Title	Fast Convergent Gait Generation for Underactuated Spoked Walker with Torso
Author(s)	Xiao, Xuan
Citation	
Issue Date	2012-09
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/10762
Rights	
Description	Supervisor:Fumihiko Asano, 情報科学研究科, 修士

Fast Convergent Gait Generation for Underactuated Spoked Walker with Torso

By Xuan XIAO

A thesis submitted to
School of Information Science,
Japan Advanced Institute of Science and Technology,
in partial fulfillment of the requirements
for the degree of
Master of Information Science
Graduate Program in Information Science

Written under the direction of
Associate Professor Fumihiko Asano

September, 2012

Fast Convergent Gait Generation for Underactuated Spoked Walker with Torso

By Xuan XIAO (1010231)

A thesis submitted to
School of Information Science,
Japan Advanced Institute of Science and Technology,
in partial fulfillment of the requirements
for the degree of
Master of Information Science
Graduate Program in Information Science

Written under the direction of
Associate Professor Fumihiko Asano

and approved by
Associate Professor Fumihiko Asano
Professor Chong Nak-Young
Associate Professor Ryo Maezono

August, 2012 (Submitted)

Contents

List of Figures

List of Tables

Keywords: Limit cycle walking, Rimless wheel, Deadbeat control, Gait generation, Uneven terrain.

Abstract

A rimless spoked wheel or simply rimless wheel (RW) is investigated as the most simplified model of passive dynamic walking. In McGeer's early study on passive-dynamicwalkers, the stability of a RW was analyzed based on the return map. It was also clarified that a passive-dynamic gait of a RW is always one-periodand asymptotically stable because it automatically achieves both constraint conditions for restored mechanical energy and impact posture. The constraint on impact posture is necessary to make the energy-loss coefficient constant. The restored mechanical energy is always kept constant because the step length is constant. It is then shown that the kinetic energy immediately before impact converges to the steady value. The generated passive-dynamic gait is obviously natural and energy-efficient.

On the other hand, applications of inherent passive dynamics to efficient active walking on level ground have been actively investigated. Active dynamic walkers with small actuators are called limit cycle walkers in distinction from passive dynamic walkers, and they can walk on level ground efficiently exploiting their natural dynamics. During the last decade, many energy-efficient walkers have been developed and nowadays they are familiar in the field of robotic bipedal locomotion. The control design, however, has been a continuous process of trial and error and the limit cycle stability remains unclear.

Based on the previous research, fortunately, it has been clarified that a fast convergent gait can be generated by applying a simple output following control to a desired-time trajectory, and that a deadbeat gait can be generated by modifying the system parameters. The deadbeat gait is a walking gait whose discrete state error converges to zero only in one step. It can be said that the deadbeat gait is the optimal one in terms of the convergence speed. Such a fast convergent gait has a tremendous advantage in limit cycle walking on uneven terrain.

Based on the observations, I propose a novel approach: continuous-time output deadbeat control (CODC) based on the discrete-time output deadbeat control (DODC). I introduce the model of an underactuated spoked walker with a torso that can exert the joint torque between the stance leg and torso. The walker can walk forward by controlling the torso and using the reaction moment. In this thesis, first, I propose CODC in which the control input is defined as a linear function of time. By applying this method, the control output which is defined as the relative angle between the stance leg and torso is settled to zero and the mechanical energy lost at impact is successfully restored during the stance phases. Through analysis, I find that a deadbeat gait can be generated by changing the control interval or desired settling time. Second, I investigate the properties of the generated gait and compare the gait efficiency with that generated by DODC through numerical simulations to prove the advantages of CODC. At last, I show some advantages in the gait generated by CODC on uneven terrain and discuss the problems.

Chapter 1

Introduction

1.1 Background

A rimless spoked wheel or simply rimless wheel (RW) is investigated as the most simplified model of passive dynamic walking. In McGeer's early study [1] on passive-dynamic walkers, the stability of a RW has been analyzed based on the return map as a simplified walking. It was clarified that a passive-dynamic gait of a RW is always one-period and asymptotically stable because it automatically achieves both constraint conditions for restored mechanical energy and impact posture [3]. The constraint on impact posture is necessary to make the energy loss coefficient constant. The restored mechanical energy is always kept constant because the step length is constant. It is then shown that the kinetic energy immediately before impact converges the steady value. The rimless wheel captures some essential mechanical features of human walking such as foot collision, falling-and-catching and inverted pendulum behaviour of walking.

On the other hand, applications of inherent passive dynamics to active walking on level ground have been actively investigated. Active dynamic walkers with small actuators are called limit cycle walkers [2]in distinction from passive dynamic walkers and they are nowadays familiar. During the last decade, many energy-efficient walkers have been developed [4] [6]. The control design, however, has been a continuous process of trial and error and the limit cycle stability remains unclear. Making the model walk stably is always what we want to achieve. As we expect, the deadbeat gait is the optimal one in terms of the convergence speed. Such a fast convergent gait has a tremendous advantage in limit cycle walking on uneven terrain. The deadbeat gait is a walking gait whose discrete state error converges to zero only in one step. Searchers have developed a lot of methods how to achieve fast convergent gait generation such as swing leg retraction helps walking stability for biped walker and so on. On the other hand, based on Evolutionary Algorithms and Neural Networks, learning vector quantization also has been proved to be highly robust in the training process yielding a remarkably high recognition accuracy of gait patterns. But we hope to achieve fast fast convergent gait generation by simple control methods. Fortunately, based on the previous research, it has been clarified that a fast convergent gait can be generated by applying a simple output following control to a desired-time trajectory, and that a deadbeat gait can be generated by modifying the system parameters. Recently Coleman et al have proposed a method for deriving the transition functions for the state error in passive dynamic walking of a rimless wheel. Based on this method we develop discrete-time output deadbeat control. As a big gait, we could prove how to achieve fast convergent gait generation mathematically.

1.1.1 Discrete-time output deadbeat control

Similar with desired-time trajectory control, we set a simple discrete function instead of the complex one as the output control $v(t)$ as Fig 1.1. On the other hand we set

$$y := \theta_1 - \theta_2 - \frac{\alpha}{2} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{\theta} - \frac{\alpha}{2} \tag{1.1}$$

for achieving $\ddot{y} = v(t)$. With the simple function, we can get the transition functions for the state error after some calculation. At last we could prove how to achieve fast convergent gait generation mathematically. I will explain the process of calculation in Chapter 3 and 4 by continuous-time output deadbeat control in details. However, there is a problem of discrete-time deadbeat control that a discrete control is not a good idea because a sudden changing of torque is difficult to control. I hope to develop some continuous and more energy-efficient control.

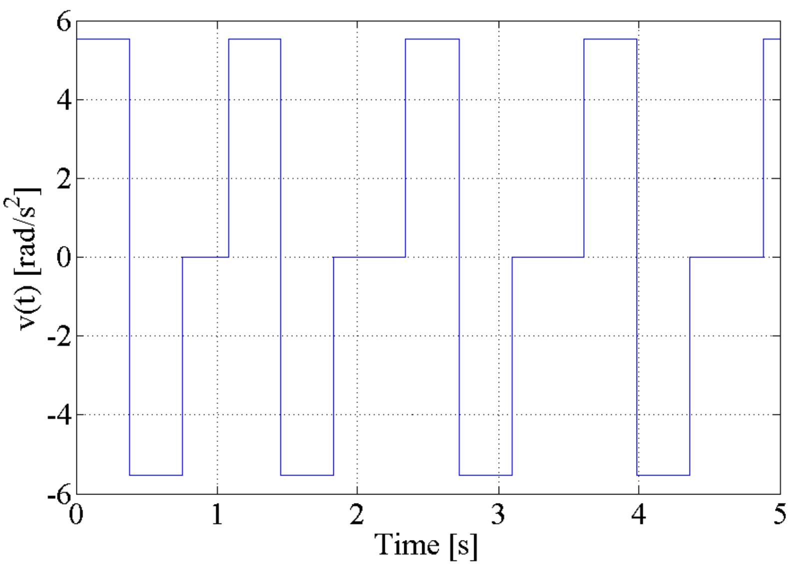


Figure 1.1: Time-evolution of $v(t)$ by DODC

1.2 Thesis Goal

This thesis is a research on the continuous-time deadbeat control of the underactuated spoked walker with torso. The main goal is to develop a simple and continuous control to achieve the fast convergent gait generation. I hope I can mathematically prove $\dot{\theta}'_{\mathrm{eq}}$ decreases with the increase of T_{set} in my control as the same as the discrete-time deadbeat control, which means I can find the deadbeat gait generation mathematically.

The second goal is to make the comparison between the continuous-time deadbeat control and the discrete-time deadbeat control. I want to prove that the continuous-time deadbeat control is more efficient than the discrete-time deadbeat control. I will also make the comparison on the walking speed and the angular velocity. I hope to find some more advantages of continuous-time deadbeat control.

At last I want to show the performs of going upstairs and downstairs by deadbeat gait control. We will try to develop a better adaptability such as abilities of overcoming the upstairs and going downstairs stably to get a larger improvement.

1.3 Structure of Thesis

This thesis has six chapters totally. The second chapter describes the model and the basic equation of motion. The third and fourth chapters show us the control synthesis and the prove of the deadbeat gait generation mathematically and the results of simulations. The fifth chapter describes the comparisons between the continuous-time deadbeat control and the previous control. The simulation of going upstairs and downstairs will also be shown in this chapter. In the last chapter I will make the conclusions and discuss some future work.

Chapter 2

Modeling and Analysis

As the generated gait is natural and energy efficient, we choose the rimless wheel as the walking model. Instead of walking on the slope, we add a torso on the rimless wheel to make the model walk on the ground level. This chapter will introduce the underactuated spoked walker with torso and show us some basic equations of motion.

2.1 Underactuated Rimless Wheel with Torso

Fig. 2.1 shows the model of a planar underactuated rimless wheel with a torso. As we see the model consists of a rimless wheel whose mass is m_1 [kg] and a torso whose mass is m_2 [kg]. We assume that this can exert a joint torque between the stance leg and torso to make the model walk stable. We also assume that the model has eight leg frames and the relative angle between neighboring frames, α, is $\pi/4$ [rad]. The CoM positions of both frames are at the joint.

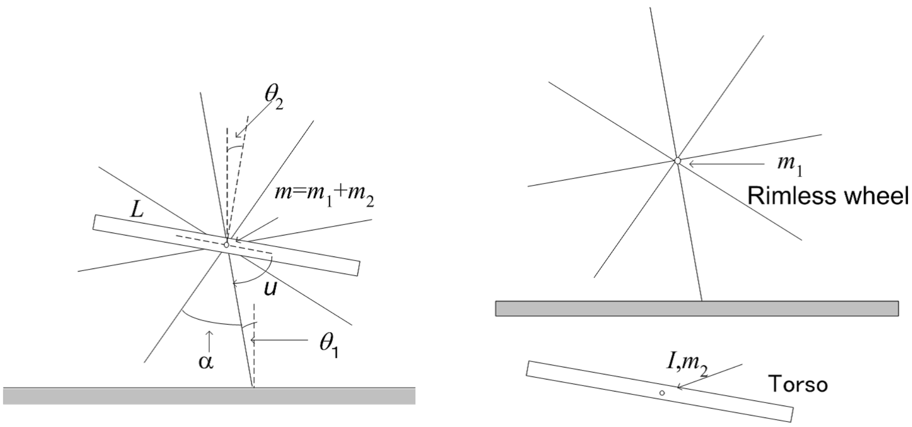


Figure 2.1: Modeling and analysis

Table 2.1: Parameters of model

m_1	Mass of rimless wheel	[kg]
m_2	Mass of torso	[kg]
m	Mass of model	[kg]
L	Length of legs	[m]
α	Angular between two legs	[rad]

2.2 Equation of Motion and Linearization

2.2.1 Dynamic equation

I choose $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^{\mathrm{T}}$ as the the generalized coordinate vector. The equation of motion then becomes

$$\boldsymbol{M}_0\ddot{\boldsymbol{\theta}} + \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{S}u, \tag{2.1}$$

where

$$\boldsymbol{M}_0 = \begin{bmatrix} ml^2 & 0 \\ 0 & I \end{bmatrix}, \quad \boldsymbol{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} -mgl\sin\theta_1 \\ 0 \end{bmatrix}, \boldsymbol{S} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \tag{2.2}$$

Here, $m := m_1 + m_2$ [kg] is the total mass of the robot, I [kg·m^2] is the inertia moment around the CoM, l [m] is the leg length of the radius of the rimless wheel, g [kg/s^2] is the acceleration of gravity.

2.2.2 Collision equations

The transition equation for the angular positions is defined as

$$\boldsymbol{\theta}^+ = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{\theta}^-, \tag{2.3}$$

where the superscripts "$-$" and $+$ denote immediately before and immediately after impact. Here, the angular positions at impact satisfy

$$\theta_1^- = -\theta_1^+ = \frac{\alpha}{2}, \quad \theta_2^- = \theta_2^+.$$

We assume that $\dot{\theta}_1^- = \dot{\theta}_2^-$ is achieved immediately before impact in every step and that the torso is mechanically locked to the rimless wheel (body frame). The transition equation for the angular velocities then becomes

$$\dot{\boldsymbol{\theta}}^+ = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{ml^2\cos\alpha + I}{ml^2 + I}\dot{\theta}_1^-. \tag{2.4}$$

2.2.3 Iuput-output linearization

By linearizing Eq (2.1) around $\theta_1 = \dot{\theta}_1 = 0$, we get

$$\boldsymbol{M}_0\ddot{\boldsymbol{\theta}} + \boldsymbol{G}_0\boldsymbol{\theta} = \boldsymbol{S}u, \tag{2.5}$$

where

$$\boldsymbol{M}_0 = \begin{bmatrix} ml^2 & 0 \\ 0 & I \end{bmatrix}, \quad \boldsymbol{G}_0 = \begin{bmatrix} -mgl & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

On the other hand, I choose

$$y := \theta_1 - \theta_2 - \frac{\alpha}{2} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{\theta} - \frac{\alpha}{2} \tag{2.6}$$

as the control output. The second order derivative of y with respect to time becomes:

$$\ddot{y} = \boldsymbol{S}^{\mathrm{T}}\ddot{\boldsymbol{\theta}} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\left(\boldsymbol{S}u - \boldsymbol{G}_0\boldsymbol{\theta}\right) = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{S}u - \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{G}_0\boldsymbol{\theta} \tag{2.7}$$

Then I can consider the following control input $v(t)$ for achieving $\ddot{y} = v(t)$.

$$u = \frac{v(t) + \boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{G}_0\boldsymbol{\theta}}{\boldsymbol{S}^{\mathrm{T}}\boldsymbol{M}_0^{-1}\boldsymbol{S}} = \frac{ml^2 I}{ml^2 + I}\left(v(t) - \frac{g}{l}\theta_1\right) \tag{2.8}$$

This implies that only θ_1 is necessary to control the relative angle or control output.

In the subsequent sections, I design an output following of deadbeat control to the linearized model.

Chapter 3

Control Synthesis

The problem I am facing now is how to design $v(t)$. I have tried the the desired-time trajectory tracking control in the mid-term presentation and also tried the discrete-time deadbeat control recently. I can mathematically show that I can get the deadbeat gait generation by DODC. I will explain it in detail in the next chapter. Now I hope to develop a continuous-time deadbeat control to make the $v(t)$ continuous. The continuous-time control will be very similar with the discrete-time deadbeat control. I think it will have the same property as the discrete-time deadbeat control.

3.1 Continuous-time Output Deadbeat Control

First of all, I try to design $v(t)$ as a linear function:

$$v(t) = \begin{cases} v_0 \left(\frac{T_{\mathrm{set}}}{2} - t \right) & (0 \leq t < T_{\mathrm{set}}) \\ 0 & (t \geq T_{\mathrm{set}}) \end{cases} \tag{3.1}$$

T_{set} is the time period of control input which we can control. The step period must be longer than T_{set} so that the control input can be finished in one step. When the step period is longer than T_{set}, the settling-time condition is satisfied and then the model falls down as a 1-DOF rigid body. Now our problem is how to get the value of v_0. Then I design the equation as:

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \tag{3.2}$$

I can denote Eq (3.2) as:

$$\dot{\boldsymbol{Y}} = \hat{\boldsymbol{A}}\boldsymbol{Y} + \hat{\boldsymbol{B}}v(t); \tag{3.3}$$

The details of the matrices are :

$$\boldsymbol{Y} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}, \hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \hat{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{3.4}$$

Now we need some basic definitions to describe the state vectors.

3.1.1 Basic definitions

1. Let i be the step number.
2. The model starts walking from the impact posture; this is defined as the 0-th collisions and steps are contextually counted.
3. The motion between the 0-th and 1-st collision is called the "first step". The subsequent collision and steps are contextually counted.
4. Let t_i [s] be the absolute time of the i-th collision. The i-th step period is defined as $T_i = t_{i+1} - t_i$ [s].
5. The state vectors just before and just after impact, $\boldsymbol{x}(t_i^-)$ and $\boldsymbol{x}(t_i^+)$ are simply denoted as $\boldsymbol{x}_i^-$ and $\boldsymbol{x}_i^+$.

3.1.2 Solution of input control

As the linear differential equation, we can get the the general solution as :

$$\boldsymbol{Y}_{i+1} = e^{\hat{\boldsymbol{A}}t}\boldsymbol{Y}_i + \int_0^t e^{\hat{\boldsymbol{A}}(t-s)}\hat{\boldsymbol{B}}v(s)\mathrm{d}s \tag{3.5}$$

When $t > T_{\mathrm{set}}$, $v(t) = 0$. I can simplify the Eq (3.4) as :

$$\boldsymbol{Y}_{i+1} = e^{\hat{\boldsymbol{A}}T_i}\left(\boldsymbol{Y}_i + \int_0^{T_{\mathrm{set}}} e^{-\hat{\boldsymbol{A}}s}\hat{\boldsymbol{B}}v(s)\mathrm{d}s\right) \tag{3.6}$$

Since we have the details about the equation above, we can calculate the $e^{\hat{\boldsymbol{A}}T_i}$ and $\int_0^{T_{\mathrm{set}}} e^{-\hat{\boldsymbol{A}}s}\hat{\boldsymbol{B}}v(s)\mathrm{d}s$ by mathematics software :

$$e^{\hat{\boldsymbol{A}}T_i} = \begin{bmatrix} 1 & T_i \\ 0 & 1 \end{bmatrix}, \int_0^{T_{\mathrm{set}}} e^{-\hat{\boldsymbol{A}}s}\hat{\boldsymbol{B}}v(s)\mathrm{d}s = \begin{bmatrix} \dfrac{vT_{\mathrm{set}}^3}{12} \\ 0 \end{bmatrix}. \tag{3.7}$$

With the boundary conditions when $T = 0$ and $T = T_{\text{set}}$ below:

$$\begin{bmatrix} y_i^+ \\ \dot{y}_i^+ \end{bmatrix} = \begin{bmatrix} -\dfrac{\alpha}{2} \\ 0 \end{bmatrix},$$

and

$$\begin{bmatrix} y_{i+1}^- \\ \dot{y}_{i+1}^- \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha}{2} \\ 0 \end{bmatrix}$$

We can get Eq (3.8) at last:

$$\begin{bmatrix} \dfrac{\alpha}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & T_i \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -\dfrac{\alpha}{2} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{vT_{\text{set}}^3}{12} \\ 0 \end{bmatrix} \right) \tag{3.8}$$

Finally, we get the value of v_0 as :

$$v_0 = \frac{12\alpha}{T_{\text{set}}^3}$$

3.2 Simulation Results

Now I have achieved the $v(t)$ of continuous-time control:

$$v(t) = \begin{cases} \dfrac{12\alpha}{T_{\text{set}}^3} \left(\dfrac{T_{\text{set}}}{2} - \text{t} \right) & (0 \leq t < T_{\text{set}}) \\ 0 & (t \geq T_{\text{set}}) \end{cases} \tag{3.9}$$

I can get some simulations of CODC by MATLAB. The parameters are as the tables below. Then I will show you some basic figures when $T_{\text{set}} = 0.7$ [s].

Table 3.1: Parameters of model

m_1	m_2	L	I_0	α
1.0	1.0	1.0	1.0	$\frac{\pi}{4}$
$[kg]$	$[kg]$	$[m]$	$[kgm^2]$	$[rad]$

Table 3.2: Parameters of initial condition

θ_1	θ_2	$\dot{\theta}_1$	$\dot{\theta}_2$	T_{set}
$-\dfrac{\alpha}{2}$	0	0.85	0.85	0.7
[rad]	[rad]	[rad/s]	[rad/s]	[s]

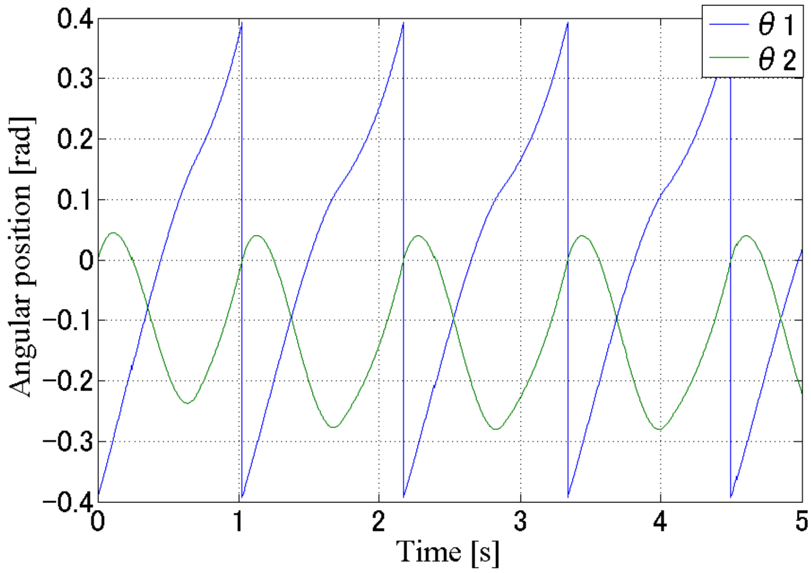


Figure 3.1: Time-evolution of angular positions in level gait generated by CODC where $T_{\text{set}} = 0.7$ [s]

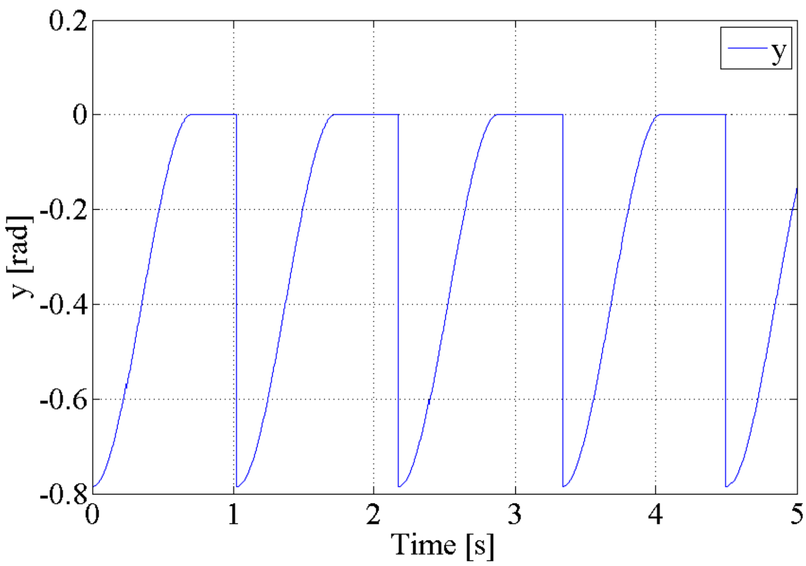


Figure 3.2: Time-evolution of y in level gait generated by CODC where $T_{\text{set}} = 0.7$ [s]

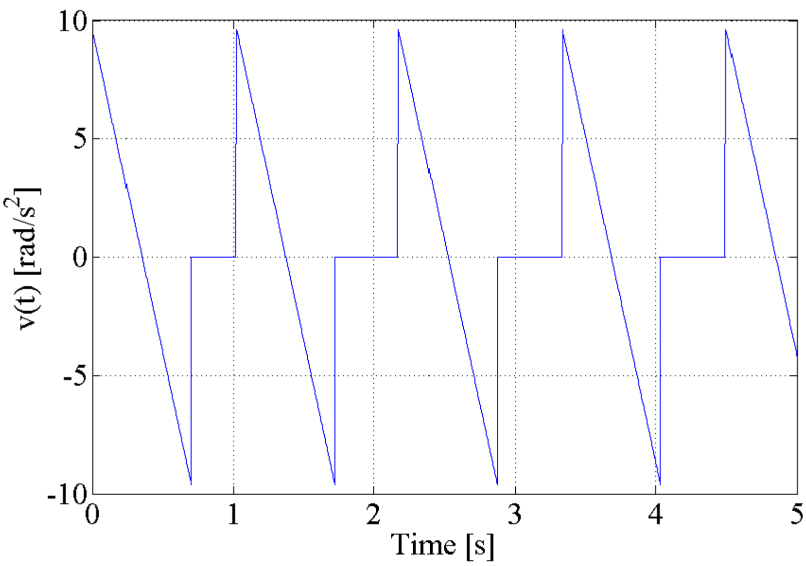


Figure 3.3: Time-evolution of $v(t)$ in level gait generated by CODC where $T_{\mathrm{set}} = 0.7$ [s]

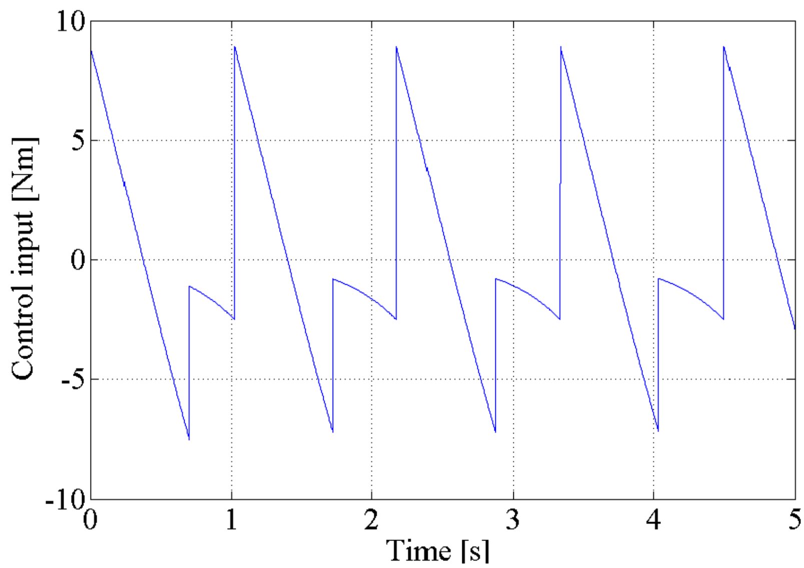


Figure 3.4: Time-evolution of u in level gait generated by CODC where $T_{\mathrm{set}} = 0.7$ [s]

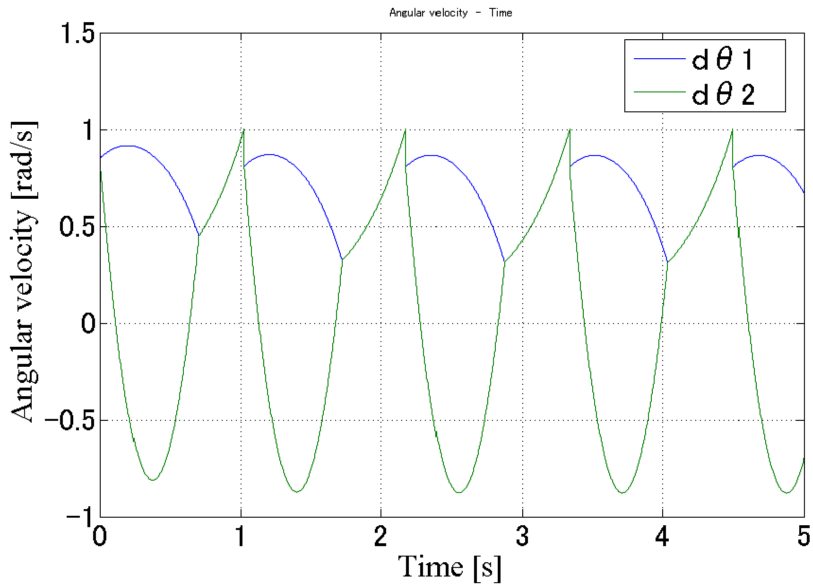


Figure 3.5: Time-evolution of angular velocity in level gait generated by CODC where $T_{\mathrm{set}} = 0.7$ [s]

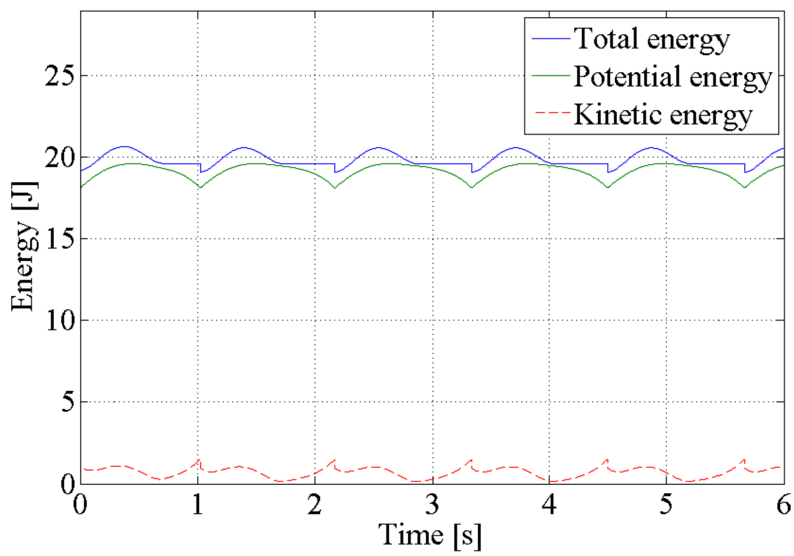
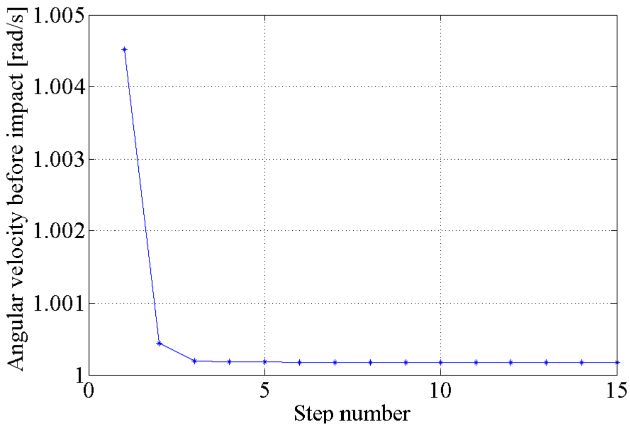
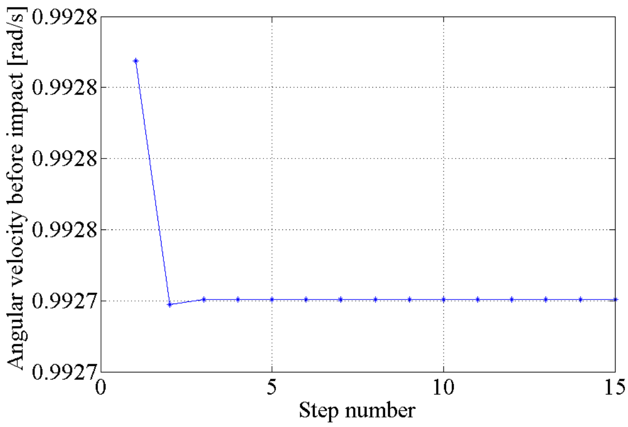


Figure 3.6: Time evolution of energies in level gait generated by CODC where $T_{\mathrm{set}} = 0.7$ [s]



(a) Velocity immediately before impact where $T_{\text{set}} = 0.7$ [s]



(b) Velocity immediately before impact where $T_{\text{set}} = 0.76$ [s]

Figure 3.7: T_{set} between 0.7 [s] and 0.76 [s] to get deadbeat gait generation.

Chapter 4

Deadbeat Gait Generation

Now I will try to mathematically prove that a deadbeat gait can be generated by the continuous-time deadbeat control. The section 4.1 is based on a new thesis of F.Asano [8]. I will prove a deadbeat gait can be generated by CODC in the rest section. At last of this chapter I will show the simulations of deadbeat gait generation by the continuous-time deadbeat control.

4.1 Analysis of Deadbeat Gait Generation

4.1.1 Linearization of system

Following Eq (2.3) and Eq (2.6):

$$ml^2\ddot{\theta}_1 = mgl\theta_1 + u, \tag{4.1}$$

$$u = \frac{ml^2 I}{ml^2 + I}\left(v(t) - \frac{g}{l}\theta_1\right) \tag{4.2}$$

I put the Eq (4.1) and Eq (4.2) together and then I can get a new equation:

$$ml^2\ddot{\theta}_1 = mgl\theta_1 + \frac{ml^2 I}{ml^2 + I}\left(v(t) - \frac{g}{l}\theta_1\right), \tag{4.3}$$

After the simplification, I can get :

$$ml^2\ddot{\theta}_1 = \left(ml^2 - \frac{ml^2 Ig}{(ml^2 + I)l}\right)\theta_1 + \frac{ml^2 I}{ml^2 + I}v(t) \tag{4.4}$$

Then, the linear equation becomes :

$$\ddot{\theta}_1 = \frac{mgl^2}{(ml^2+I)l}\theta_1 + \frac{I}{ml^2+I}v(t) \tag{4.5}$$

The state space realization then becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix} = \begin{bmatrix}0 & 1\\ \dfrac{mgl^2}{(ml^2+I)l} & 0\end{bmatrix}\begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{I}{ml^2+I}\end{bmatrix}v(t). \tag{4.6}$$

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}v(t), \tag{4.7}$$

Where :

$$\boldsymbol{X} = \begin{bmatrix}\theta_1\\ \dot{\theta}_1\end{bmatrix}, \boldsymbol{A} = \begin{bmatrix}0 & 1\\ \dfrac{ml^2g}{(ml^2+I)l} & 0\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix}0\\ \dfrac{I}{ml^2+I}\end{bmatrix},$$

4.1.2 Derivation of state error system

Based on the equation above, I can get the general solution of state vector immediately before the $(i+1)$-th impact, $\boldsymbol{x}_{i+1}^-$ as:

$$\boldsymbol{X}_{i+1}^- = e^{\boldsymbol{A}T_i}\boldsymbol{X}_i^+ + \int_0^{T_i} e^{\boldsymbol{A}(T_i-s)}\boldsymbol{B}v(s)\mathrm{d}s \tag{4.8}$$

When $t > T_{\mathrm{set}}$, $v(t) = 0$. I can simplify Eq (4.8) as :

$$\boldsymbol{X}_{i+1}^- = e^{\boldsymbol{A}T_i}\left(\boldsymbol{X}_i^+ + \int_0^{T_{\mathrm{set}}} e^{-\boldsymbol{A}s}\boldsymbol{B}v(s)\mathrm{d}s\right) \tag{4.9}$$

Here I define $\boldsymbol{\eta}$ as:

$$\boldsymbol{\eta} = \int_0^{T_{\mathrm{set}}} e^{-\boldsymbol{A}s}\boldsymbol{B}v(t)\mathrm{d}s \tag{4.10}$$

Then I put Eq (4.10) into Eq (4.9) to get:

$$\boldsymbol{X}_{i+1}^{-} = e^{\boldsymbol{A}T_i}\left(\boldsymbol{X}_i^{+} + \boldsymbol{\eta}\right). \tag{4.11}$$

In a steady gait, Eq (4.11) should satisfy :

$$\boldsymbol{X}_{\text{eq}}^{-} = e^{\boldsymbol{A}T^{*}}(\boldsymbol{X}_{\text{eq}}^{+} + \boldsymbol{\eta}). \tag{4.12}$$

Now I define the state error vector just before the $i-th$ impact as:

$$\Delta\boldsymbol{X}_i^{-} = \boldsymbol{X}_i^{-} - \boldsymbol{X}_{\text{eq}}^{-}$$

Similarly we define :

$$\Delta T_i = T_i - T^{*}.$$

Eq (4.11) is then written as :

$$\boldsymbol{X}_{i+1}^{-} = e^{\boldsymbol{A}(T^{*}+\Delta T)}(\boldsymbol{X}_{\text{eq}}^{+} + \boldsymbol{\eta} + \Delta\boldsymbol{X}_i^{+}) \tag{4.13}$$

After the simplification Eq (4.13) becomes:

$$\boldsymbol{X}_{i+1}^{-} = e^{\boldsymbol{A}\Delta T}(\boldsymbol{X}_{\text{eq}}^{-} + \Delta\boldsymbol{X}_i^{+} e^{\boldsymbol{A}T^{*}}) \tag{4.14}$$

Based on the approximation of :

$$e^{\boldsymbol{A}\Delta T_i} = \boldsymbol{I}_2 + A\Delta T_i$$

and I ignore the error terms higher than the second order, the Eq (4.14) is rearranged as :

$$\boldsymbol{X}_{i+1}^{-} = \boldsymbol{X}_{\text{eq}}^{-} + \boldsymbol{A}\boldsymbol{X}_{\text{eq}}^{-}\Delta T_i + e^{\boldsymbol{A}T^{*}}\Delta\boldsymbol{X}_i^{+} \tag{4.15}$$

According to the boundary conditions we have known:

$$\boldsymbol{X}_{i+1}^{-} = \begin{bmatrix} \frac{\alpha}{2} \\ \dot{\theta}_{i+1}^{-} \end{bmatrix}, \boldsymbol{X}_{\text{eq}}^{-} = \begin{bmatrix} \frac{\alpha}{2} \\ \dot{\theta}_{\text{eq}}^{-} \end{bmatrix} \tag{4.16}$$

Here, I define $\boldsymbol{p} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and I multiply Eq (4.15) by $\boldsymbol{p}$, I can get the result as below:

$$\frac{\alpha}{2} = \frac{\alpha}{2} + \boldsymbol{p}A\boldsymbol{X}_{\mathrm{eq}}^{-}\Delta T_i + \boldsymbol{p}e^{\boldsymbol{A}T^*}\Delta\boldsymbol{X}_i^{+} \tag{4.17}$$

I can prove that $\boldsymbol{pAX}_{\mathrm{eq}}^{-} = \dot{\theta}_{\mathrm{eq}}^{-} \neq 0$. So in the end ΔT_i can be solved as :

$$\Delta T = -\frac{\boldsymbol{p}e^{\boldsymbol{A}T^*}\Delta\boldsymbol{X}_i^{+}}{\boldsymbol{pAX}_{eq}^{-}} \tag{4.18}$$

On the other hand, $\Delta\boldsymbol{X}_{i+1}^{-} = \boldsymbol{X}_{i+1}^{-} - \boldsymbol{X}_{\mathrm{eq}}^{-}$ Based on the Eqs (4.15) and (4.18), we can get :

$$\Delta\boldsymbol{X}_{i+1}^{-} = \boldsymbol{Q}\Delta\boldsymbol{X}_i^{-}, \boldsymbol{Q} = \left(\boldsymbol{I}_2 - \frac{\boldsymbol{AX}_{\mathrm{eq}}^{-}\boldsymbol{p}}{\boldsymbol{pAX}_{\mathrm{eq}}^{-}}\right)e^{\boldsymbol{A}T^*} \tag{4.19}$$

$\boldsymbol{Q}$ is the transition matrix. Since I only need to consider about the angular velocity, I define $\boldsymbol{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and at last I can get the equation of transition function, $\bar{Q}$:

$$\Delta\dot{\theta}_{1(i+1)}^{-} = \bar{\boldsymbol{Q}}\Delta\dot{\theta}_{1i}^{-},\ \bar{Q} = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{Q}v \tag{4.20}$$

4.2 Solution of Deadbeat Gait Generation

Now let us get the solution of the control. Based on the conditions I have known and some complex matrices I can get from some mathematics software, some details matrices of $\bar{Q}$ are as below:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ \hat{\omega}^2 & 0 \end{bmatrix}, (\hat{\omega} = \sqrt{\frac{ml^2 g}{(ml^2 + I)l}}) \tag{4.21}$$

$$e^{\boldsymbol{A}T^*} = \begin{bmatrix} \cosh(\hat{\omega}T^*) & \frac{\sinh(\hat{\omega}T^*)}{\hat{\omega}} \\ \hat{\omega}\sinh(\hat{\omega}T^*) & \cosh(\hat{\omega}T^*) \end{bmatrix} \tag{4.22}$$

We can get :

$$\bar{Q} = \cosh(\hat{\omega}T_i) - \frac{\hat{\omega}\theta_{1\mathrm{eq}}^{-}}{\dot{\theta}_{1\mathrm{eq}}^{-}}\sinh(\hat{\omega}T_i) \tag{4.23}$$

On the other hand, as we know,

$$\boldsymbol{X}_{\text{eq}}^{+} + \boldsymbol{\eta} = e^{-\boldsymbol{A}T^{*}} \boldsymbol{X}_{\text{eq}}^{-} \tag{4.24}$$

$$\begin{bmatrix} \theta_{1\text{eq}}^{+} \\ \dot{\theta}_{1\text{eq}}^{+} \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \cosh(\hat{\omega}T_i) & -\dfrac{\sinh(\hat{\omega}T_i)}{\hat{\omega}} \\ -\hat{\omega}\sinh(\hat{\omega}T_i) & \cosh(\hat{\omega}T_i) \end{bmatrix} \begin{bmatrix} \theta_{1\text{eq}}^{-} \\ \dot{\theta}_{1\text{eq}}^{-} \end{bmatrix}, \tag{4.25}$$

At last what I want to get is:

$$\bar{Q} = 0 \Leftrightarrow \dot{\theta}_{1\text{eq}}^{+} + \eta_2 = 0 \tag{4.26}$$

As we have defined,

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \int_0^{T_{\text{set}}} e^{-\boldsymbol{A}t} \boldsymbol{B} v(t) \mathrm{d}t \tag{4.27}$$

Here I only need to consider about the angular velocity. Based on the conditions I have known I can calculate the result as :

$$\eta_2 = -\frac{12\alpha I(2 - 2\cosh(\hat{\omega}T_{\text{set}}) + \hat{\omega}T_{\text{set}}\sinh(\hat{\omega}T_{\text{set}})}{2ml^2\omega^2 T_{\text{set}}^3}. \tag{4.28}$$

Where :

$$\hat{\omega} = \frac{l\sqrt{m}\omega}{\sqrt{I + ml^2}}, \omega = \sqrt{\frac{g}{l}}.$$

We also found that: $\dfrac{\partial \eta_2}{\partial T_{\text{set}}} < 0$, which means η_2 is decreasing as T_{set} increases. We get the figure of T_{set}-evolution of η_2 as below.

4.3 Simulation Results

It is complex to calculate the stable angular velocity just after the impact.We hope to get it by simulation. We can get the figures about stable angular velocity and T_{set} as Fig 4.2. We can also find that when $T_{\text{set}} = 0.742$ [s], $\bar{Q} = 0$. At last in the simulation when $T_{\text{set}} = 0.747$ [s], the model can get the deadbeat gait generation. we can get some basic figures of deadbeat gait generation from the simulations by MATLAB.

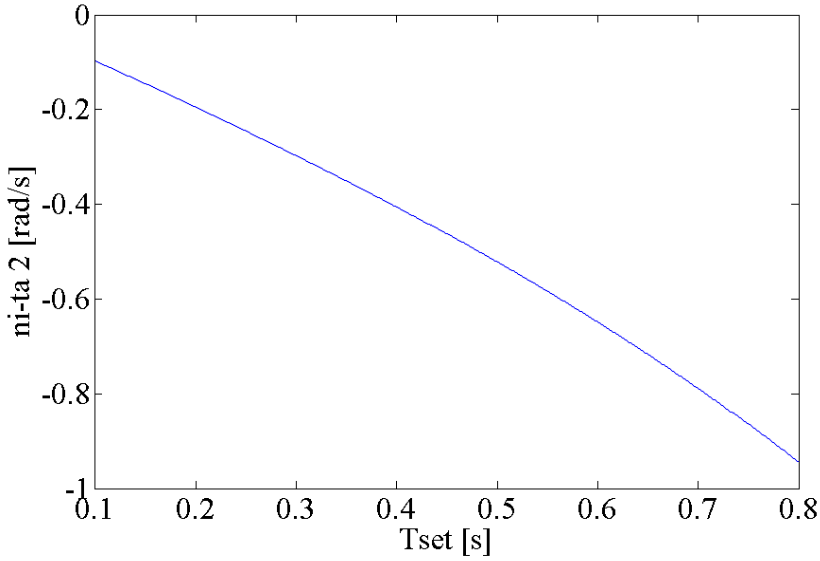


Figure 4.1: T_{set}-evolution of η_2

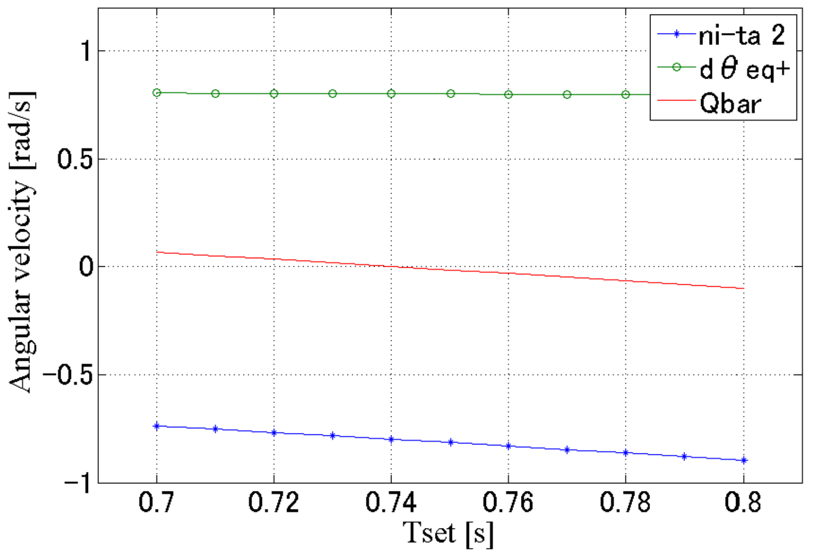


Figure 4.2: η_2, θ^{+}_{eq}, and $\bar{Q}$ with respect to T_{set}

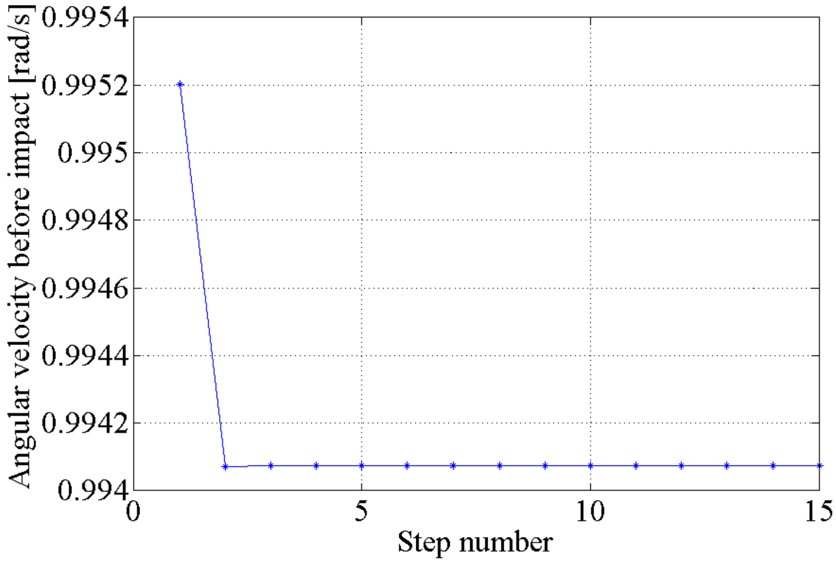


Figure 4.3: Velocity immediately after impact where $T_{\rm set} = 0.747$ [s]

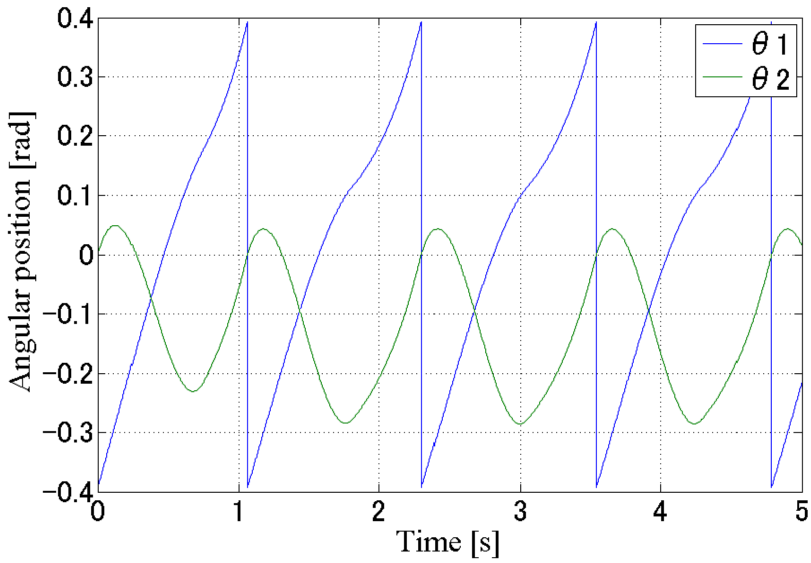


Figure 4.4: Time-evolution of angular positions when deadbeat gait generation

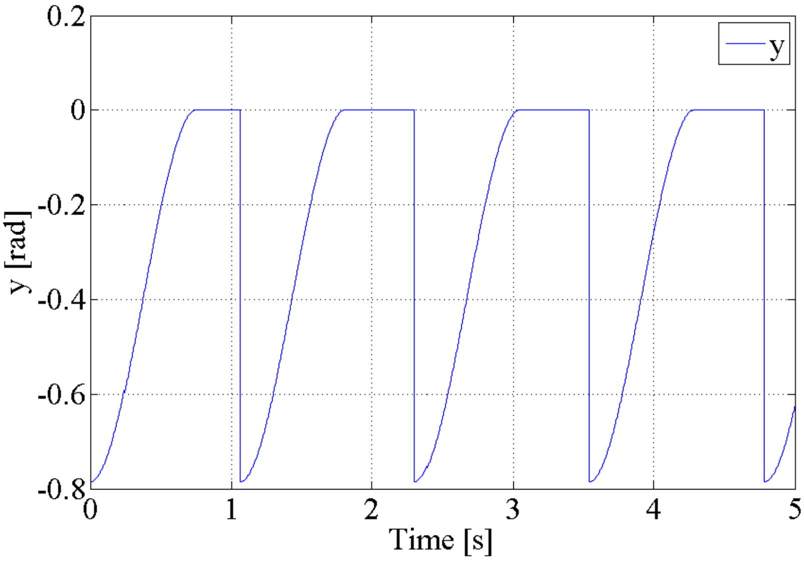


Figure 4.5: Time-evolution of y when deadbeat gait generation

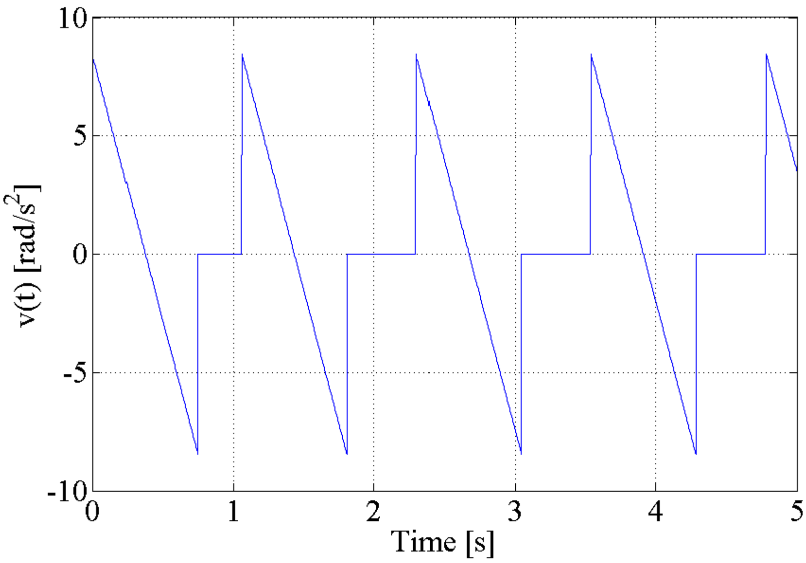


Figure 4.6: Time-evolution of $v(t)$ when deadbeat gait generation

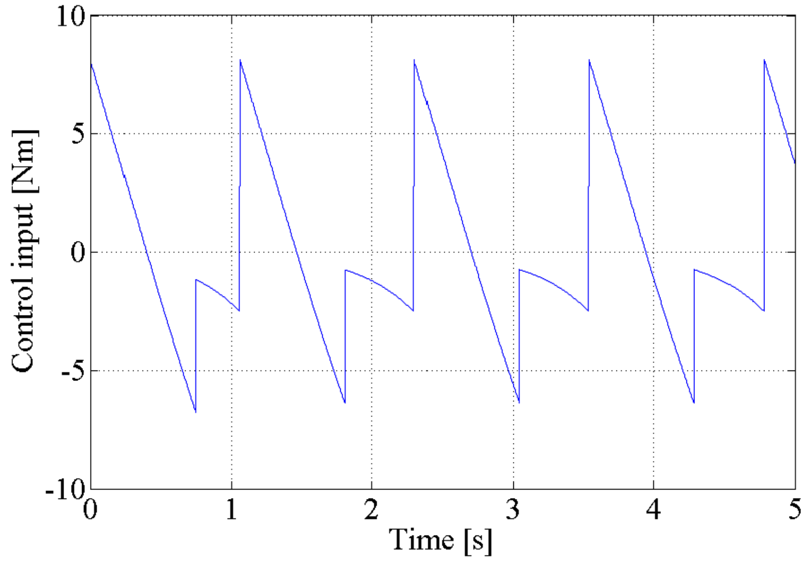


Figure 4.7: Time-evolution of u when deadbeat gait generation

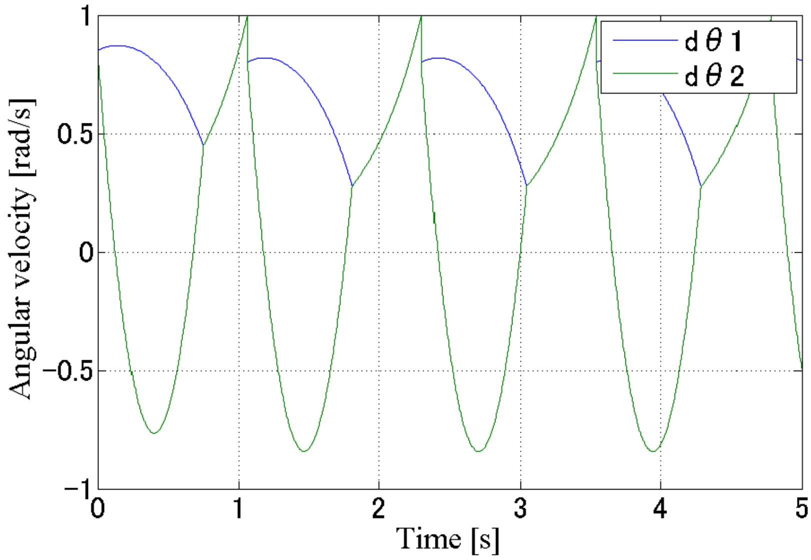


Figure 4.8: Time-evolution of angular velocity when deadbeat gait generation

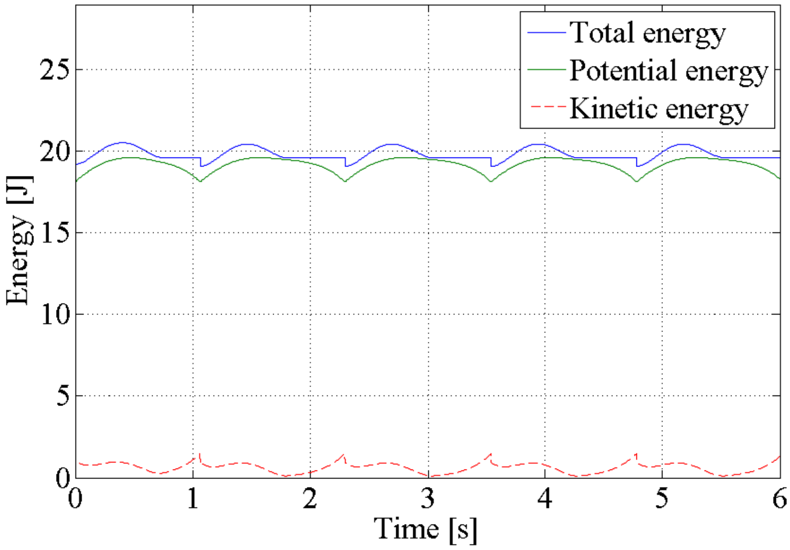


Figure 4.9: Time-evolution of energies when deadbeat gait generation

Chapter 5

Experiments and Evaluation

Until now I have learn three kinds of control methods. It has been mathematically proved that the discrete-time deadbeat control and the continuous-time deadbeat control can get the deadbeat gait generation. In this chapter first I will make the comparisons among these three control methods on some basic parameters. Then I will make the comparisons between the discrete-time deadbeat control and the continuous-time deadbeat control on some gait properties such as step period and energy efficiency. At last I will try some experiment about the model by continuous-time deadbeat control walking on the uneven ground.

5.1 Comparisons of Basic Figures

Now I have tried three kinds of control methods:
Desired-time trajectory control

$$v(t) = \begin{cases} \dfrac{20*6\alpha}{T_{\mathrm{set}}^5}t^3 - \dfrac{12*15\alpha}{T_{\mathrm{set}}^4}t^2 + \dfrac{6*10\alpha}{T_{\mathrm{set}}^3}t & (0 < t < T_{\mathrm{set}}) \\ 0 & (t > T_{\mathrm{set}}) \end{cases} \tag{5.1}$$

Discrete-time deadbeat control

$$v(t) = \begin{cases} \dfrac{4\alpha}{T_{\mathrm{set}}^2} & (0 < t < \dfrac{T_{\mathrm{set}}}{2}) \\ -\dfrac{4\alpha}{T_{\mathrm{set}}^2} & (\dfrac{T_{\mathrm{set}}}{2} < t < T_{\mathrm{set}}) \\ 0 & (t > T_{\mathrm{set}}) \end{cases} \tag{5.2}$$

Continuous-time deadbeat control

$$v(t) = \begin{cases} \dfrac{12\alpha}{T_{\mathrm{set}}^3}(\dfrac{T_{\mathrm{set}}}{2} - t) & (0 < t < T_{\mathrm{set}}) \\ 0 & (t > T_{\mathrm{set}}) \end{cases} \tag{5.3}$$

With these figures we can find the similarities and differences among these three control. Instead of making the comparison with the same T_{set}, I want to make the comparison when all the model get the deadbeat gait generation. Then the figures of steady velocity looks almost the same except the values. Fig 5.1 is steady velocity of continuous-time deadbeat control. The angular position and y are almost the same in these three controls just as Fig 5.1. I also just show the figures of the continuous-time deadbeat control here. I will show the comparisons among three deadbeat gait generations from Fig 5.4 to Fig 5.7. We can find the difference of $v(t)$ and u among these control methods. As we see, the u are all discontinuous when the walker has an impact. I design CODC as a compromise of desired-time trajectory control and DODC. It is simple and almost continuous. Based on Fig 5.5, we can understand why the angular velocity and the total energy of CODC changes smoothly.

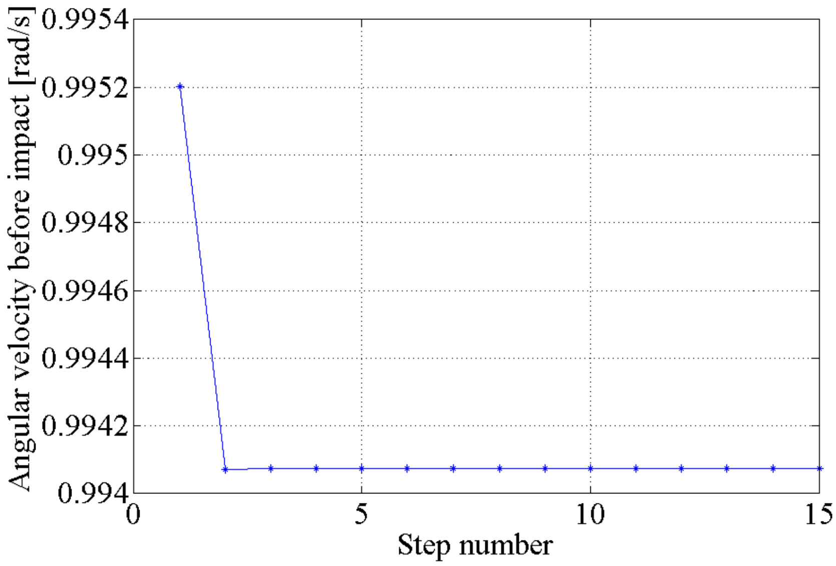


Figure 5.1: Velocity immediately before impact when deadbeat gait generation by CODC

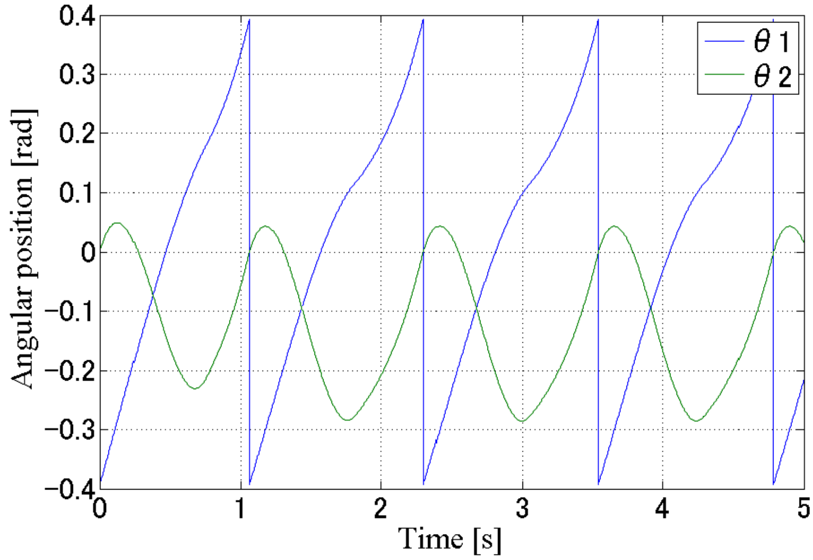


Figure 5.2: Time-evolution of angular positions when deadbeat gait generation by CODC

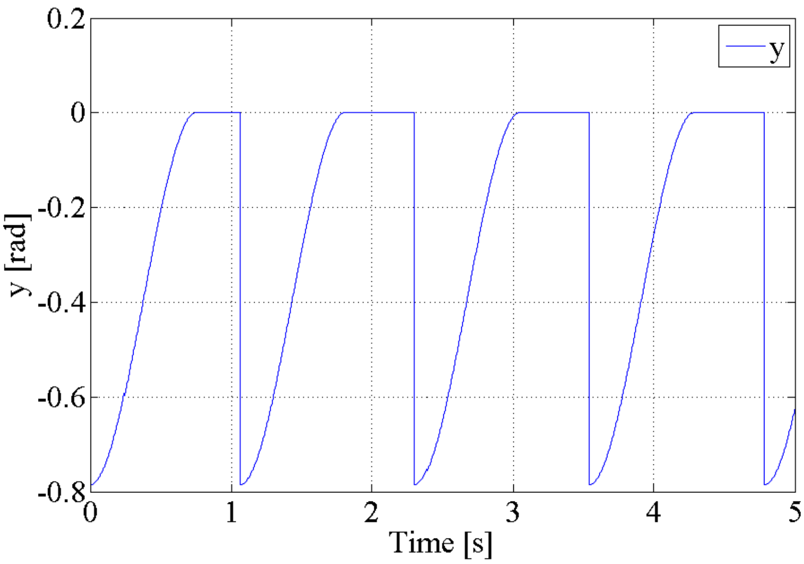
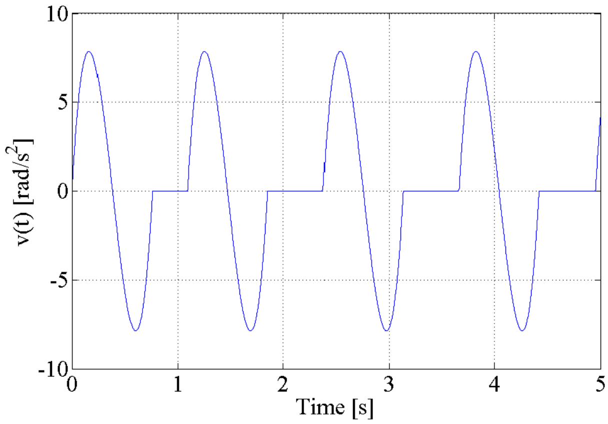
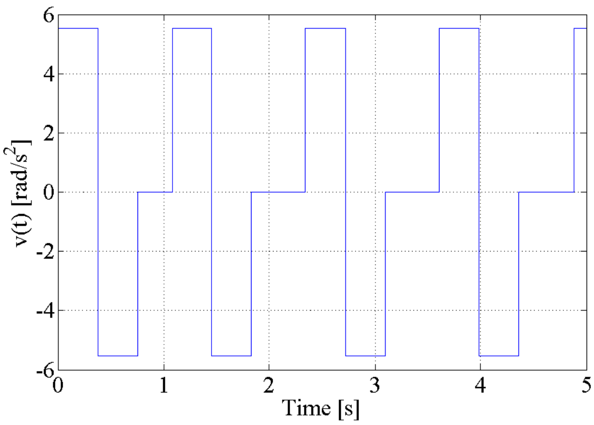


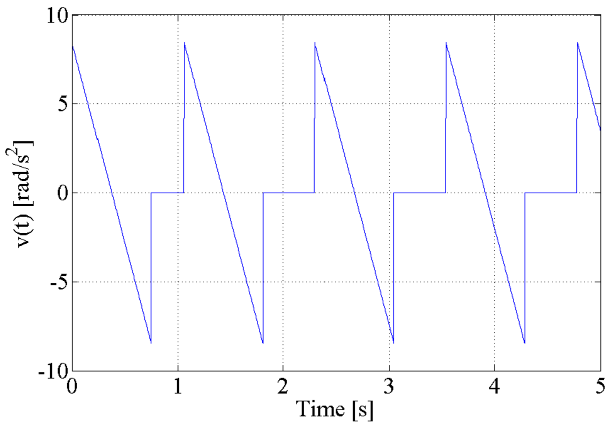
Figure 5.3: Time-evolution of y when deadbeat gait generation of continuous-time deadbeat control



(a) Desired-time trajectory

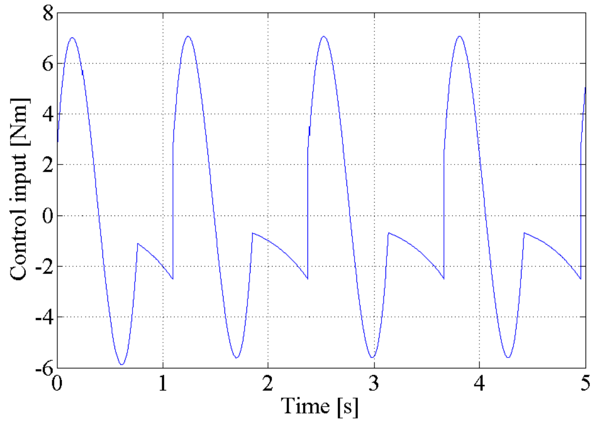


(b) Discrete-time deadbeat

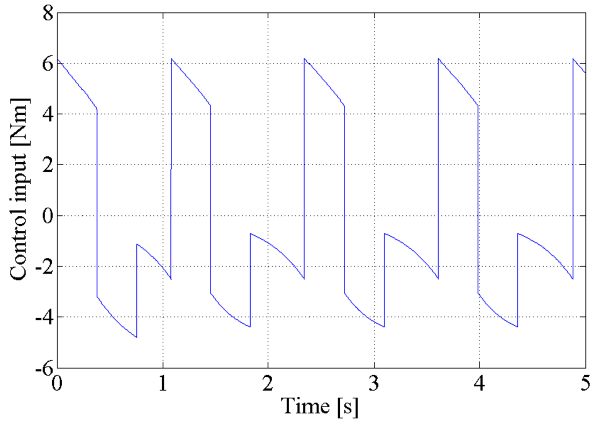


(c) Continuous-time deadbeat

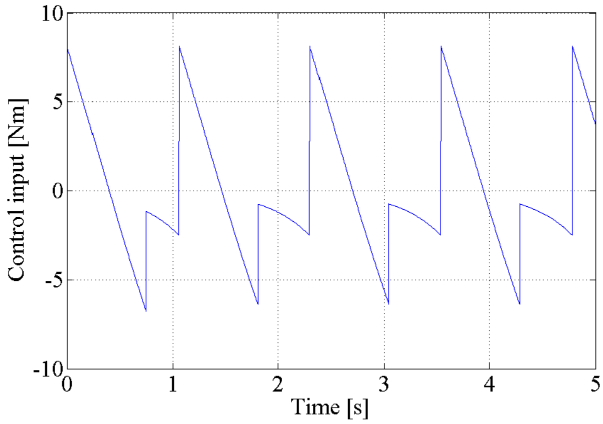
Figure 5.4: Time-evolution of $v(t)$ of three deadbeat gait generations



(a) Desired-time trajectory

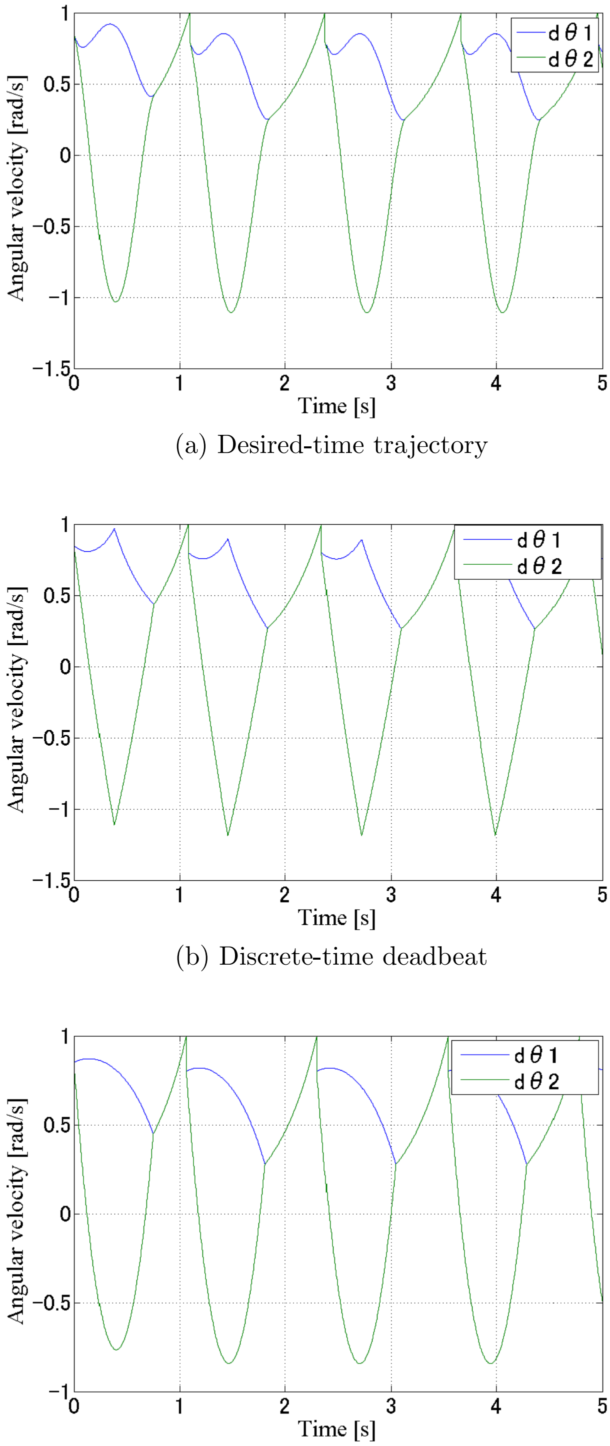


(b) Discrete-time deadbeat



(c) Continuous-time deadbeat

Figure 5.5: Time-evolution of u of three deadbeat gait generations

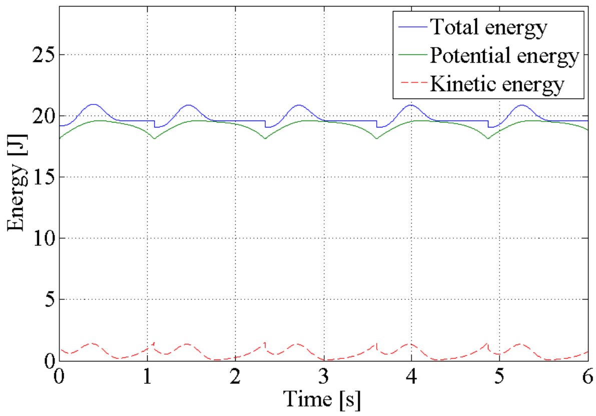


(a) Desired-time trajectory

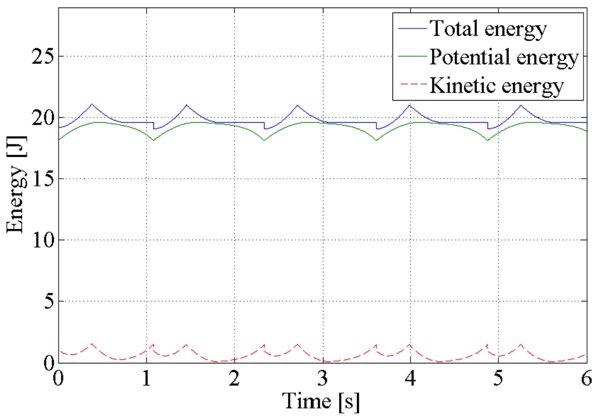
(b) Discrete-time deadbeat

(c) Continuous-time deadbeat

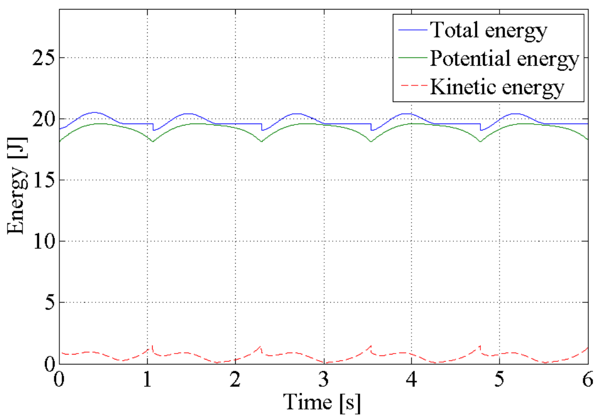
Figure 5.6: Time-evolution of angular velocity of three deadbeat gait generations



(a) Desired-time trajectory



(b) Discrete-time deadbeat



(c) Continuous-time deadbeat

Figure 5.7: Time-evolution of energies of three deadbeat gait generations

5.2 Comparison of Gait Properties

In this part I didn't want to consider about the desired-time trajectory control because it is a very complex control compared with the other two control systems. I will make the comparisons between the discrete-time deadbeat control and the continuous-time deadbeat control to prove the continuous-time deadbeat control walk faster and have higher energy efficiency.

5.2.1 Walking speed

I want to make the comparison on the step period and the walking speed that when we set T_{set} of the discrete-time deadbeat control and the continuous-time deadbeat control as the same value. The walking speed here refers to how fast the model can walk. We use the equation below to value the walking speed of the model.

$$\text{walking speed} = \frac{\text{step length}}{\text{step period}}$$

As the step length is constant with the same model, the walking speed only depends on the step periods. Now we will make the comparison when DODC and CODC have the same $\bar{Q}$.

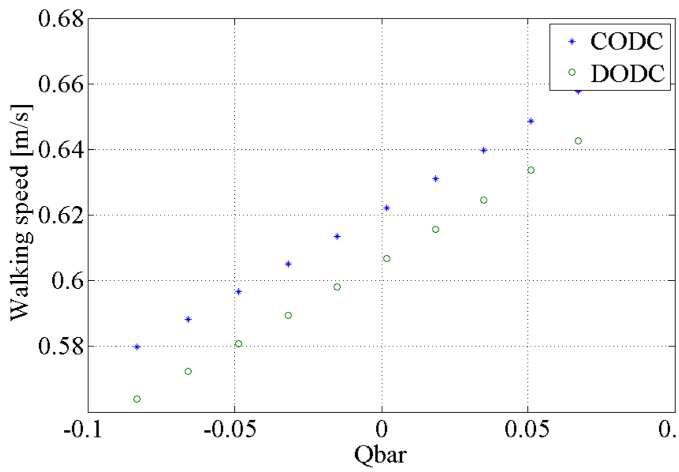


Figure 5.8: Walking speed when DODC and CODC

The results are very interesting: the continuous-time deadbeat control have a shorter step period than the discrete-time deadbeat control. That means with the same T_{set}, the Continuous-time deadbeat control have a higher average walking speed than Discrete-time deadbeat control! Of course, just a little faster.

5.2.2 Energy efficiency

Now we define another equation to value the mechanical work. Assuming that we use the electricity to drive the model and all the electricity can be changed into the mechanical energy. W here will show us that how much electricity will be cost in one step when the model is walking stably [9].

$$W = \int_0^{T^*} (\left|u\dot{\theta}_1 - u\dot{\theta}_2\right|)\mathrm{d}t,$$

We can get the values of u and angular velocity from the figures above. We find that when $t = \dfrac{T_{\text{set}}}{2}$, the angular velocity of θ_1 and θ_2 are almost largest. And we also find that the u of continuous-time deadbeat control is much lower than the u of the discrete-time deadbeat control. Here I want to prove that the continuous-time deadbeat control is more energy efficient than the discrete-time deadbeat control. According to Tad McGeer [1], we could use SR to measure efficiency.

$$SR = \frac{W}{mg \times \text{step length}}.$$

As the mass of model and the length of step are all the same, we can value the SR based on W. Here is the figure of W when DOCD and CODC achieve the same $\bar{Q}$. We found that the W of discrete-time is much higher than the W of CODC in the simulation. We can get the figure of SR as below. I can conclude that the continuous-time deadbeat control is much more efficient than the discrete-time deadbeat gait control.

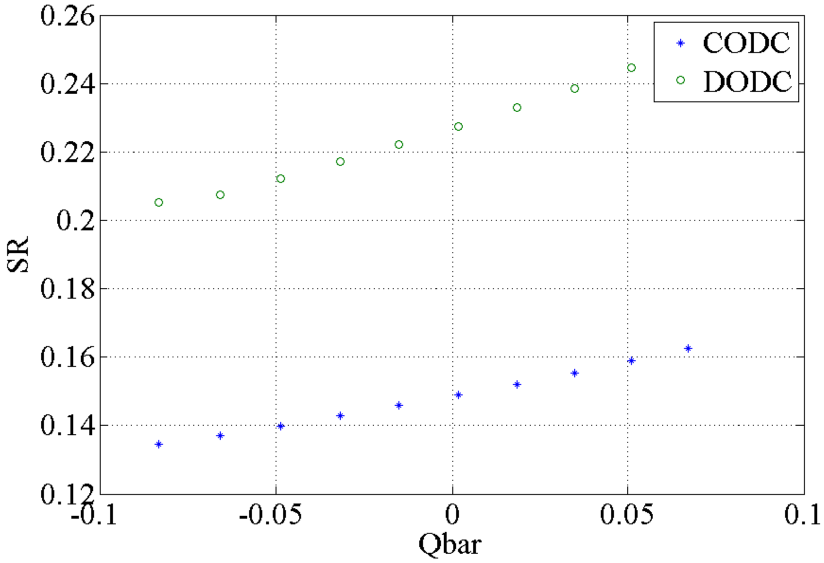


Figure 5.9: $\bar{Q}$-evolution of SR when by DODC and CODC

5.3 Extension to Uneven Ground

Although the generated gait is natural and energy efficient, the adaptability and robustness are low. When we get the deadbeat gait generation, the model will get stable after two impacts. We want to do the experiments how will the model perform when going upstairs and going downstairs.

5.3.1 Requirement of initial velocity

In the simulation of chapter 4, we find that we cannot set the initial velocity too large or too low. When the initial velocity is very low, the model will not have enough energy to overcome the middle position. When the initial velocity is too large, the time of first step period will less than T_{set}. In this situation, we cannot finish the control in one step so we cannot get the deadbeat gait generation. In the simulation of chapter 4, based on many experiments, we find we need to set the initial velocity as 0.72 [s]< $\dot{\theta}_0$ <1.01 [s] to get the deadbeat gait generation. We think that when the length of legs are shorter, the orbit of COM will get flat and the energy that the model need to finish one step will get less. With this idea we try to set the length of legs, L as 0.7 [m] instead of 1 [m]. We find we can set $T_{\text{set}} = 0.535$ [s] to get the deadbeat gait generation. We can see the figure of steady angular velocity below.

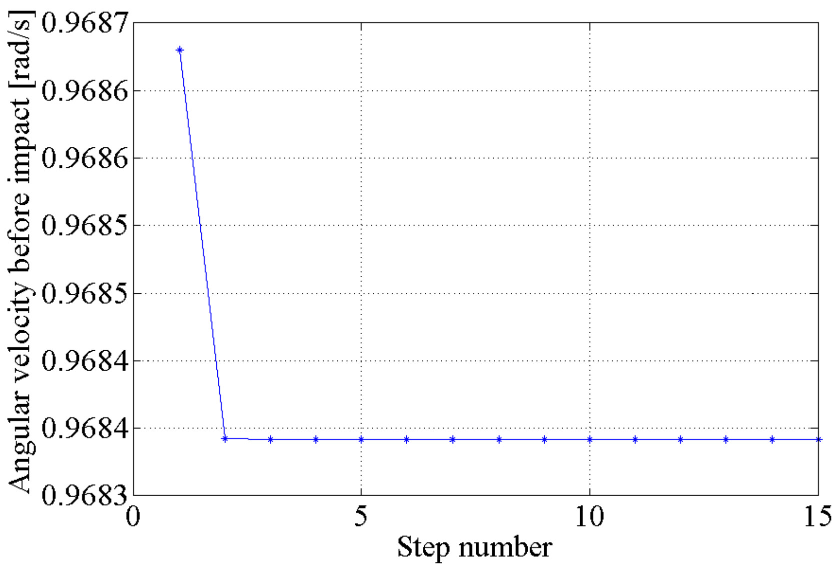


Figure 5.10: Velocity immediately before impact when $L = 0.7$ [m] and $T_{\text{set}} = 0.535$ [s]

Although the process is too complex to analyse because not only the length of legs but also the control input are different, we almost get the result as we hope. In the simulations I find that when I set the initial velocity as 0.52 [rad/s], the model can still overcome the middle position. The value is much lower than 0.72 [rad/s], the lower limit of the longer

longer leg model. I also get the same conclusion from the experiments when length of legs of deadbeat gait generation are 0.8 [m] and 0.9 [m]. Based on these experiments we can assume that the model with shorter legs overcome the stair more easily when we use the continuous-time deadbeat control. I will try to prove this conclusion from the experiments of next section. In the future work I will try to prove the result mathematically. Here I just use the result which is based on many experiments. On the other hand, what is the upper limit of initial velocity is too complex to analysis. I hope to solve the problem in the future.

5.3.2 Experiments

In the experiment we arrange the model go upstairs at the fifth impact and go downstairs at then seventh impact. The parameters are as the table below. We find that the continuous-time deadbeat control perform poor. We find that the model can only overcome the stairs with heigh of 0.01 [m].

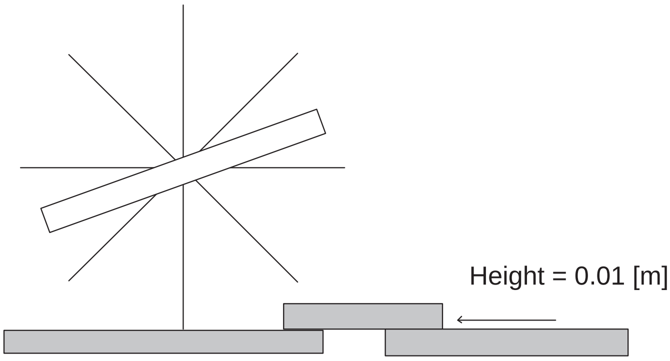


Figure 5.11: Model of Experiment 1

Table 5.1: Parameters of initial condition in Experiment 1

L	T_{set}	Height of stair
1.0	0.747	0.01
[m]	[s]	[m]

Here are the result of this Experiment 1. I can find that the model finished the uneven ground perfectly. As we see in Fig 5.12, the model goes upstairs at fifth step so we get a lower angular velocity before impact. After a step on the same stair, we make the model go downstairs at seventh step. The model get steady after two steps, which means the model could reject from some uncertainties quickly. In Fig 5.14 and 5.15 we find after the fifth impact, the lowest kinetic energy is almost zero in this step. So if the heigh of stair is larger than 0.01 [m], the model cannot overcome the stairs. In this experiment the requirement of the ground is high.

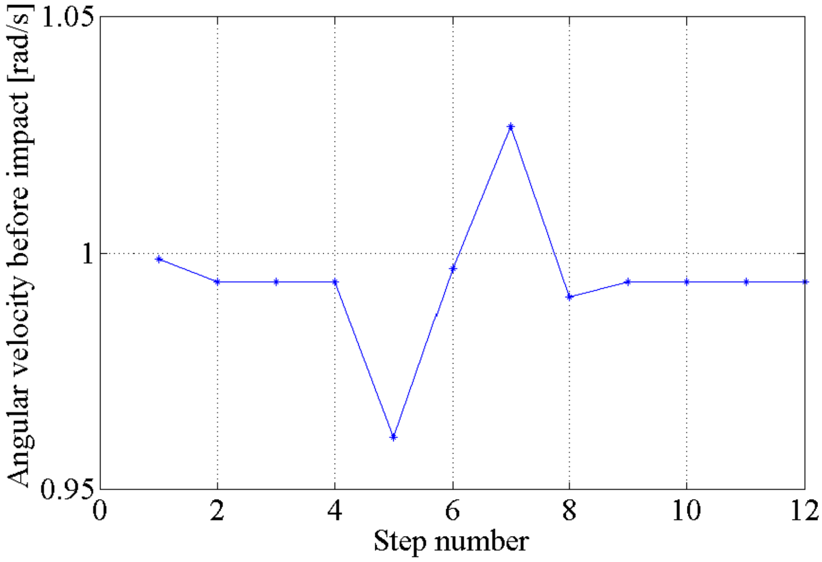


Figure 5.12: Velocity immediately before impact when deadbeat gait generation goes upstairs and downstairs (L=1.0 [m])

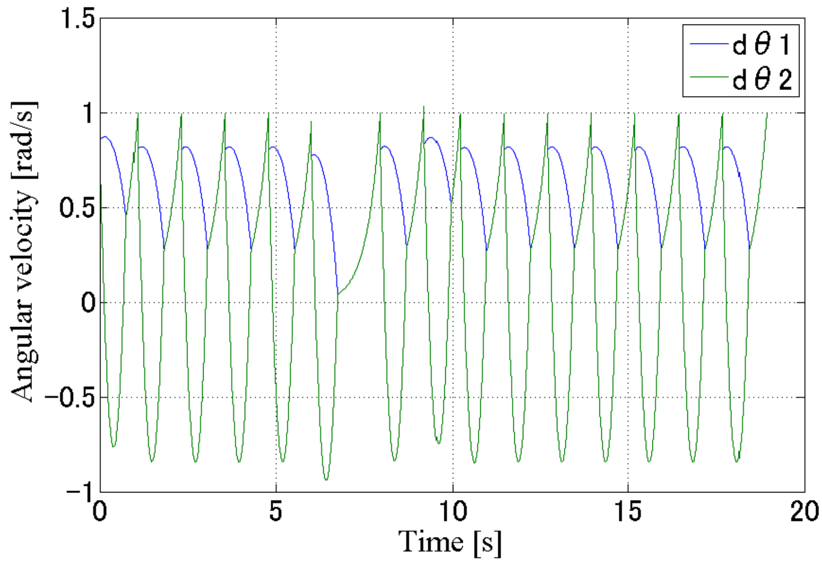


Figure 5.13: Time-evolution of angular velocity when deadbeat gait generation goes upstairs and downstairs (L=1.0 [m])

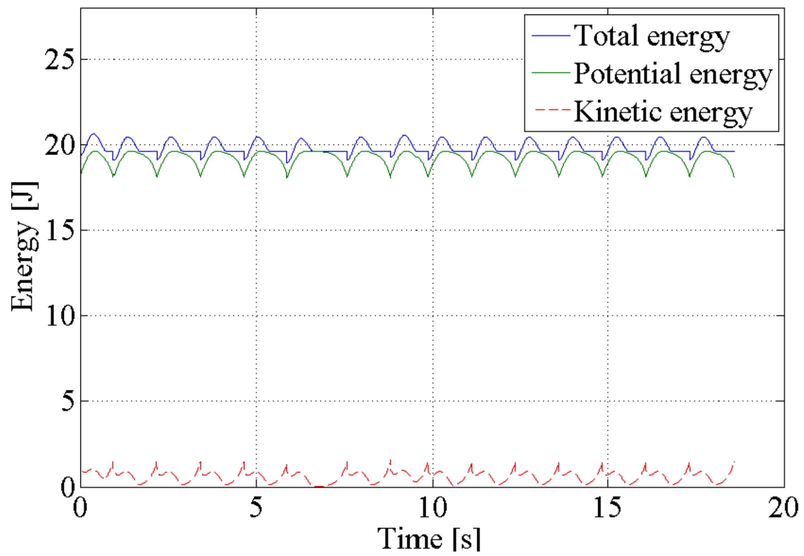


Figure 5.14: Time-evolution of energy when deadbeat gait generation goes upstairs and downstairs (L=1.0 [m])

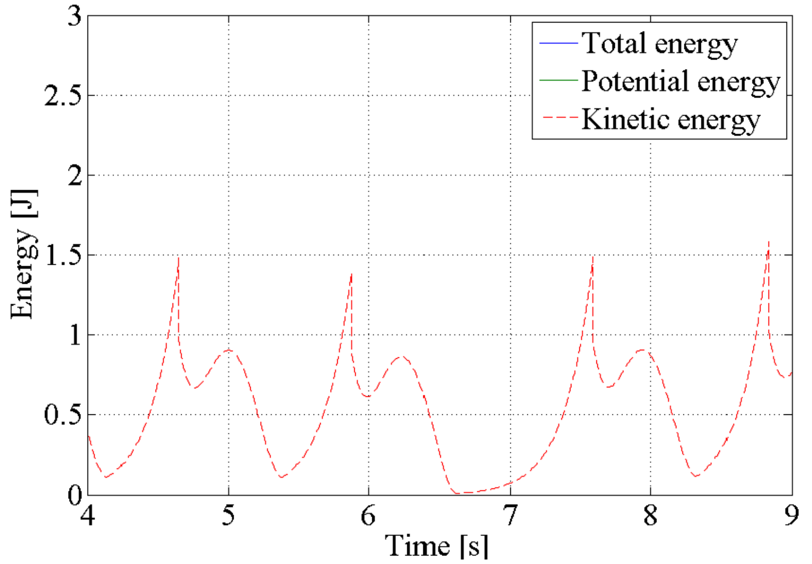


Figure 5.15: Time-evolution of energy when model goes upstairs.

In the second experiment, we only make the model go downstairs. With the same parameters as the first experiment, even if we set the heigh of stair as 0.03 [m], the model can still keep deadbeat gait generation. We add a second stair at the seventh impact and then the model will go downstairs continuously as we see in the figures below. We find that the model perform perfectly when going downstairs. Here are the figures of simulation. As we see in Fig 5.17 and 5.18, the velocity before impact when going downstairs are the same. So we can prove the model go downstairs stably. On the other hand, the heigh of stairs is much higher than that in experiment 1. We can conclude that in this condition the model perform well when going downstairs but perform bad when going upstairs.

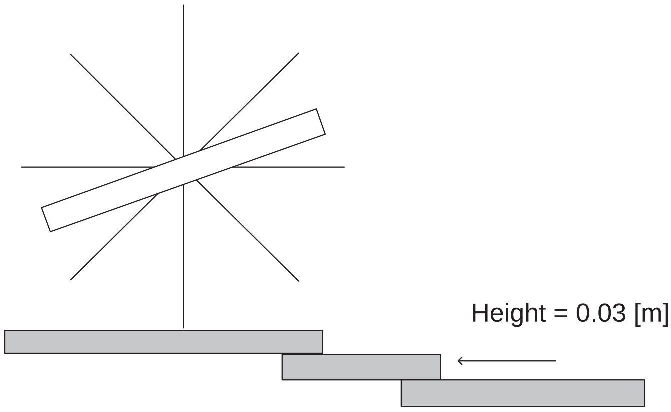


Figure 5.16: Model of Experiment 2

Table 5.2: Parameters of initial condition in Experiment 2

L	T_{set}	Height of stair
1.0	0.747	0.03
[m]	[s]	[m]

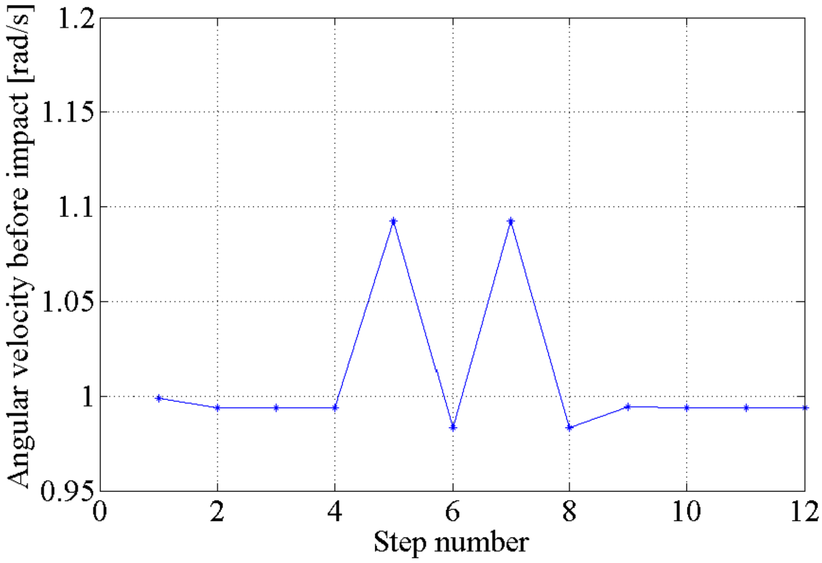


Figure 5.17: Velocity immediately before impact when the deadbeat gait generation goes downstairs

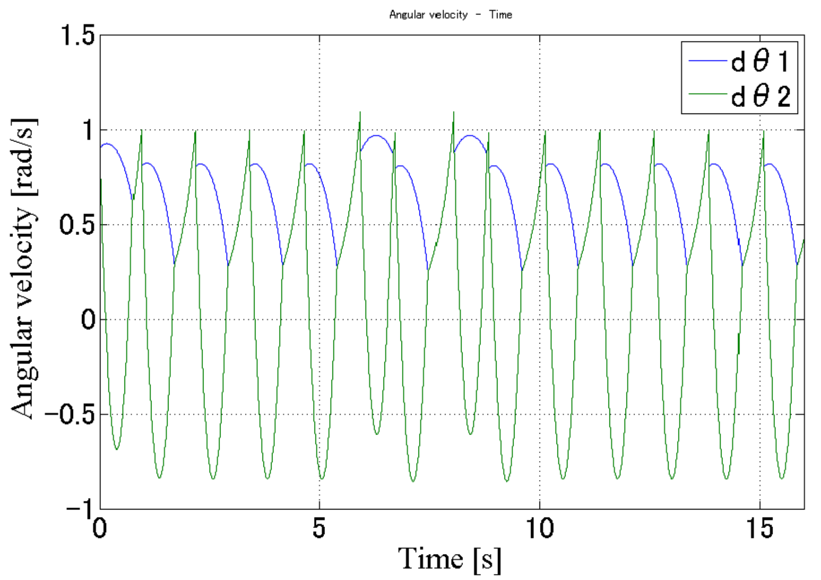


Figure 5.18: Time-evolution of angular velocity when deadbeat gait generation goes downstairs

In the experiment 3, we set the $T_{\text{set}} = 0.5$ [s]. The model can overcome the stair but it is far from the deadbeat gait generation. The ability of rejection is very low. In this condition, we set a lower T_{set} to get a higher control input. So the walker can overcome higher stairs. On the other hand, with a lower T_{set}, the walker could finish control input in one step more easily. The requirement of initial velocity get lower for the walker. Which means the walker could perform well when going downstairs. In this experiment the walker overcome the stair with height of 0.03 [m]. But we can see from Fig 5.20 that the walker need a lot of steps to get steady velocity.

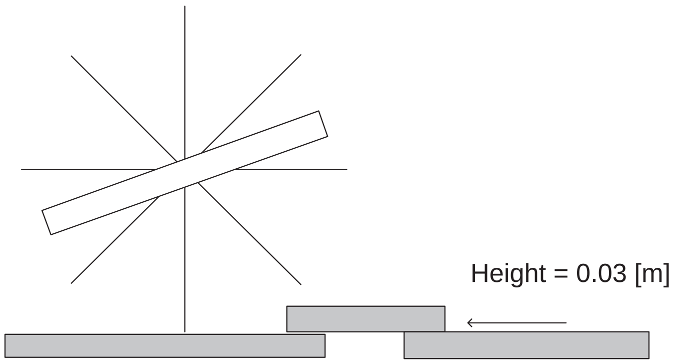


Figure 5.19: Model of Experiment 3

Table 5.3: Parameters of initial condition in Experiment 3

L	T_{set}	Height of stair
1.0	0.5	0.03
[m]	[s]	[m]

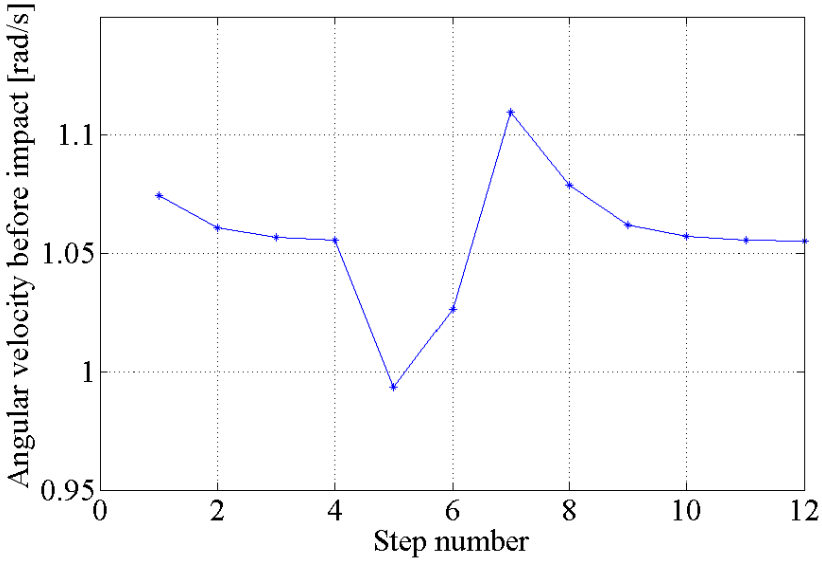


Figure 5.20: Velocity immediately before impact of asymptotical gait generation when $T_{\mathrm{set}} = 0.5$ [s]

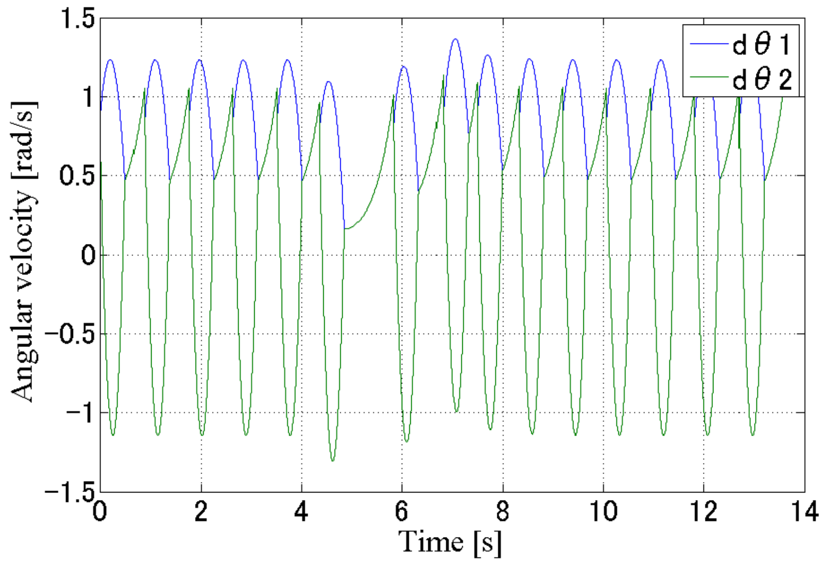


Figure 5.21: Time-evolution of angular velocity of asymptotical gait generation when $T_{\mathrm{set}} = 0.5$ [s]

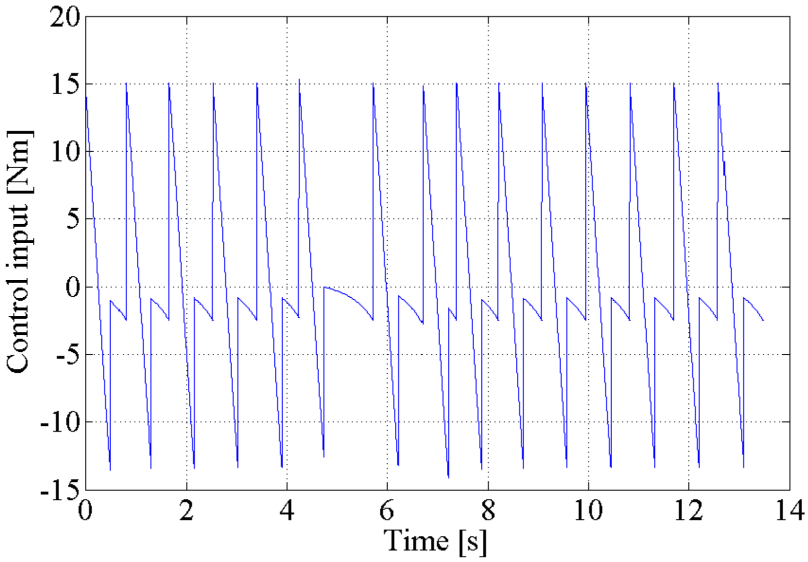


Figure 5.22: Time-evolution of u of asymptotical gait generation when $T_{\mathrm{set}} = 0.5$ [s]

In the Experiment 4, I try to set the length of legs as 0.7 [m] and set the T_{set} as 0.535 [s] to get another deadbeat gait generation. On this condition, we try to make the model go upstairs and find that the model could overcome the stair with heigh of 0.02 [m]. As we hope the shorter legs perform better when going upstairs. From Fig 5.24 we can conclude that the model goes upstairs stably, which means we have improve the ability of overcoming stairs. But when the model goes downstairs, the velocity of model is too fast to finish a T_{set}. This situation is also should be avoided.

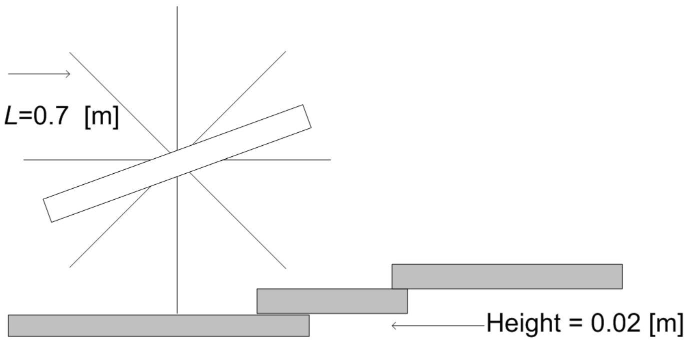


Figure 5.23: Model of Experiment 4

Table 5.4: Parameters of initial condition in Experiment 4

L	T_{set}	Height of stair
0.7	0.535	0.02
[m]	[s]	[m]

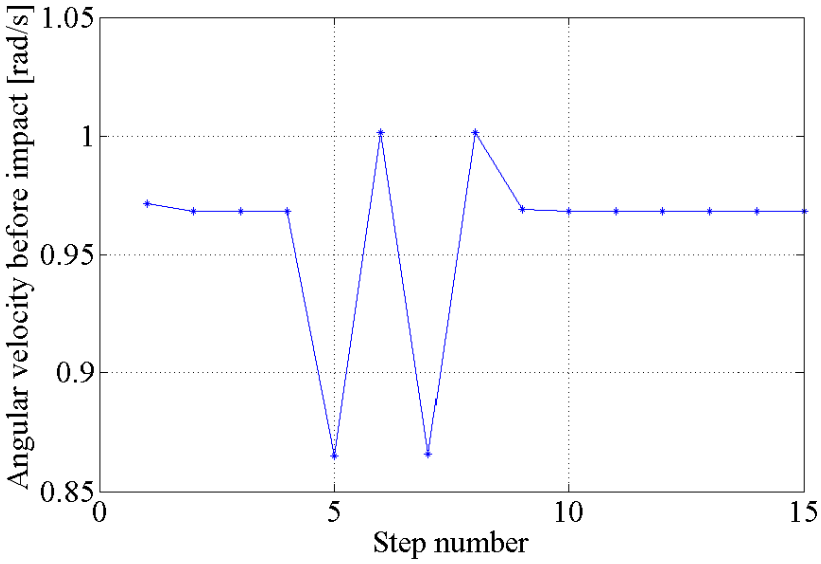


Figure 5.24: Velocity immediately before impact when deadbeat gait generation with shorter legs (L=0.7 [m])

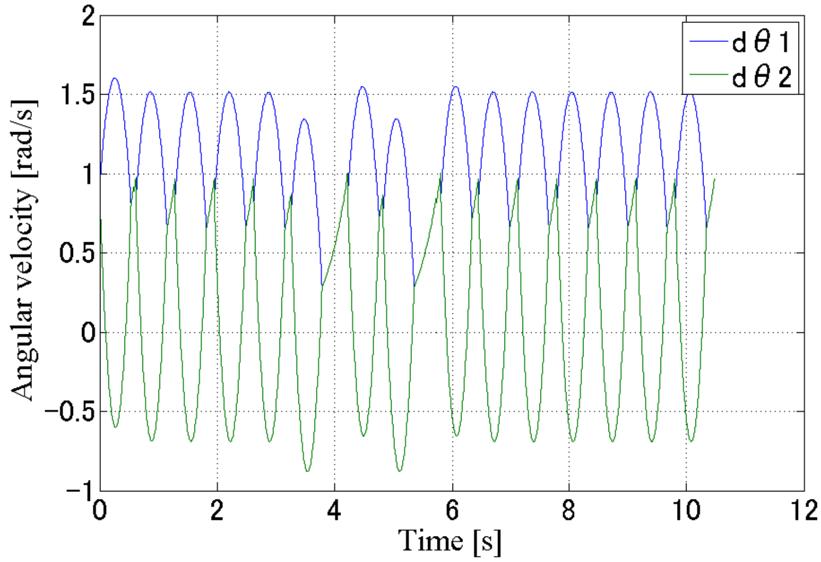


Figure 5.25: Time-evolution of angular velocity when deadbeat gait generation with shorter legs (L=0.7 [m])

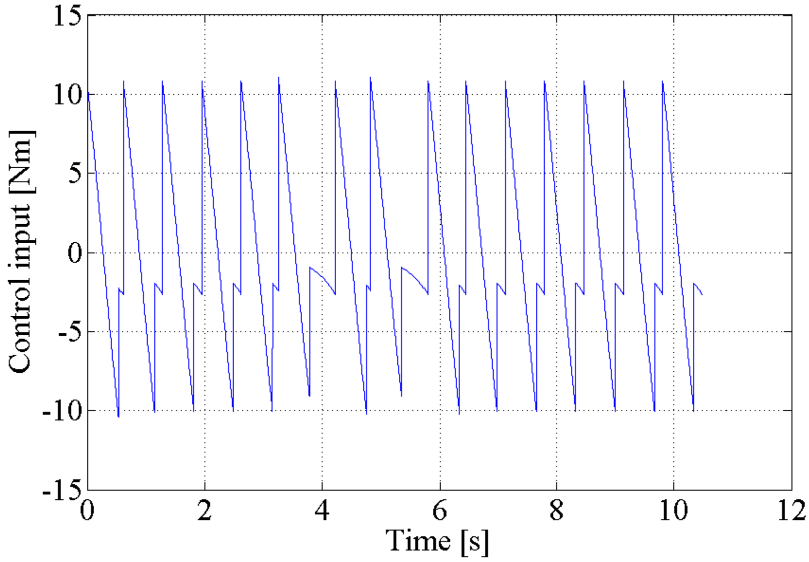


Figure 5.26: Time-evolution of u when deadbeat gait generation with shorter legs (L=0.7 [m])

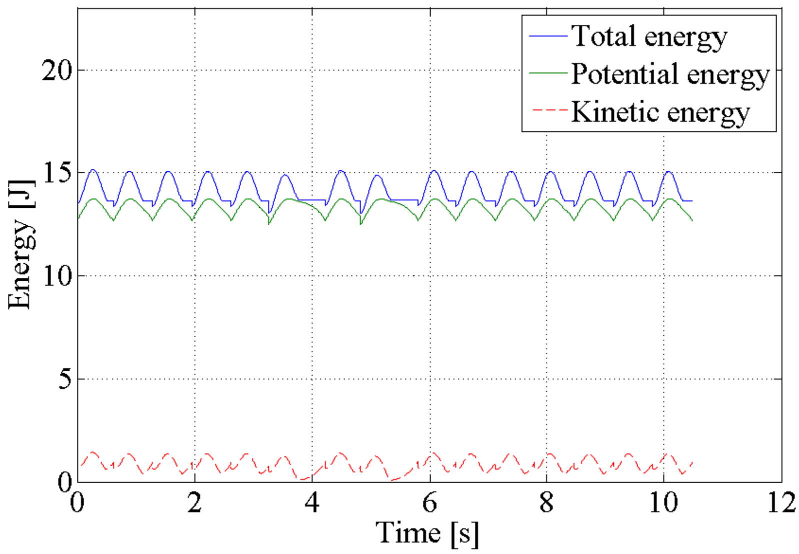


Figure 5.27: Time-evolution of energy when deadbeat gait generation with shorter legs (L=0.7 [m])

Based on these experiments, we can conclude that:

1. If I set T_{set} as a small value, I can make the model overcome the stairs but it is after some steps that the model can get stable.

2. The deadbeat gait generation can fast get stably after going upstairs and downstairs. But the heigh of the stairs cannot be too large.

3. When I change the length of the legs of model, the ability of going upstairs gets better. But I still need some more investigations about the ability of going downstairs stably .

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis developed the continuous-time output deadbeat control (CODC) based on the discrete-time deadbeat control. As the advantages, CODC is very simple and the input is almost continuous. CODC also achieve faster walking speed and higher energy efficiency than DODC. On the other hand, we still need to face some problems such as control input is not continuous and we need to measure θ_1 to calculate the control torque u when the walker is moving. I still need some investigations to achieve the experiment machine. In this thesis I achieved the simulations by MATLAB. In the simulation experiments I compared some gait properties of CODC with DODC. I also designed some experiments how the model with the continuous-time control will perform when going upstairs and downstairs. These are the conclusions I get from this thesis:

1. I develop the CODC and mathematically prove that we can get the deadbeat gait generation with this control.
2. I can confirm from the experiments that CODC is faster and more energy-efficient than DODC.
3. I have proved that the deadbeat gait generation can walk on the uneven ground stably if the condition of the ground is not too bad.
4. The deadbeat gait generation with shorter legs perform better when going upstairs. We can choose the length of legs depending on the condition of the ground to make the model walk on the uneven ground better.

6.2 Future Work

There are still some problems we need to face. First, I cannot mathematically prove the relationship among T_{set}, the length of legs and the heigh of stairs. Some more experiments are necessary. Second, I also need some more investigations about other parameters such as mass of model, the inertia of torso and so on. But maybe the system will get too complex to design.

Besides these problems I am also interested in that how the model will perform when walking on the soft ground and how to achieve deadbeat gait generation with higher speed. I hope I will solve these problems in the future.

Acknowledgements

First, I want to express my special thank to my supervisor, professor Fumuhiko Asano for his constant encouragement and guidance in my research and daily life. Without his consistent and illuminating instruction, this thesis could not have reached its present form. Second, I would like to express my gratitude to my senior fellows, Junji Kawamoto , Daiki Tanaka, Masataka Ohshima and Yoshiki Yasuda, for their kindly help and guidance in my research. I really appreciate all the lab members' kindness. Third, I would like to express my gratitude to professors, teachers and friends in JAIST. I really appreciate their kindly help and encouragement in my daily life. At last, I would like to express my love to my family. It is their encouragement and support that make me have the confidence to finish my research. I love my family.

Reference

[1] T. McGeer, "Passive dynamic walking," *Int. J. of Robotics Research*, vol. 9, no. 2, pp. 62-82,1990.

[2] Fumihiko Asano, "Efficiency and optimality of two-period limit cycle walking," *Advanced Robotics*, Vol. 26, No. 1–2, pp. 155–176, Jan. 2012.

[3] Fumihiko Asano, Junji Kawamoto, "Passive Dynamic Walking of Viscoelastic-legged Rimless Wheel," *Proceedings of the 2012 IEEE International Conference on Robotics and Automation*, pp. 2331-2336, May 16, 2012.

[4] Fumihiko Asano, Zhi-Wei Luo, "Asymptotically stable biped gait generation based on stability principle of rimless wheel," *Robotica*, Vol. 27, Issue 6, pp. 949-958, Oct. 2009.

[5] Michael J. Coleman, "Dynamics and stability of a rimless spoked wheel: a simple 2D system with impacts," *Dynamical Systems*, pp. 215-238, Feb. 2010.

[6] M. Garcia, A. Chatterjee, A. Ruina, M. Coleman: "The simplest walking model: Stability, complexity, and scaling," *ASME J. of Biomechanical Enegineering*, Vol. 120, No. 2, pp281–288, 1998.

[7] M. W. Whittle: Gait analysis: an introduction, *Butterworth-Heinemann*, 2001.

[8] Fumihiko Asano, "Stability analysis of underactuated bipedal gait using linearized model," *IEEE-RAS International Conference on Humanoid Robots*, pp. 282–287, Oct. 2011.

[9] Joseph H Solomon, Marttijn Wisse, Mitra JZ Hartmann, "Fully interconnected, linear control for limit cycle walking," *Adaptive Behavior*, pp. 18(6) 492-506, 2010.