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Outage Analysis of Correlated Source Transmission in Block Rayleigh Fading Channels

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Abstract—A goal of this paper is to theoretically derive the outage probability of the correlated source transmission in block Rayleigh fading channels. The correlation between the two information sources is assumed to be expressed by the bit flipping model, where the information bits transmitted from the second transmitter are the flipped version of the information bits transmitted from the first transmitter with a probability $p_e$. The source sequences are independently encoded by channel encoders, and then transmitted to the destination block-by-block via different time- or frequency-slots. The channels are assumed to be suffering from independent block Rayleigh fading. This paper then shows that the outage probability of this system can be expressed by double integrals with respect to the probability density functions (pdf) of the instantaneous signal-to-noise power ratios (SNRs) of those channels, where the range of the integration is determined by the Slepian-Wolf coding theorem. The most significant finding made by this paper is that the asymptotic diversity order is one so far as $p_e$ is non-zero, and diversity order 2 can be achieved only if $p_e = 0$. The major applications of this paper’s results include outage evaluation of extract-and-forward (EF) relay systems allowing intra-link (source-relay link) errors, sensor networks, and wireless mesh networks. The latter half of this paper provides results of outage probability calculations for one-way EF relay scenario utilizing the concept of the technique presented in this work.

I. INTRODUCTION

The Slepian-Wolf theorem is well known as an efficient technique for the lossless compression of the correlated sources. In the distributed source coding models, as shown in Fig. 1, each of the data streams is separately encoded and the both encoded data streams are finally jointly decoded by a single decoder. According to the fundamental contribution of Slepian and Wolf in [1], it has been proven that by exploiting the correlation knowledge of the data streams at the destination, the distributed source coding can achieve the same compression rate as the optimum single encoder which compresses the sources jointly.

A goal of this paper is to derive theoretical outage probability of correlated source transmission in block Rayleigh fading channels. This paper assumes the scenario where the correlation model of the sources can be expressed as the bit flipping model [2], as $b_2 = b_1 \oplus e$ and $P(e = 1) = p_e$, where $p_e$ is the bit flipping probability. The source information to be transmitted from the first and the second transmitters is channel-encoded for error protection, and modulated according to the modulation format used. This paper assumes the separation of the Source-Channel coding, supported by Shannon’s Separation theorem [3], and hence providing practical channel coding and modulation schemes is out of the scope.

In block fading channels, the channel gain changes, frame-by-frame. Therefore, given the channel coding and modulation schemes, sometimes the gains of either one of the two or the both channels are faded below the transmission chain requires. This paper theoretically derives the outage probability of the correlated source transmission based on the assumption described above. It is shown in this paper that the outage probability can be expressed by double integrals with respect to the pdf of the instantaneous SNRs of the channels, where the range of the integration is determined by the Slepian-Wolf coding theorem. The most significant finding of this paper is that the second order diversity can be achieved only if $p_e = 0$, and otherwise, even though the decay of the outage probability curve is equivalent to the second order diversity when the received average SNR is low, but according to the increase in the average SNR, it gradually changes and asymptotically the decay becomes equivalent to the case of no diversity. The latter half of this paper focuses on a one-way EF relay as an application of the system investigated in the first half of this paper.

Unlike the conventional decode-and-forward (DF) relay system, EF only extracts the information part from the received signal, re-encodes the erroneous data, and transmits the channel coded data to the destination. Performance comparison between the theoretical outage and the frame-error-rate (FER) of the proposed EF relay system using bit interleaved coded
modulation with iterative detection (BICM-ID), is briefly designed in section IV. The impact of relay locations on the outage probability is also investigated. With $p_e = 0$, representing an extreme case, the outage probability achieved by the proposed system is compared with that achieved by maximum ratio combining (MRC), where the signals from the two sources are first MRC-combined, and then the processing specified by the transmission chain is performed.

This paper is organized as follows. First of all, the system model based on the Slepian-Wolf theorem is briefly introduced in Section II. The outage probability is then derived in Section III. Furthermore, the proposed EF relay system is detailed in IV. Finally, we present the numerical results and provide performance assistance in Section V.

II. SYSTEM MODEL

A system model assumed in this paper is shown in Fig. 1, where $b_1$ is the source information bit to be transmitted from the first transmitter, and $b_2$ is to be transmitted from the second transmitter. According to the Slepian-Wolf theorem, the achievable rate region is constituted as an unbounded polygon, comprised of parts 3, 4, 5 and 6 shown in Fig. 2. The source information can be recovered only when the compressed rate pair falls into this area. For instance, if $b_1$ is compressed at the rate $R_1$ which equals to its entropy $H(b_1)$, then $b_2$ can be compressed at the rate $R_2$ which is less than its entropy $H(b_2)$, but must be greater than their conditional entropy $H(b_2 | b_1)$, or vice verse. Specifically, the pair of compression rates $R_1$ and $R_2$ satisfy three equations [1]:

$$R_1 \geq H(b_1 | b_2),$$

$$R_2 \geq H(b_2 | b_1),$$

$$R_1 + R_2 \geq H(b_1, b_2).$$

where $H(b_1, b_2)$ denotes the joint entropy of the correlated source information streams. For the binary symmetric ($P(1) = P(0) = 0.5$) sources adopted in our paper, we have $H(b_1) = H(b_2) = 1$, $H(b_1 | b_2) = H(b_2 | b_1) = H(p_e)$, $H(b_1, b_2) = 1 + H(p_e)$ with $H(p_e) = -p_e \log_2(p_e) - (1 - p_e) \log_2(1 - p_e)$. The threshold signal-to-noise ratio is given by

$$\text{SNR}[H] = 2^{R_e} - 1.$$  

where $R_e$ represents the rate which takes into account of the channel coding and the modulation scheme.

III. OUTAGE DERIVATION

As shown in Fig. 2, the entire rate region for the pair of rates, $R_1$, $R_2$ can be divided into 7 parts. It is well known that the Slepian-Wolf admissible rate region is the sum of part 3, 4, 5 and 6. The part 7 also has to be included in the admissible region if the decoder at the receiver aims to only retrieve the information sequence transmitted from the first transmitter, such as relay systems. In summary, the total achievable rate of the system, where only the source information from the first transmitter is of interest, includes part 3, 4, 5, 6 and 7. On the contrary, areas 1 and 2 represent the region unachievable by the Slepian-Wolf theorem. By using equation Eq. (4), the entropy values can be transformed to the SNR domain, which helps to constitute the achievable SNR region.

Basically, when the instantaneous SNR pair $\gamma_1$ and $\gamma_2$ fall outside the admissible region, the outage event happens and the error-free communication can not be guaranteed at arbitrary transmission rate. Consequently, under this system scenario, the definition of the outage probability of the transmission system utilizing the Slepian-Wolf theorem is given by

$$P_{o, su} = P_1 + P_2,$$  

where $P_1$ and $P_2$ are the probabilities that the instantaneous SNR pair $\gamma_1$ and $\gamma_2$ fall into the unachievable areas 1 and 2, respectively, shown in Fig. 2. Based on the achievable rate region, we can mathematically derive $P_1$ as:

$$P_1 = \int_{\gamma_1 = \text{SNR}[0]}^{\text{SNR}[H(b_1, b_2)]} \int_{\gamma_2 = \text{SNR}[0]}^{\text{SNR}[\infty]} P(\gamma_1, \gamma_2) d\gamma_1 d\gamma_2,$$  

where $P(\gamma_1, \gamma_2)$ are the joint pdf of $\gamma_1$ and $\gamma_2$ as the instantaneous SNRs of the two channels, and $P(i), i = 1, 2,$ represents the pdf of the instantaneous SNR of the independent channel i. Assuming that $\gamma_1$ and $\gamma_2$ are statistically independent,

$$P_1 = \int_{\gamma_1 = \text{SNR}[0]}^{\text{SNR}[H(b_1, b_2)]} P(\gamma_1) d\gamma_1 \int_{\gamma_2 = \text{SNR}[0]}^{\text{SNR}[\infty]} P(\gamma_2) d\gamma_2,$$  

however, the integration of $P(\gamma_2)$ from 0 to infinity equals to 1.0. Hence, in block Rayleigh fading channels, Eq. (7) can be simplified to

$$P_1 = \int_{\gamma_1 = \text{SNR}[0]}^{\text{SNR}[H(b_1, b_2)]} P(\gamma_1) d\gamma_1.$$  

where

$$P(\gamma_1) = P(\gamma_1 | \text{SNR}[0]) P(\text{SNR}[0])$$

is the pdf of the instantaneous SNR of the independent channel i. Assuming that $\gamma_1$ and $\gamma_2$ are statistically independent.
\[ P_1 = \int_1^{\text{SNR}[H(b_1|b_2)]} P(\gamma_1) d\gamma_1 \]
\[ = \int_1^{\text{SNR}[H(b_1|b_2)]} \frac{1}{\Gamma_1} \exp(-\frac{\gamma_1}{\Gamma_1}) d\gamma_1 \]
\[ = 1 - \exp\left(-\frac{H(p_e)}{\Gamma_1}\right). \]  
(8)

Similarly, \( P_2 \) can be derived as follows:

\[ P_2 = \int_1^{\text{SNR}[H(b_1)]} \int_1^{\text{SNR}[H(b_1|b_2)-H(\gamma_1)]} P(\gamma_1, \gamma_2) d\gamma_1 d\gamma_2. \]  
(9)

where \( H(\gamma_1) = \frac{1}{\Gamma_1} \log_2(1 + \gamma_1) \) transforms the instantaneous SNR to its corresponding entropy, which is an inverse of Eq. (4). Then we have

\[ P_2 = \int_1^{\text{SNR}[H(b_1)]} P(\gamma_1) d\gamma_1 \]
\[ = \int_1^{\text{SNR}[H(b_1|b_2)]} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \left\{ \exp\left(-\frac{\gamma_1}{\Gamma_1}ight) - 1 \right\} d\gamma_1. \]  
(10)

For the calculation of Eq. (10) with respect to \( \gamma_1 \), the trapezoidal numerical integration method is used, which generates high enough accuracy.

For comparison, the outage probability of the MRC scheme is also presented below, assuming \( p_e = 0 \). The output of the MRC combiner is a weighted sum of all branches. After the MRC-combining, the pdf of the instantaneous SNR in the block Rayleigh fading channel can be defined as [4]

\[ P_{y_2}(\gamma) = \frac{\gamma^{M-1} \exp\left(-\frac{\gamma}{\Gamma}\right)}{\Gamma^M (M-1)!}. \]  
(11)

where \( M \) represents the diversity order. The outage probability of MRC is equal to the probability that the instantaneous combined SNR after combining is less than a given threshold. Consequently, the outage probability of the MRC scheme with diversity \( M \) can be expressed as follows:

\[ P_{\gamma_2}(\gamma) = \frac{\gamma^{M-1} \exp\left(-\frac{\gamma}{\Gamma}\right)}{\Gamma^M (M-1)!}. \]  
(12)

In this paper, only the cases of \( M = 1, 2 \) are taken into account.

IV. EXTRACT-AND-FORWARD RELAY SYSTEM

This section applies the Slepian-Wolf transmission system to a DF relay system, where the relay does not aim to perfectly recover the original information transmitted by the source, but it only “extracts” the source information, even though the relay knows that extracted sequence may contain some errors. In this sense, the proposed technique is referred to as “Extract-and-Forward” (EF) system. As shown in Fig. 3, the extracted sequence representing an estimate of the original information sequence, which is then interleaved and transmitted to the common destination. Obviously, the original and extracted sequences are correlated, where in this paper it is assumed that the errors caused in the source-relay (SR) channel can be expressed by the bit flipping model. This is reasonable because we assume block fading and no heavy decoding of the channel code is performed at the relay. Hence, we can apply the results of the previous sections when evaluating the outage probability of the proposed EF system.

As shown in Fig. 3, the original information stream \( b_1 \) at the source is broadcasted to both the relay and the destination using the first time slot. The channel gain \( G_1 \) of the (source-destination) SD link is normalized to \( G_1 = 1.0 \), and the gain \( G_2 \) of the (relay-destination) RD link, relative to \( G_1 \), depends on the relay location and is given as:

\[ G_2 = \left( \frac{d_1}{d_2} \right)^\alpha, \]  
(13)

where \( d_1 \) and \( d_2 \) denote the distances from the source and the relay to the common destination node, respectively. Without the loss of generality, \( d_1 \) is also normalized to 1.0. \( \alpha \) represents
the path loss exponent, which is set to 3.52 according to [5]. Therefore, the received signals from the source node \((y_1)\) and the relay node \((y_2)\) can be expressed as:

\[
y_1 = \sqrt{G_1} \cdot h_1 \cdot b_1 + n_1, \tag{14}
\]

\[
y_2 = \sqrt{G_2} \cdot h_2 \cdot b_2 + n_2, \tag{15}
\]

where \(n_1\) and \(n_2\) are the zero-mean white additive Gaussian noises. It is assumed that both \(n_1\) and \(n_2\) have the same variance \(\sigma^2\). \(h_1\) and \(h_2\) are the complex channel gains of the SD and RD channels, respectively, and are constant over each block because of the block fading assumption. Consequently, the instantaneous SNR \(\gamma_i = G_i |h_i|^2 E_{s,i}/N_{0,i}\), where \(E_{s,i}\) and \(N_{0,i}\) represent the per-symbol signal power and the noise spectrum power density, both of the \(i\)-th channel, respectively. The pdf of \(\gamma_i\) is given by [6]

\[
P(\gamma_i) = \frac{1}{\Gamma_i} \exp(-\gamma_i/\Gamma_i), \tag{16}
\]

where \(\Gamma_i = G_i E_{s,i}/N_{0,i}\), denoting the normalized average SNR of the \(i\)-th channel. As defined before, we assume that the transmission channels 1 and 2 suffer from independent Rayleigh block fading without any channel correlation.

Substituting Eq. (16) into Eq. (8) and Eq. (9), we obtain \(P_1\) and \(P_2\), respectively for the proposed EF relay system.

V. NUMERICAL RESULTS

A. Same Distances of SD and RD

In this sub-section, the relay and source nodes are assumed to have the same distance to the destination node, \(d_1 = d_2\), and therefore the average SNRs \(\Gamma_1\) and \(\Gamma_2\) are always identical, according to Eq. (13). Fig. 4 plots the outage comparison between the Slepian-Wolf coding system and the MRC scheme with \(p_e = 0\) value as a parameter. For the MRC case, we assume \(p_e = 0\) and present the outage probability with diversity \(M = 1, 2\) only for comparison.

Note that when calculating the outage probability, the \(R_e\) value that taken into account the channel coding and modulation scheme used in practice, was set at \(R_e = 1\). This is because we assume source-channel separation for each channel, supported by Shannons separation theorem [3]. However, when making the performance comparison assuming practical transmission chain, such as that provided in subsection V-C, \(R_e\) should be set at the actual channel code rate and modulation format used in the system.

It is found from Fig. 4 that even though the EF system forwards the extracted original information which may contain errors, the destination can recover the original data using the correlation knowledge. Hence, the outage probability with the proposed EF relay system is better than the conventional DF system where when the relay detects error \((p_e \neq 0)\), it does not forward the data, as represented by the line with the index MRC, \(M = 1\). In fact, the line with the index MRC, \(M = 1\) is equivalent to the line with the index SW, \(p_e = 0.5\), which corresponds to the case where the two sources transmitted from the source and the relay are completely uncorrelated. As the \(p_e\) value drops, the outage probability also decreases. Interestingly, the decay of the outage probability is equivalent to the second order diversity when the received average SNR is low, but according to the increase in the received SNR, it gradually changes and asymptotically the decay becomes equivalent to case of no diversity. This asymptotic performance tendency can be mathematically proven, which is provided in Appendix A.

The second order diversity can be achieved only if \(p_e = 0\) over entire range of the average SNR. It is found from Fig. 4 that with \(p_e = 0\), the outage probability with the proposed EF performance is better than that with the MRC combining technique. In fact, their mathematical expressions are different, of which proof is provided in Appendix B. However, the EF’s superiority over MRC, shown in Fig. 4, is only the result of numerical calculation, and providing mathematical proof for the superiority of EF over MRC for \(p_e = 0\) is still left as an open question.

B. Different Relay Locations

In our discussion so far, the source and relay are assumed to have the same distance to the destination. While in this subsection, we will further discuss the different cases by changing the relay’s location in the Slepian-Wolf system. We consider the following 4 cases: (A), \(d_1 = d_2\); (B), \(d_1 = 2d_2\); (C), \(d_1 = 2d_2\); (D), \(d_1 = 4d_2\).

The average SNRs \(\Gamma_1\) and \(\Gamma_2\) of the channels 1 and 2, respectively, at each location scenario becomes as follows: given the path loss parameter \(\alpha\) equal to 3.52, \(\Gamma_1 = \Gamma_2\) in the location A; \(\Gamma_1 = \Gamma_2 + 4.4\) dB in location B; \(\Gamma_1 = \Gamma_2 + 10.6\) dB in location C; \(\Gamma_1 = \Gamma_2 + 21.19\) dB in location D.

The Fig. 5 shows the outage comparison for the different relay scenarios when \(p_e = 0.005\), with the x-axis representing the average SNR of the channel 1. The difference can be found
C. Practical Applications

In [7], an EF relay system using BICM-ID technique is presented as a practical application of the proposed system, where the correlation knowledge is utilized via the \textit{vertical} iterations at the destination. Both of the source and relay are assumed to have the same distance towards to destination. Readers may look at [7] for more detailed about this technique.

The performance of the system based on [7] is compared with the outage probability calculated using the method shown in this paper. The results are shown in Fig. 6 in terms of FER and outage probability versus average SNR of SR channel, where $R = 1$ for the technique show in In [7]. The FER curve with the BICM-ID based EF relay system is roughly 2 dB away from the outage curve. This is because the extrinsic information transfer (EXIT) curve of the channel code used in the BICM-ID is not exactly matched to the demapper EXIT curve, and hence there is a loss in information rate. The rate loss appears in the form of the 2dB SNR loss from the theoretical outage.

VI. CONCLUSION

In this work, the outage probability of a correlated source transmission system based on the Slepian-Wolf theorem has been analyzed over the block Rayleigh fading channel. The bit-flipping model was used to express the correlation model between the two sources, and the outage probability was theoretically calculated assuming that the receiver aims to decode the information sequence transmitted from the first transmitter. We then applied the results of the outage probability analysis to extract-and-forward (EF) strategy, where the relay only extracts the erroneous information sequence, interleavers, channel-encodes and transmits it to the destination. It has been shown that the proposed EF technique achieves better performances than the conventional decode-and-forward (DF) technique, so far as the error probability $p_e$ of the source-relay link is zero.

Furthermore, the impact of relay locations were evaluated, where the average SNR of the two channels are different. It has been found that in all the location scenarios considered, smaller outage probability can be achieved with the proposed EF relay than the conventional DF scheme.

APPENDIX A

Let $x = H(p_e)/\Gamma_1$, the exponential function can be Maclaurin-expanded as:

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} \approx 1 - x.$$  \hspace{1cm} (17)

According to Eq. (8), $P_1$ can be estimated as

$$P_1 = 1 - \exp(-x) \approx x.$$ \hspace{1cm} (18)

Obviously, when the average SNR becomes large, the value of $P_1$ is proportional to $x$ and hence the diversity order asymmetrically converges into 1.

APPENDIX B

When $p_e = 0$, $H(b_1|b_2) = 0$, $H(b_1) = 1$ and $P_1 = 0$. According to Eq. (4), $\text{SNR}\{H(b_1)\} = 1$, $\text{SNR}\{H(b_1|b_2)\} = 0$.
and $H(b_2, b_1) = 1 + H(b_1|b_2) = 1$. With $R_c = 1$, the Eq. (10) can be expressed as

$$P_2 = \int_{\gamma_1=0}^{1} A \, d\gamma_1.$$  \hspace{1cm} (19)

with A being

$$A = \frac{1}{\Gamma_1} \{ \exp \left( -\frac{\gamma_1}{\Gamma_1} \right) - \exp \left( -\frac{\gamma_1}{\Gamma_1} - \frac{2[1-\log_2(1+\gamma_1)]}{\Gamma_2} \right) \}. \hspace{1cm} (20)$$

Eq. (12) can then be expressed as

$$P_o,MRC = \int_{\gamma_1=0}^{1} B \, d\gamma_1.$$  \hspace{1cm} (21)

with B being

$$B = \frac{\gamma_1}{\Gamma_1^2} \exp \left( -\frac{\gamma_1}{\Gamma_1} \right). \hspace{1cm} (22)$$

Hence, it is found that with $p_e = 0$, Eq. (19) and Eq. (21) have the same integration range, but $A \neq B$. Hence, the outage probability of the Slepian-Wolf transmission system is different from that with the MRC scheme, both having the diversity 2.

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