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Description	

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Utilization of 2-D Markov Source Correlation using Block Turbo Codes

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***Abstract*—This paper proposes a joint source-channel coding technique using high rate block turbo codes where two-dimensional (2-D) source correlation of binary Markov source is well exploited. A modified Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm is used in both horizontal and vertical decoders to exploit the 2-D source correlation. Based on extrinsic information transfer (EXIT) chart analysis and trajectory evaluation, we found out that the log-likelihood ratio (LLR) correlation for the block turbo codes increases as the source correlation becomes stronger and this prevents iterative decoding from convergence. A memory-1 accumulator with random interleaver is added to the design to minimise the LLR correlation and to achieve better performance when the source correlation is very strong. Further improvement can be achieved by replacing the memory-1 accumulator with a longer memory code to further reduce the statistical dependency of the LLR. It is shown that the proposed technique can achieve significant improvement over the standard system where the source correlation is not exploited.**

I. Introduction

In classic communication systems, the optimisation of source coding and channel coding has been performed independently according to the Shannon separation theorem [1]. In a point-to-point communications, the separation theorem hold as long as the entropy rate is less than the channel capacity. However, this theorem is based on the assumption that infinite length codes are used for the source and channel coding processes and therefore, it is too idealistic to be applied in practical communication systems having latency restrictions. Furthermore, most existing source coding schemes are not fully optimised and hence, there will be residual redundancy left after source encoding. These drawbacks have motivated considerable number of works which invoke the ideas of joint source-channel coding (JSC).

In 1993, a breakthrough in channel coding was made by Berrou *et al,* where a new class of channel coding scheme, called turbo codes was proposed [2]. Due to its excellent performance, several works have considered the utilization of the turbo codes in JSC systems. JSC technique shown in [3]–[5] characterizes the source based on Markov model and exploits the memory structure inherent within the source during decoding process at the receiver. Encoder optimization and modification for the utilization of the source statistic are considered in [6], [7] in addition to decoding algorithm modifications. Recently, to exploit the source statistic in serial concatenated convolutional codes (SCCC), a modified BCJR algorithm has been proposed in [8], where the superiority in performance is demonstrated without requiring high computational complexity. The techniques that have been proposed so far only exploit the source correlation in one direction, which is referred to as one dimensional (1-D) source correlation.

In this work, a novel JSC technique exploiting 2-D source correlation is proposed. The 2-D source correlation in horizontal and vertical directions is exploited at the channel decoder. The modified version of the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm in [8] is used at horizontal and vertical decoders to exploit the 2-D correlation. The proposed JSC model uses block turbo codes (BTC), which is suitable for the utilization of high rate codes, as the constituent channel codes, and also can achieve good bit error rate (BER) performance [9]. We show in this paper that in the case of highly correlated source, adding an inner code to the standard BTC-based design will further improves the BER performance.

The remainder of the paper is organized as follows. Brief description of the system model based on the standard BTC is presented in section II. Section III describes the performance improvement achieved by adding a rate-1 inner code in the standard BTC design. The simulation result for image transmission over a channel suffering from additive white Gaussian noise (AWGN) using the proposed method is presented in section IV. Finally, concluding remarks are provided in Section V.

II. System Description

First order binary Markov source is considered in this work to model the source behaviour. We first describe the source in 1-D case and later, we will extend it to the 2-D case.

At a time t, the Markov source U_t is determined by its value at the time $t-1$, according to the conditional probability $Pr\{U_t|U_{t-1}\}$. In a binary case, there are four possible transitions which can be conveniently represented by

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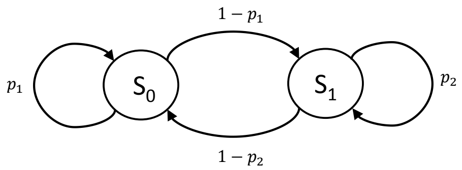

Fig. 1. Two-state Markov chain. State S_i emits binary output i, $i \in \{0, 1\}$

the state transition matrix:

$$\mathbf{A} = [a_{i,j}] = \begin{bmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{bmatrix}, \tag{1}$$

where p_1 and p_2 are the transition probabilities as shown in Fig. 1. S_0 and S_1 are referred as the state that emits "0" and "1", respectively.

The entropy rate of stationary first order Markov source [10] can be calculated by:

$$H(S) = -\sum_{i,j \in \{0,1\}} \mu_i a_{i,j} \log a_{i,j}, \tag{2}$$

where μ_i is the stationary state distribution.

In the case of the 2-D source, the current value of the source $U_{t,n}$ is determined by two values, which are the previous value in horizontal direction $U_{t-1,n}$ and vertical direction $U_{t,n-1}$. This can be viewed as the coupling of two first order Markov chain. Based on the couple Markov chain (CMC) model in [11], the transition matrix $\mathbf{B}$ for 2-D source can be obtained by using the 1-D source transition matrices $\mathbf{A}^H$ and $\mathbf{A}^V$ for the horizontal and vertical directions, respectively, as

$$\begin{aligned} \mathbf{B} = [b_{i,j,k}] &= Pr\{U_{t,n} = k | U_{t,n-1} = i, U_{t-1,n} = j\} \\ &= \frac{a^H_{i,k} \cdot a^V_{j,k}}{\sum_{f=0}^{1} (a^H_{i,f} \cdot a^V_{j,f})}, \quad i, j, k \in \{0, 1\}, \end{aligned} \tag{3}$$

where $a^H_{i,k}$ and $a^V_{j,k}$ are the elements of the matrices $\mathbf{A}^H$ and $\mathbf{A}^V$, respectively. The entropy rate for the 2-D case can be defined as

$$\begin{aligned} H(U_{t,n}|U_{t,n-1}, U_{t-1,n}) = H(U_{t,n-1}, U_{t-1,n}|U_{t,n}) + H(U_{t,n}) \\ - H(U_{t,n-1}, U_{t-1,n}), \end{aligned} \tag{4}$$

which, by assuming $U_{t-1,n}$ and $U_{t,n-1}$ are independent given $U_{t,n}$, can be reduced to

$$\begin{aligned} H(U_{t,n}|U_{t,n-1}, U_{t-1,n}) &= H(U_{t-1,n}|U_{t,n}) + H(U_{t,n-1}|U_{t,n}) \\ &\quad + H(U_{t,n}) - H(U_{t,n-1}, U_{t-1,n}), \end{aligned} \tag{5}$$

$$\begin{aligned} &= H_H(S) + H_V(S) + H(U_{t,n}) \\ &\quad - H(U_{t,n-1}, U_{t-1,n}). \end{aligned} \tag{6}$$

$H_H(S)$ and $H_V(S)$ can be obtained by using (2) for horizontal and vertical directions, respectively, while $H(U_{t,n})$ and $H(U_{t,n-1}, U_{t-1,n})$ can be determined empirically. Note that in the case of symmetric Markov chain for both directions, $H(U_{t,n}) = 1$.

The proposed scheme is based on the diagram shown in Fig. 2 where BTC is used as channel coding. It is worth mentioning that the BTC structure used in this paper is different from product codes, whereby the redundancy part that is used for checking the parity of the parity of both codes is omitted. The 2-D source has a length of $K_1 \times K_2$, whereby K_1 and K_2 indicate the numbers of columns and rows of the 2-D source, respectively. Two Bose, Chaudhuri, Hocquenghem (BCH) codes, C_1 and C_2 are used to encode the information sequence of each row and each column, respectively. C_1 adds parity bits to the length K_1 information sequence to produce length N_1 codeword for each row. In the same way as C_1, the length N_2 sequence is produced by C_2 for each column. C_1 can be represented by the parameter set (N_1, K_1, D_1) and C_2 by (N_2, K_2, D_2) where D_1 and D_2 are the minimum Hamming distances of C_1 and C_2, respectively. In this paper, we assume identical BCH codes are used for C_1 and C_2 and therefore $K_1 = K_2 = K, N_1 = N_2 = N$ and $D_1 = D_2 = D$. Block interleaver, π, with size of $K \times K$ is used for C_2 to encode the information sequence column by column. The use of block interleaver is important to maintain the source correlation in the vertical direction. The information bit sequences together with the parity sequences of each row and each column are multiplexed and modulated using binary-phase shift keying (BPSK), and transmitted to an AWGN channel where the noise variance is σ^2.

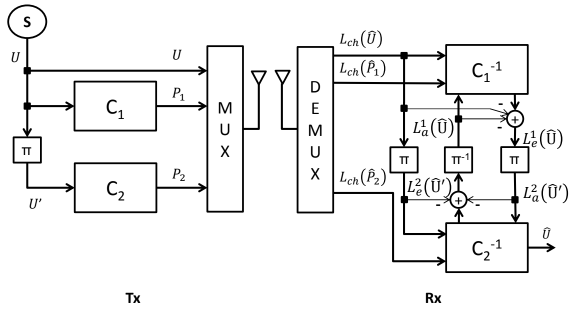

Fig. 2. BTC with decoder C_1^{-1} and C_2^{-1} use modified BCJR algorithm to exploit the 2-D source correlation during decoding process

At the receiver, two channel decoders C_1^{-1} and C_2^{-1} are used to decode the information where the decoders iteratively exchange the extrinsic information. In order to exploit the source correlation during the channel decoding process, the standard BCJR algorithm [12] is modified. The modified BCJR algorithm proposed by Xiaobo *et al.* for 1-D correlated source in [8] is used in this work. The modified BCJR algorithm is applied to the decoders, where C_1^{-1} utilises the correlation in the horizontal direction while C_2^{-1} utilises the source correlation in the vertical direction. By using the modified BCJR algorithm, extra information from the source correlation is gained by the decoders and hence it is expected that the technique outperforms the system using the standard BCJR where the source correlation is not exploited.

A. BER Performance

Simulations were conducted to evaluate the BER performance of the proposed scheme. When evaluating the per-

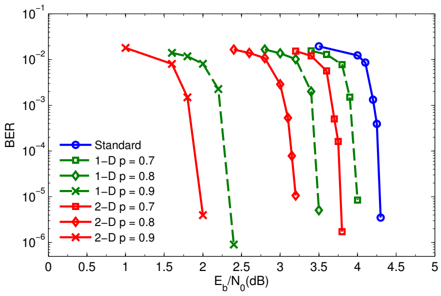

Fig. 3. BER performance with various source correlation after 12 iterations of BTC using BCH$(255, 247, 3)$ as the constituent code, $R_c = 0.94$

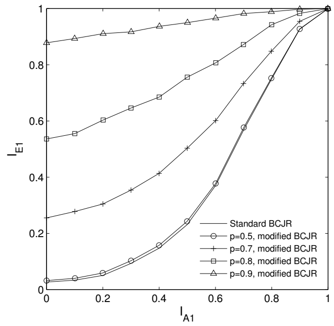

Fig. 4. EXIT chart of decoder C_1^{-1} with various p values using BCH$(255, 247, 3)$ as the constituent code at $E_b/N_0 = 3.5$ dB

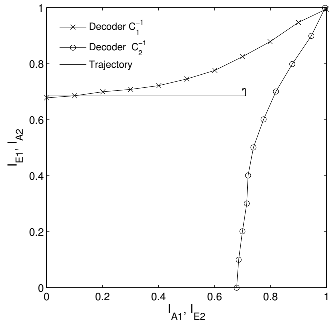

Fig. 5. EXIT chart and trajectory for the iterative decoding of BTC utilising 2-D source correlation $p = 0.9$ at $E_b/N_0 = 0.5$ dB

formance of the proposed scheme with various correlation strengths, symmetric Markov source was assumed i.e., $p_1 = p_2 = p$, and the same correlation is used in the horizontal and vertical directions. The p value is used to indicate the strength of the source correlation where $p = 0.5$ corresponds to the weakest correlation due to equal chance of having value 0 and 1 given the previous value. The correlation becomes stronger as the p value deviates from 0.5 either towards 0 or 1. Hence we only consider $1 > p \geq 0.5$ since the correlation is symmetric with $0 < p \leq 0.5$. BCH$(255, 247, 3)$ is used as the constituent code C_1 and C_2 for the BTC, and with this setting the overall code rate of the system $R_c = 0.94$. Both decoders use the modified BCJR algorithm in order to utilise the source correlation. As shown in Fig. 3, the stronger the source correlation, the better BER can be achieved. On the contrary, the standard system which exploits no correlation information in the BCJR algorithm does not achieve any gains in BER performance. For 1-D case, only C_2 decoder uses the modified BCJR algorithm for the utilisation of source correlation in one direction. Fig. 3 clearly indicates that the proposed scheme outperforms both the 1-D and standard systems. The gain achieved compared to the standard scheme becomes larger as the p value approaches 1 because more information can be gained through the iterations, as the correlation becomes stronger.

B. EXIT Chart Analysis

The additional information gained by exploiting the source correlation can be represented by the extrinsic information transfer (EXIT) chart [13]. Fig. 4 shows the EXIT chart at $E_b/N_0 = 3.5$ dB for the standard and the proposed schemes with various p values. The EXIT function of decoder C_1^{-1} and C_2^{-1} are expressed as

$$I_{E1}(\hat{U}) = T(I_{A1}(\hat{U}), E_b/N_0), \quad (7)$$

$$I_{E2}(\hat{U}) = T(I_{A2}(\hat{U}), E_b/N_0), \quad (8)$$

where I_{E1} and I_{E2} are the extrinsic mutual information for C_1^{-1} and C_2^{-1}, respectively and the a priori mutual information is denoted by I_{A1} and I_{A2} for C_1^{-1} and C_2^{-1}, respectively.

It is found that with our proposed 2-D scheme, the EXIT chart does not indicate the actual BER performance of the system. This can be seen in Fig. 5 where we have plotted the trajectory together with the EXIT curves for $p = 0.9$ at $E_b/N_0 = 0$ dB. Although the tunnel is open until the (1,1) mutual information (MI) point, the trajectory is stuck in the middle point. This phenomenon happens because the extrinsic log-likelihood ratio (LLR) between the output and input of each constituent decoder is highly correlated after first iteration.

III. Proposed Technique

A. BTC with Accumulator

In order to reduce the statistical dependency in the 2-D case, this paper proposes the use of a rate-1 accumulator (ACC) C_3 which is a memory-1 recursive systematic convolutional (RSC) code with generator polynomial $(G_r, G) = (3, 2)_8$, together with random interleaver when designing the BTC, as shown in Fig. 6 and Fig. 7 for the transmitter and receiver block diagrams, respectively. The proposed structure can be seen as combination of serial and parallel concatenated codes to form a hybrid concatenated codes. In the standard system where the decoders use the standard (not modified) BCJR algorithm, BER performance is degraded as shown in Fig. 8 with the use of ACC. When both decoders use the modified algorithm, the performance also degrades with $p = 0.7$. However the BER improvement by having an ACC can be seen as the source correlation becomes stronger, as shown in Fig. 8 for $p = 0.8$ and $p = 0.9$. The gain compared to the performance of using the standard BTC structure increases as the correlation

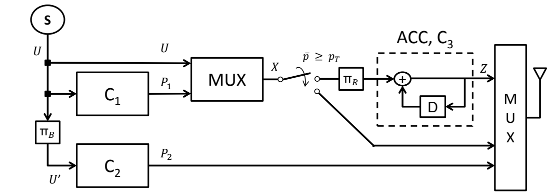

Fig. 6. Transmitter structure of the proposed scheme with ACC

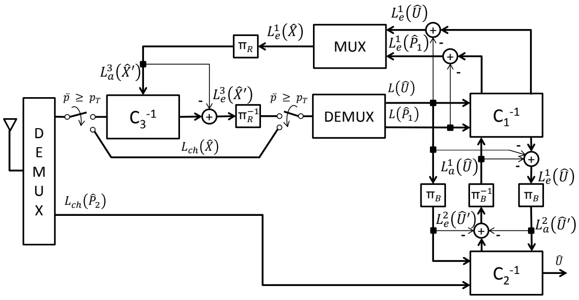

Fig. 7. Receiver structure of the proposed scheme

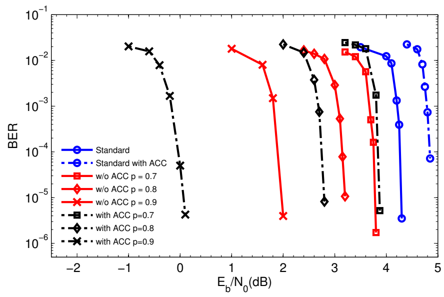

Fig. 8. Comparison of BER performance of the standard BTC design (without ACC) and the proposed scheme with ACC with various source correlation, employing 12 iterations

becomes stronger. Therefore, switches are included in the design in order to make sure that any degradation will not occur even if the source correlation becomes less than the threshold value p_T. The optimal p_T value is determined empirically; with BCH$(255, 247, 3)$, we have found out that $p_T = 0.74$ and this is the point where the system with the ACC starts to achieve better performance than without it.

The proposed scheme with BCH$(255, 247, 3)$ has code rate $R_c = 0.94$, and when an Entropy 1 random sequence is decoded with the standard BCJR decoder, the performance of the standard BTC shown in Fig. 3 at BER 10^{-5} is about 2.81 dB away from the Shannon limit (SL) as shown in Table I. In the case of source with memory, the SL can be found from the capacity equation for AWGN per dimension, $C = \frac{1}{2}\log_2(1 + \frac{2E_bR_c}{N_0})$ and based on condition $H(U_{t,n}|U_{t,n-1}, U_{t-1,n})R_c \le C$ we have

$$(\frac{E_b}{N_0})_{lim} = \frac{2^{2H(U_{t,n}|U_{t,n-1},U_{t-1,n})R_c} - 1}{2R_c}. \quad (9)$$

However, the shaping loss from the Gaussian SL tends to be larger when comparing the performance of high rate

TABLE I
GAP OF THE PROPOSED SYSTEM TO THE SHANNON AND CCC LIMITS FOR DIFFERENT SOURCE CORRELATION

p	H	SL(dB)	CCC(dB)	Gap-SL(dB)	Gap-CCC(dB)
Standard	1.00	1.54	3.96	2.81	0.39
0.7	0.78	-0.27	0.43	4.05	3.35
0.8	0.54	-2.66	-2.10	5.46	4.90
0.9	0.26	-6.66	-6.63	6.66	6.63

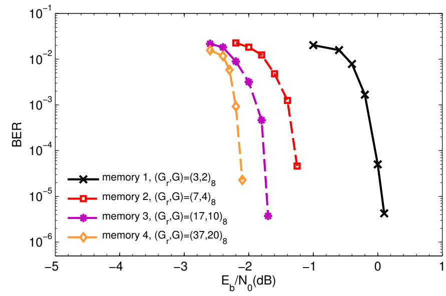

Fig. 9. BER performance of different RSC codes for C_3 utilising 2-D source correlation with $p = 0.9$, employing 12 iterations

codes (i.e. BTC) with the SL. In this case, constellation constraint capacity (CCC) [14] should be used as a limit of a system utilizing high rate codes. In the case of source with memory, $(\frac{E_b}{N_0})_{lim}$ for CCC can be obtained by applying the same condition $H(U_{t,n}|U_{t,n-1}, U_{t-1,n})R_c \le C$ to the CCC equation in [14]. The gap of the proposed scheme to the SL and the CCC (BPSK) for various p values are summarized in Table I. The difference between the SL and CCC becomes smaller as the source correlation becomes stronger but the distance of the proposed scheme to both limits becomes larger with stronger source correlation. The ACC with random interleaver helps in reducing the LLR correlation but is still unable to keep the LLR correlation completely uncorrelated especially in the case of strong source correlation.

B. BTC with Longer Memory Inner Codes

The ACC used as the inner code C_3 in the proposed design is replaced by a longer memory RSC code in this sub-section. Fig. 9 shows the BER performance when using various RSC codes of different memory lengths, as the rate-1 inner code C_3, with $p = 0.9$ and both decoders C_1^{-1} and C_2^{-1} use the modified BCJR algorithm. It can be seen that better performance can be achieved with longer memory codes; with memory-2 $(G_r, G) = (7, 4)_8$ code, the improvement at BER 10^{-5} over the system with the memory-1 ACC is 1.2 dB. The improvement of 1.7 dB with memory-3 code, and 2.1 dB with memory-4 code.

From the EXIT chart and trajectory illustrated in Fig. 10, it can be observed that the trajectory is no longer stuck in the middle point and reaches the (1,1) MI point when using longer memory inner codes. This shows that when using a longer memory code, the problem with the LLR correlation investigated before can be significantly reduced, resulting in better BER performance hence reducing the gap to the CCC

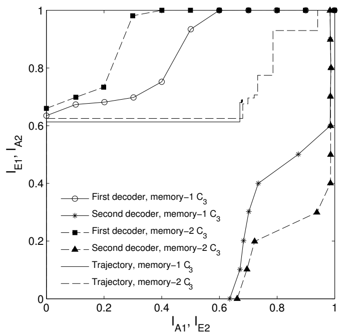

Fig. 10. Comparison of EXIT curve (projection) and trajectory for C_1^{-1} and C_2^{-1} at $E_b/N_0 = -1.2$ dB when using RSC code memory-1 $(3,2)_8$ and memory-2 $(7,4)_8$ as the constituent code for C_3 with $p = 0.9$

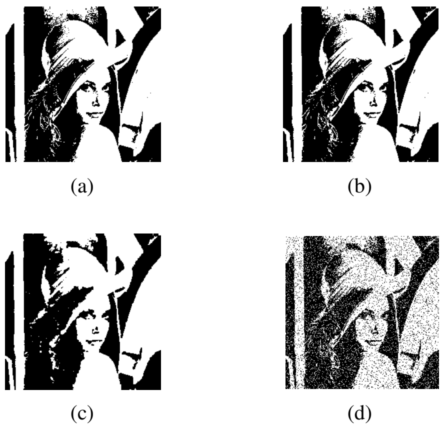

Fig. 11. Image source with $\bar{p} = 0.92$ at $E_b/N_0 = -1$ dB , (a) transmitted image (b) 2-D ACC (0% error) (c) 2-D without ACC (2.74% error) (d) standard (11.02% error)

limit. However in the case of weak source correlation, longer memory inner codes degrade further the BER performance.

IV. Image Transmission Simulation

We have conducted a simulation for transmitting an image through the proposed scheme. The image has been quantized to form a binary matrix before it is fed into the encoder. The transition probability p of each row and column of the matrix is measured, and the average p, $\bar{p}$, is calculated. Based on the proposed block diagram in Fig. 6 and Fig. 7, $\bar{p}$ is used to determine beforehand whether to fed into or bypass the ACC. Fig. 11(a) shows the transmitted image with size 247×247 pixel dimensions and $\bar{p} = 0.92$, where the source information is fed into the ACC when using the proposed scheme since $\bar{p} \geq p_T (= 0.74)$. Fig. 11(b) shows the result of the image at the receiver with $E_b/N_0 = -1$ dB after 15 iterations by using the proposed 2-D scheme. The result is compared with the other schemes as shown in Fig. 11(c) and Fig. 11(d) for 2-D without ACC and standard BTC without utilising the source correlation, respectively. The proposed scheme with ACC achieves the highest image quality with 0% error and the standard BTC without utilising the source correlation results in the lowest quality with error of 11.02%.

V. Conclusion

In this paper, a novel technique, 2-D source correlation exploited by using block turbo codes, has been proposed. The two decoders for the constituent codes use the modified version of the BCJR algorithm in order to utilise the source correlation in the horizontal and vertical directions, by assuming the source correlation property is known to the receiver. Unfortunately the expected BER performance can not be achieved, and the trajectory is not consistent with the EXIT chart due to the LLR correlation. Therefore, we have investigated the use of an inner code to reduce the effects of the LLR correlation and showed that in the case of strong source correlation, the longer the memory of the inner code the better the performance, and hence it reduces the gap to the theoretical limit. The use of the inner code only provides advantage with strong source correlation, but on the contrary it causes degradation of performance with weak source correlation. An adaptive system has been proposed to bypass or pass through the inner code based on the average correlation of the 2-D source. Finally, we have demonstrated the effectiveness of the proposed technique by the image transmission simulation.

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