac{P_1}{N + P_2\cos^2\theta}\right) p(\theta) d\theta, \\ \mathcal{R}_2 &\leq \int_0^{2\pi} \log_2\left(1 + \frac{P_2}{N + P_1\cos^2\theta}\right) p(\theta) d\theta, \\ \mathcal{R}_1 + \mathcal{R}_2 &\leq \frac{(A+B)}{2}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} A &= \int_0^{2\pi} \log_2\left\{\left(1+\frac{P_1}{N}\right)\left(1+\frac{P_2}{N+P_1\cos^2\theta}\right)\right\} p(\theta) d\theta, \\ B &= \int_0^{2\pi} \log_2\left\{\left(1+\frac{P_2}{N}\right)\left(1+\frac{P_1}{N+P_2\cos^2\theta}\right)\right\} p(\theta) d\theta, \end{aligned}$$

and $p(\theta)$ is the distribution of phase difference θ.

The Slepian-Wolf/Shannon theorem states that the condition to achieve arbitrarily low bit-error-rate (BER) for two correlated sources, b_1 and b_2, is given by (1). By using (13) we have

$$R_c \mathcal{H}(b_1, b_2) \leq \frac{(A+B)}{2}, \tag{14}$$

with R_c being the the channel coding rate. The total MAC capacity in (14) is maximum at $\theta = 90°$; it is minimum at $\theta = 0°$ to result in the capacity of multiple input single output (MISO) channel as

$$\begin{aligned} R_c \mathcal{H}(b_1, b_2) &\leq \frac{1}{2} \cdot 2\log_2\left(1 + \frac{P_1 + P_2}{N}\right) \\ &\leq \log_2(1 + \gamma_1 + \gamma_2). \end{aligned} \tag{15}$$

The outage capacity for each user can be calculated as

$$\mathcal{C}_{out} = \frac{1}{2}\mathcal{H}(b_1, b_2) = \frac{(A+B)}{4R_c}. \tag{16}$$

Since coding rate $R_{c_1} = R_{c_2} = R_c = 1/2$, the SW capacity limit for source correlation $\rho = 1.00$ is $\eta_{SW} = 0.5$ bit/channel use, while for $\rho = 0.00$ is $\eta_{SW} = 1$ bit/channel use.

[9]When p_e is unknown to the receiver, it can be estimated using the technique in [6]. However, the estimation quality can be improved by the use of *a posteriori* LLR as shown in [10].

V. Performance Evaluation

A series of computer simulation was conducted to verify the effectiveness of the proposed SW-ISM structure over fading MAC. Binary sequence b_1 is generated randomly with length of 10,000 bits[10] and was randomly flipped with probability of $p_e = \{0, 0.01, 0.1, 0.49\}$ to have source correlations $\rho = \{1.00, 0.98, 0.8, 0.02\}$, respectively. The length of interleaver Π_0 is also 10,000 bits.

The binary sequences b_1 and b_2 are then independently encoded by the same memory-2 rate 1/2, $R_{c_1} = R_{c_2} = 1/2$,[11] non-systematic non-recursive convolutional codes (NSNRCC) with a generator polynomial $G = [7, 5]_8$, resulting in two independent sequences x and y, each having a length of 20,000 bits. The sequences x and y are further independently interleaved by random interleaver Π_1 and Π_2, respectively,[12] and then doped accumulated by D-ACC with the doping rates $P = \{6, 12\}$. The value P should be selected such that the EXIT curve of the ISM demapper is matching with the EXIT curve of the joint decoders.

For the computational complexity consideration, in this paper we use notation $\alpha(H\beta V\eta)$ pattern, where each β *HI*s are followed by η *VI*s, and the whole process is repeated α times. The total iterations is $\alpha(\beta + \eta)$ for each source.[13] We show that in the next section the SW-ISM demapper with $10(H1V1)$ can achieve close enough performance to the Slepian-Wolf/Shannon limit.

A. BER Performance

The BER performance, evaluated in terms of average BER (averaged over fading realizations as well as the two sources) vs. average SNR γ_1, γ_2, is plotted in Fig. 4. The theoretical BER curves of maximum ratio combining (MRC) with the first and the second diversity orders, $M = 1$ and $M = 2$ [11], are also plotted for comparison. To obtain reliable results, the simulation was conducted with 50,000 frames each having 10,000 information bits results in total of 5×10^8 bits.

As shown in Fig. 4, BER performance of SW-ISM with correlation of $\rho = 0.02$ is closer to the BER curve of theoretical MRC with first order diversity. However, when the correlation of $\rho = 0.98$, the performance is similar to the second order diversity at lower SNR up to SNR = 11.2 dB, before it turns to follow the first order diversity's curve tendency.[14]

The agreement of SW-ISM in Fig. 4 with the first order diversity theoretical curve indicates that the proposed system works even though with a very weak source correlation. However, the improvement of the *VI* loop is very significant when the correlation is high. The improvement given by strong

[10]This length is assumed to be enough to simulate the number of flipped bits e, especially for $p_e = 0.01$ because $0.01 \times 10,000 = 100$ flipped bits is enough to make the difference with $p_e = 0$.

[11]These coding rates are unchanged by the D-ACC since $R_{dacc} = 1$.

[12]The length of interleavers Π_1 and Π_2 is, therefore, also 20,000 bits.

[13]The performance may be improved when computational complexity is not a concern so that we can perform as many iterations as we wish.

[14]This is clear since the diversity order is integer, while the correlation $\rho = 0.98$ does not mean that the two sources are identical. Therefore, the second order diversity can not be achieved for $\rho \neq 1.00$.

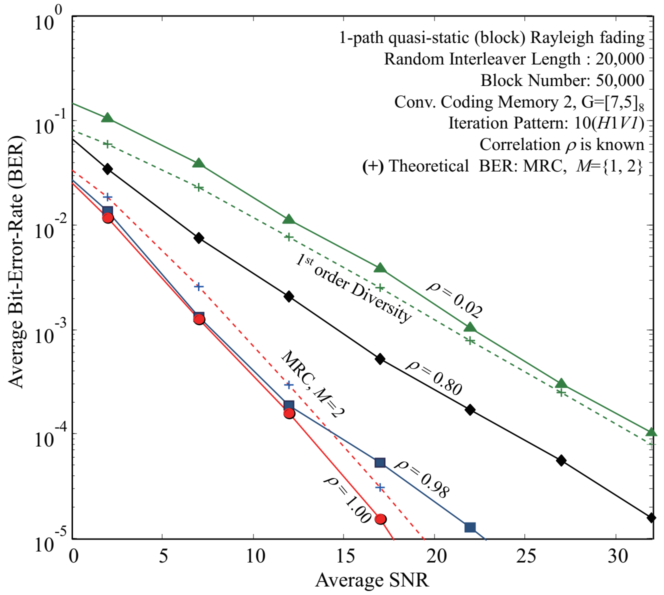

Fig. 4. BER performance of SW-ISM structure over frequency-flat Rayleigh fading channels

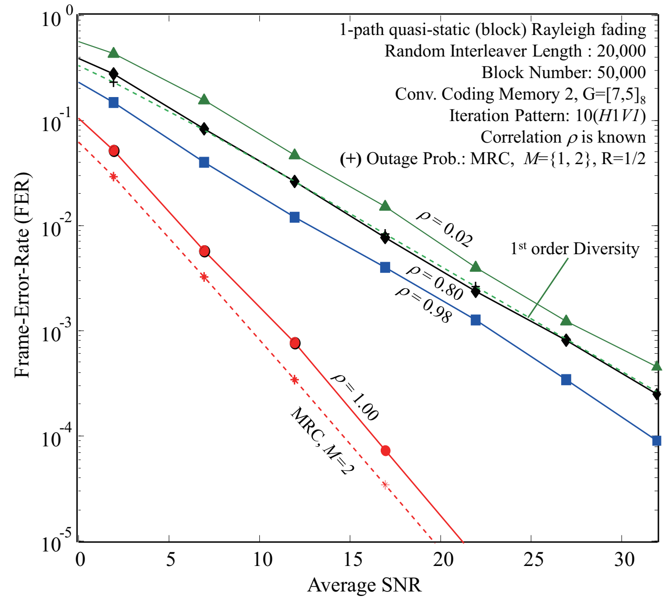

Fig. 5. FER performance of SW-ISM structure over frequency-flat Rayleigh fading channels

correlation such as with $\rho = \{0.98, 1.00\}$ is about 19.71 dB at BER= 2×10^{-4} over MRC without *VI* loop. The coding gain of SW-ISM gradually decreases as the correlation is decreased. With $\rho = 0.80$, the improvement is 10.154 dB.

B. FER Performance

Fig. 5 shows the frame-error-rate (FER) performance vs. average SNR γ_1, γ_2 of the proposed SW-ISM structure with the same parameters as in Fig. 4. The FER results shown in Fig. 5 support the results presented in Fig. 4, and also consistent to the theoretical outage curves of MRC with the first and the second order diversity. The theoretical outage probability of MRC is plotted from [11] where the threshold SNR γ_0 is the theoretical SNR of the Shannon limit.

In this paper, our decoder is assisted by the D-ACC that provide clear turbo-cliff with very low error-floor, and hence, our proposed ISM demapper's FER performance is almost equivalent to to the outage probability P_{out}, i.e.,

$$\mathrm{FER} = P_{out} = \Pr(\mathcal{C}_{out} < \eta_{SW}), \tag{17}$$

with $\mathcal{C}_{out}$ being the outage capacity shown in (16). It is also confirmed from Fig. 5 that the FER performance curves obtained by the computer simulation agree with the theoretical outage probability of MRC.

VI. Conclusions

This paper investigated in frequency-flat fading MAC the proposed simple coding structure, SW-ISM structure, with single transmission phase where the receiver has only one antenna. The proposed SW-ISM works even with very simple decoder (memory-2 convolutional code). The doped accumulator is used to both flexibly adapt for the correlation property and to reduce the error-floor. The improvement achieved by the proposed SW-ISM structure by exploiting the source correlation is significant, up to 19.1 dB gain, from the performance of the almost uncorrelated sources. *The structure has potential applications in relaying systems that allow errors between the source-relay links and/or sensor networks with only single phase transmission.*

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