Abstract—This paper proposes an iterative spatial demapping (ISM) scheme for simultaneous data exchange in three-way relaying systems where each user does not have direct links to the other users. Based on a priori knowledge provided by the decoders, ISM demapper makes the simultaneous data exchange only within two transmission phases by the help of a relay. The proposed structure is very simple since only two 2-state (memory 1) convolutional codes (CC) are used by each user: first CC is with rate-1 as doped accumulator (inner encoder) and the other is with rate-1/2 (outer encoder). The performances are evaluated by computer simulations in additive white Gaussian noise (AWGN) and two cascaded frequency-flat block Rayleigh fading channels. Bit-error-rate (BER) performance results show that clear turbo cliff can be achieved, with a transmit power close enough to the theoretical limit, for each user in AWGN channels. Extrinsic information transfer (EXIT) chart analysis is also presented to evaluate the convergence property of the iterative decoding algorithm. Since the proposed technique is spectrally efficient, it has a plenty of potential applications for future multi-way relaying systems where power and spectrum efficiency as well as energy consumption due to signal processing are of significant importance.

I. INTRODUCTION

Nowadays the development of portable wireless devices that enables conference calls in which each user in a group requires sending and receiving data to and from all the other members in the group has attracted attentions as a societal needs. Such situation is exemplified by an emergency rescue team in a devastated areas. In a satellite communication systems, the similar situation may happens between satellite and users on the earth, where the coverage and quality of the direct link is limited from the satellite to the users. To increase spectrum efficiency and extend the coverage, a practical solution is to introduce a relay terminal. The terminal is critical especially when each user does not have direct link to the other users or the satellites.

Studies on cooperative communications among multiple users with the help of relay(s) are presented, for example, in [1] where it is stated that feedback is still beneficial in a multiuser multi-way relay where the relay has a common output for each user. Ref. [2] provides the theoretical analysis where the relay has its own message to each user, while in [3] the practical iterative decoding technique is proposed by assuming that the direct link between the users is available. In [4], three dimensional network coding scheme is proposed for three-way relay channels using a cube with a Latin square-like constraint.

Many authors assume that each user transmits their messages in different dedicated time slots to avoid multiple access interference (MAI); it requires multiple time slots and scheduling strategies to achieve better performance [5]. With the insightful findings from [6] and [7], in this work, we consider data exchange in three-way relay channels as shown in Fig. 1. We propose a structure of iterative spatial demapping and decoding so that the data exchange is made possible only in with two transmission phases, where all users simultaneously transmit their information to the relay (Phase 1) and the relay broadcast their message as common output of the three-way relay channel (Phase 2). We refer Phase 1 and Phase 2 to as multiple access channel (MAC) and broadcast (BC) phases, respectively.

II. SYSTEM MODEL

We focus on single carrier transmission. Fig. 1 shows the system model considered in this paper. During MAC Phase, each user $U_i$ transmits message $s_{i,t} \in \{1, 2, 3\}$ simultaneously as indicated by the solid line. The relay (R) amplify-and-forwards (AF) the message from each user and broadcast the
composite signal $s_R$ to all users during BC Phase as indicated by the dashed line.\textsuperscript{1} It should be noted here that AF relaying is preferable in this situation to minimize the complexity of multiuser detection.\textsuperscript{2}

In this paper, we assume the messages $s_1, s_2,$ and $s_3$ are all binary phase shift keying (BPSK) modulated symbols with $\mathbb{E}[s_1^2] = \mathbb{E}[s_2^2] = \mathbb{E}[s_3^2] = 1$ having transmit powers $P_1, P_2,$ and $P_3,$ respectively. The complex channels coefficient between users and relay in MAC Phase and BC Phase, $h_{i1, h_{i2, h_{i3}}}$ and $h_{i1, h_{i2, h_{i3}}, h_{i3}}$, respectively, are assumed to be known to each corresponding destination;\textsuperscript{3} the destination in MAC Phase is the relay $R_i$, while in BC Phase is $U_i, i \in \{1, 2, 3\}$. No pre-processing before the transmission is assumed.

The received signal at the relay is expressed as
\begin{equation}
y_R = h_1^m \sqrt{P_1} s_1 + h_2^m \sqrt{P_2} s_2 + h_3^m \sqrt{P_3} s_3 + z_r, \quad (1)
\end{equation}
where $z_r \sim \mathcal{CN}(0, \sigma_r^2)$ is zero mean complex additive white Gaussian noise (AWGN) component with variance of $\sigma_r^2$ at the relay. The relay amplify-and-forwards the received composite signal $y_R$ to form symbols $s_R$ to be relayed with an amplifying factor $G$, as
\begin{equation}
s_R = \sqrt{G} y_R, \quad (2)
\end{equation}
where $P_r$ is the transmit power of the relay. By assuming that $\mathbb{E}[|h_i|^2] = 1$, we can re-express the amplifying factor $G$ in (2) as
\begin{equation}
G = \frac{P_r}{|y_R|^2} = \frac{P_r}{P_1 + P_2 + P_3 + \sigma_r^2}. \quad (3)
\end{equation}

Each user is assumed to know $G$ and is possible to subtract-off its own signal from the receive composite signal so that the received signal at each user $U_i$ is
\begin{align*}
y_i &= h_{i1} s_R - h_{i1} \sqrt{G} h_1^m \sqrt{P_1} s_1 + z_i, \\
&= h_{i1} \sqrt{G} \sum_{n=1,n\neq i}^3 h_{n1}^m \sqrt{P_n} s_n + h_{i1} \sqrt{G} z_i, \\
&= h_{i1} \sqrt{G} \sum_{n=1,n\neq i}^3 h_{n1}^m \sqrt{P_n} s_n + (h_{i1} \sqrt{G} + 1) z_i, \quad (4)
\end{align*}
where it was assumed that the variances of the noise components, $z_i, i \in \{1, 2, 3\}$, of each user and the relay are identical, i.e., $\sigma_i^2 = \sigma_r^2$, $z_i \sim \mathcal{CN}(0, \sigma_r^2)$.

We assume that each user transmit their signals with the same power $P_1 = P_2 = P_3 = P$; the variance of AWGN with each user is also assumed to be $\sigma_i^2 = \sigma_r^2 = \sigma_2^2 = 1$. Throughout the paper, we consider an amplification factor $G = 2$.\textsuperscript{4} From (1) and (3), the equivalent signal-to-noise power ratio
\begin{equation}
\gamma_i = \frac{PP_r}{3P + P_r + 1} = \frac{GP}{G + 1} = \frac{2P}{3}, \quad (5)
\end{equation}
which is smaller compared to the point-to-point (P2P) communication where with the same transmit power $P$, $\gamma_i = P$.

\section{Problem Formulation}

The challenge of three-way relaying with only two transmission phases is the demapping of superpositioned symbols received at the relay during the MAC Phase transmission because the channel $h_{i1}, h_{i2}, h_{i3}$ are complex random variable following the Rayleigh fading distribution. With the definition that the phase difference between the two channels are expressed as
\begin{equation}
\theta_{ii} = \angle(h_i, h_i). \quad (6)
\end{equation}
Obviously, $\theta_{ii} = 0, i \in \{1, 2, 3\}$ results in the worst performance.

Fig. 2(a)-(f) shows examples of the received signal constellations (only when $G = 1$ and $P = 1$) at the relay $R$ and at the destination $U_{ij}$ for a channel realization and for 1,000 random channel realizations; it is confirmed from Figs. 2(b), 2(d), 2(f) that detection of symbols at each user is simpler compared with the detection at relay (Figs. 2(a), 2(c), 2(e)) because each user can subtract-off their own symbols that reduces the computation of symbol detections. This fact motivates us to use amplify-and-forward protocol at the relay. However, the
difficulty still remains especially when the phase difference \( \theta_{jk} = 0 \) or when the number of users increases. In this paper, we propose a solution based on the insightful findings from [6] and [7] to perform iterative spatial demapping at each user in a three-way relaying systems.

IV. PROPOSED DECODING STRATEGY

A. Source

Our proposed decoding strategy for three-way relay systems is shown in Fig. 3. Three sources \( b_1, b_2, \) and \( b_3 \) are transmitted from three separated single antennas simultaneously. The bitstream \( b_i \) is convolutionally encoded using a 2-state rate-1/2 CC with generator polynomial \([2, 3]_8\), denoted as \( C_1 \), interleaved by \( \Pi_1 \), doped accumulated, and BPSK-modulated to produce BPSK symbol \( s_1 \).

The rate-1 doped accumulator (D-ACC) is applied so that the convergence tunnel open until a point very close to the \((1,1)\) mutual information (MI) point in the extrinsic information transfer (EXIT) chart. As shown in Fig. 1, D-ACC is performed after the interleaver \( \Pi_1 \). The structure of D-ACC is very simple since it is composed of a memory-1 systematic recursive convolutional codes (SRC) with octal code generator of \([3, 2]_8\) followed by heavy puncturing of the coded bits such that coding rate = 1. With doping rate \( Q_i, i \in \{1, 2, 3\} \), the D-ACC at user \( U_i \) replaces every \( Q_i \)-th systematic bits \( (b_i) \) with the accumulated coded bit. For the decoding of D-ACC at the receiver, denoted as \( D_{dacc} \), it uses the Bahl-Cokce-Jelinek-Raviv (BCJR) algorithm immediately after the demapping.\(^5\)

B. ISM Demapper

Let’s consider, for example, user \( U_3 \). As shown in Fig. 1(b), with the help of \textit{a priori} information about \( s_1 \) provided by the decoder \( D_1 \) in the form of \( L_{a_1, M} \), the demapper calculates the extrinsic LLR \( L_{e_2, M} \) of the symbol \( s_2 \) from the received signal \( r \) by

\[
L_{e_2, M} = \ln \frac{\Pr(s_2 = +1 | r)}{\Pr(s_2 = -1 | r)} + \sum_{S \in S_{+1}} \exp \left\{ - \frac{|r - \chi_1 s_1 - \chi_2 s_2|^2}{(G + 1)\sigma^2} + b_1 L_{a_1, M} \right\}.
\]

where \( \chi_1 = \sqrt{G} h_1 b^m_1, \quad \chi_2 = \sqrt{G} h_2 b^m_2 \), and \( S_{+1}, S_{-1} \) are the sets of symbols having symbol \( s_1 \) being +1 and -1, respectively, with \( s_1 = 1 - 2b_1 \). Similarly, with the help of \textit{a priori} information about symbol \( s_2 \) provided by decoder \( D_2 \) in the form of \( L_{a_2, M} \), the extrinsic LLR \( L_{e_1, M} \) of symbol \( s_1 \) is given by

\[
L_{e_1, M} = f_{ism}(\sigma_3, L_{o_2})
\]

with \( s_2 = 1 - 2b_2 \).

The extrinsic LLRs, \( L_{a_1, M} \) and \( L_{a_2, M} \), are then used in doped deaccumulator \( D_{dacc} \), deinterleaved by \( \Pi_1^{-1} \) and \( \Pi_{2}^{-1} \), respectively, to provide a \textit{a priori} LLR, \( L^f_{e_1, D_1} \) and \( L^f_{e_2, D_2} \), of the coded bits for the decoders \( D_1 \) and \( D_2 \). This algorithm also applies for users 1 and 2, \( U_1 \) and \( U_2 \), respectively.

C. Destination

The destination consists of a common ISM demapper, and two independent turbo loops because two sources are to be detected. ISM demapper performs demapping of spatial constellation from \( y_i \) with the help of \textit{a priori} information in the form of LLR provided by the decoder. The demapper output is extrinsic LLR which is then fed into \( D_{dacc} \), deinterleaved, and then decoded by the decoder \( D_1 \).

V. PERFORMANCE EVALUATION AND ANALYSIS

The performances of the proposed ISM demapper is evaluated using computer simulations by assuming a single carrier block transmissions with block length of \( K = 10,000 \) bits. Iterative decoding is performed with a pattern of 50(1\(D_1, 1M, 1D_2\), where at the \( n \)-th iteration, decoding is performed only once with an order of user \( U_1 \), ISM demapping, and user \( U_2 \). This process is repeated 50 times with a stopping criteria of \( E = 7.6 \). The reason of using 50 iterations is because the use of D-ACC, with appropriate doping ratio \( Q_i, i \in \{1, 2, 3\} \), results in a better matching condition between the EXIT curve of ISM demapper and the decoders so that even with a small convergence tunnel, the \((1,1)\) MI point is still achievable if the iteration number is enough.\(^7\)

\(^5\)The value \( E \) is a number of event when bit-error-rate (BER) at the \((n+1)\)-th iteration is larger than BER at \( n \)-th iteration.

\(^7\)Iterations with 50 (in maximum) is still reasonable in practice because the proposed decoder is very simple, i.e., memory-1 CC.
A. Theoretical Limit

For the theoretical limit analysis, in this paper we refer [8] with the number of users $K = 3$ and cluster $L = 1$. With the user’s equivalent SNR, $\gamma_i$, given by (5), the Shannon limit for each user in three-way relaying system is [8]

$$R = \frac{1}{L(K-1)} \frac{G}{G+1} \frac{PP_i}{KP + P_r + 1} = \frac{1}{2} \frac{GP_i}{G+1}.$$  \hspace{1cm} (10)

With the code rate $R = 1/2$, we have $C(2E) = 1$, where $C(\gamma_i) = \log_2(1 + \gamma_i)$.\(^8\) The theoretical power limit, $P_{lim}$, is

$$P_{lim} = \frac{3}{2} = 1.76 \text{ dB}. \hspace{1cm} (11)$$

B. BER and FER Performances

BER performance is evaluated in two conditions: (a) AWGN channel with random phase difference $\theta$ (equivalent to Rayleigh fading channel with instantaneous power control) and (b) 1-path block Rayleigh fading channel. The BER results in AWGN channel ($h_1^{m}h_1^{i} = h_2^{m}h_2^{i} = h_3^{m}h_3^{i} = 1$) are plotted in Fig. 4(a).

With this setup, it is reasonable to plot BER curve of physical-layer network coding (PLNC) [9] for reference, instead of the theoretical uncoded BPSK.\(^9\) BER curve with the theoretical uncoded physical layer network coding (PLNC), presented by [9], [10]

$$P_b = \frac{1}{2} \text{erf} \left( \frac{T}{\sqrt{N}} \right) + \frac{1}{2} \text{erf} \left( \frac{T+2}{\sqrt{N}} \right) + \frac{1}{4} \text{erf} \left( \frac{T-2}{\sqrt{N}} \right),$$  \hspace{1cm} (12)

are shown in Fig. 4(a), where $T$ is the optimum threshold given by $T = 1 + \frac{2}{\sqrt{N}} \text{ln}(1 + \sqrt{1 - e^{-\frac{N}{4}}})$ [9], and $\text{erf}(x) = 1 - \text{erfc}(x)$. However, because of the Amplify-and-Forward protocol at the relay, the noise amplification by a factor of $G$ should be considered in the PLNC theoretical curve by replacing $\frac{1}{\sqrt{N}}$ in (12) with

$$\gamma_i = \frac{1}{\sqrt{N}} = \frac{GP_i}{G+1}, \hspace{1cm} \text{Average BER}$$  \hspace{1cm} (13)

and use (13) as equivalent SNR when viewing the simulation setup with $\theta_{ri} = 0$ as being equivalent to PLNC. Both the theoretical uncoded PLNC BER and equivalent PLNC (AF with $h_i^{2} = 1, \theta_{ri} = 0$) BER curves are plotted in Fig. 4(a) to clarify the effect of noise amplification. It is observed from the figure that the coding gain in the proposed iterative decoding is $8.15 - 3 = 5.15 \text{ dB}^{10}$ from the theoretical uncoded PLNC with average equivalent SNR at BER of $2 \times 10^{-5}$. It is also found that with the proposed iterative decoding the gap to the theoretical power limit $P_{lim}$ is 1.12 dB.

In practice, we can not always assume power control. Therefore, evaluation for the channel with $h_1^{m}h_1^{i} \neq h_2^{m}h_2^{i} \neq h_3^{m}h_3^{i} \neq 1$ is necessary. With the assumption that all links

\(^8\)Gaussian codebook assumption is reasonable at low SNR.

\(^9\)It is due to the superposition of the signals since the Source-Relay link is a MMC channel. Therefore, making PLNC as reference is reasonable than the theoretical uncoded BPSK.

\(^{10}\)The bandwidth expansion due to coding rate $R = 1/2$ is taken into account.

are suffering from the frequency-flat block-Rayleigh fading, average BER curve is plotted in Fig. 4(b) vs. transmit power $P$, where the average equivalent SNR is $\bar{\gamma} = 2P/3$. It is found that the decay of the curve is the same as the the decay of theoretical symbol-error-rate (SER) curve of two cascaded channels [11]. It can also be observed that with the same equivalent SNR, the degradation in average BER of three-way relay using the proposed ISM, compared with the theoretical BER of P2P link, is 3.38 dB at average BER of $10^{-4}$.

Frame-error-rate (FER) performance in frequency-flat block-Rayleigh fading channels is also shown in Fig. 4(b). The FER performance curve also has the same decay with the SER performance curve [11], however, the degradation compared to the theoretical outage probability [12] at FER of $10^{-4}$ is 7.73 dB, because of the two cascaded frequency-flat block Rayleigh fading channels as indicated by (8).
C. EXIT Analysis

This section describes EXIT analysis to evaluate the convergence properties of the proposed iterative spatial demapper and its corresponding decoders. At destination $U_3$, Fig. 5 shows the projected EXIT curves of the combined ISM demapper and the Memory-1 CC decoder $D_2$,\textsuperscript{11} where the extrinsic MI, $I_{e_1,M}$, is calculated after performing iteration for $b_2$, given the a priori MI $I_{a_1,M}$. It is assumed in Fig. 5 that $P = 3$ dB, and $\theta_1 = 0$, $h_1^m h_3^b = h_2^m h_1^b = h_3^m h_3^b = 1$, representing the worst case. With $\theta_1 \neq 0$ the ISM demapper EXIT curves have wider convergence tunnel, and hence the performance is better with the same or even lower SNR.

In Fig. 5, the X-axis is a priori MI of the ISM demapping $I_{a_1,M} = I(x^m; L_{a_1,M}^m)$, which is equivalent to the extrinsic MI of the decoder $D_1$, $I_{e_1,D_1} = I(x; L_{e_1,D_1}^m)$, $x$ and $x^m$ are shown in Fig. 1; $L_{a_1,M}^m = \Pi_1(L_{e_1,D_1}^m)$ with $\Pi_1(\cdot)$ being the $\Pi_1$ interleaver function. Here,

$$L_{a_1,M}(k) \equiv \begin{cases} L_{a_1,M}^m(k), & k \neq nQ_1, n = 1, \ldots, K, \\ 0, & \text{otherwise}, \end{cases}$$

where $L_{a_1,M}^m(k)$ is the a priori LLR for $D_{1\text{acc}}$ and $k$ is the bit-index. Eq. (14) applies for $L_{a_2,M}(k)$ with $Q_2$. The Y-axis is the extrinsic MI, $I_{e_1,M}$, from the ISM demapper and a priori MI, $I_{a_1,D_1}$, to the decoder $D_1$.

The decay of ISM demapper EXIT curve depends on each user’s transmit power $P$, the doping rate $Q_1, Q_2, Q_3$, and the power at the amplifying factor $G$. EXIT curve shown in Fig. 5 is for $G = 2$, $Q_1 = Q_2 = Q_3 = 4$, and for user $U_3$ with $P = 3$ dB.\textsuperscript{12} Since the EXIT curve of the memory-1 CC decoder is below the ISM demapper EXIT curve, and their decay is similar, a turbo cliff in BER curve is expected, and since the convergence tunnel opens until a point very close to the (1,1) MI point, no error-floor is expected.\textsuperscript{13} A trajectory is plotted in Fig. 5 to confirm the convergence property indicated by EXIT curves. The results and analysis above confirmed that very simple memory-1 CC is enough to obtain close-capacity limit performance; it allows us to avoid the use of strong code, which, in general, requires high computational complexity.

VI. CONCLUSIONS

An iterative spatial demapping (ISM) technique has been proposed for simultaneous data exchange in three-way relaying systems with very simple structure, where only two 2-state (memory 1) convolutional codes are used by each user. Based on the a priori log-likelihood ratio (LLR) provided by the decoder, the ISM demapper can distinguish simultaneously the transmitted information, which allows the three users to exchange information simultaneously within only two transmission phases. The BER performance results as well as EXIT chart analysis show that the performance in AWGN channel has clear turbo cliff with each user’s transmit power $P = 2.875$ dB, $G = 2$, and noise variance $\sigma_R^2 = \sigma_f^2 = 1$, $i \in \{1, 2, 3\}$. However, because the channels in three-way relaying systems can be seen as a concatenation of MAC and BC channels, the degradation in average BER and SER compared with that in P2P communications are unavoidable in the exchange of benefiting the simultaneous data exchange within only two transmission phases.

REFERENCES


\textsuperscript{11}It should be noted that with an EXIT projection, the ISM demapper and the decoder $D_2$ can be seen as a new set of “decoder” by decoder $D_1$.

\textsuperscript{12}The similar curve applies for other users due to the same channel encoder, rate and power $P$.

\textsuperscript{13}At least it is unobservable at BER $\geq 10^{-5}$ as confirmed by Fig. 4(a).