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Description	

Game Information Dynamics and Its Application To Congkak and Othello

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***Abstract*—This paper is concerned with uncertainty of game outcome in Congkak and Othello. Firstly, an information dynamic model of uncertainty of game outcome is derived based on fluid mechanics. Secondly, data analyses of Congkak and Othello have been done. It is found that Congkak is a unique regional game in South-East Asia, while Othello is one of the most entertaining games in the globe. It is suggested that Shannon's entropy provides a measure of uncertainty of game outcome, but not itself. The true uncertainty is given by the present proposed model.**

Index Terms- Uncertainty of game outcome, Information dynamic model, Congkak, Othello, Entropy, Entertainment.

I. Introduction

A fundamental problem of information communication is that of reproducing at one point, either exactly or approximately, information selected at another point. Frequently the information has meaning, that is, it refers to or is correlated according to some system with certain physical or conceptual entities. The significant aspect is that actual information is one selected a set of possible information. In the present paper, the selected information is evaluation function scores in Congkak and Othello. Information of game outcome here represents the data which is the uncertainty of game outcome. We consider that information is produced as the motion of particles, for stationary particles provide only trivial information. In this regard, it has been inferred by [1] that motion of visualized fluid particles, for example, is detected by the eye almost instantaneously through light having enormous high speed, 3×10^8 m/s, and is mapped on the retina. It may be evident that during this process, motion of fluid particles is transformed into that of "information particles" by light carrying the images of fluid particles. The eye and brain may work together in collecting the light reflected from the visualized fluid particles and processing the information particles, which flow in our brain.

Shannon[2] has introduced quantities of the form

$$\mathrm{H(X)} = -\sum p_i \log p_i,$$

which plays a central role in information theory as a measure of information, choice or uncertainty. The measure H is normally called the entropy of the set of probabilities $p_1, p_2, ..., p_n$. The quantity H has a number of interesting properties which further substantiate it as a reasonable measure of information. For example, (1) H=0 if and only if all the p_i but one are zero, this one having the value of unity. Thus, only when we are certain of the game outcome, does H vanish. Otherwise, H is positive, and (2) for a given n, H is a maximum and equal to log n when all situation with the p_i are equal, i.e., $\frac{1}{n}$. This is also intuitively the most uncertainty.

The concept of intelligence transmission velocity was proposed by [3]. The velocity at which intelligence can be transmitted over a telegraph current with a given line speed, i.e., a given rate of sending signal elements is expressed approximately by the following formula,

$$W = K \log m,$$

where W is the intelligence transmission velocity, m the number of current values employed, and K a constant. By the technical term, intelligence transmission velocity here means the number of characters, representing different letters, figures, etc., which can be transmitted in a given length of time assuming that the circuit transmits a given number of signal elements per unit time. Iida and Nakagawa[4] inferred that when information velocity becomes equal to the speed of light time stops completely. Can we find what happens if the intelligence transmission velocity reaches at the speed of light?

When we speak of the capacity of a system to transmit information, some sort of quantitative measure of information must be specified [5]. In the first place, there must be a group of physical symbols, such as words, dots and dashes or the like, which convey certain meanings to the parties communicating. In any given communication, the sender mentally selects a particular symbol and by some bodily motion, as of his vocal mechanism, causes the attention of the receiver. By successive selections, a sequence of symbols is brought to the listener's attention. At each selection, all other symbols may be eliminated. As the selections proceed, more and more possible symbol sequences are eliminated, and we say that the information becomes more precise. In this study, as the most precise information of game, the evaluation function scores are used (e.g. [6]).

The main purpose of the present paper is twofold:

(1) To derive the information dynamic model of uncertainty of game outcome.

(2) To collect data of Congkak and Othello aiming at measuring their entertainment.

II. MODELING

The modeling procedure is summarized as follows:

(a) Assume a flow problem as the information dynamic model and solve it.

(b) Get the solutions, depending on the position and time.

(c) Examine whether any solution of the problem can correspond to game information.

(d) If so, visualize the assumed flow with some means. If not, return to the first step.

(e) Determine the correspondences between the flow solution and game information.

(f) Finally, obtain the mathematical expression of the information dynamic model.

The modeling procedure of information dynamics based on fluid mechanics has been established by [7]. Another information dynamics model for the flow, in Fig.1, will be constructed by following the above procedure step by step.

Flow near a flat plate which is suddenly accelerated from rest and moves in its own plane with a constant velocity is solved by [8]. For a brief sketch of the solution, see [9].

(a) Let us assume the flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U_0 as shown in Fig.1. Note that the walls are two-dimensional, horizontal and infinitely long.

Since the system under consideration has no preferred length in the horizontal direction, it is reasonable to suppose that the velocity profiles are independent of the horizontal x-direction, which means that the velocity profile $u(y)$

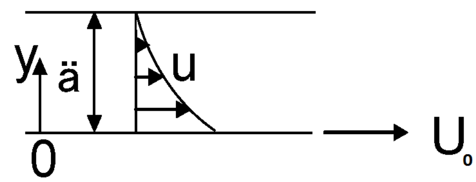

Fig. 1: A definition sketch of flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U_0

for varying distance x can be made identical by selecting suitable scale factors for u and y. The scale factors for u and y appear quite naturally as the lower wall velocity U_0 and gap between the two walls δ. Hence, the velocity profile after the time $t > 0$ can be written as the function in the following way.

$$\frac{u}{U_0} = f(\frac{y}{\delta}) \tag{1}$$

(b) Get the solutions. The velocity profile is here accounted for by assuming that the function f depends on $\frac{y}{\delta}$ only, and contains no additional free parameter. Since the fluid particles are fixed on the surface of two walls due to the viscous effect, the function must take the value of 1 on the lower wall (y=0) and the value of 0 on the upper wall (y=δ). The boundary conditions are:

t $\leq$ 0: $\frac{u}{U_0}$ = 0 for $0 \leq \frac{y}{\delta} \leq 1$

t $>$ 0: $\frac{u}{U_0}$ =1 for $\frac{y}{\delta}$=0; $\frac{u}{U_0}$=0 for $\frac{y}{\delta}$=1.

When writing down an approximate solution of the present flow, it is necessary to satisfy the above boundary conditions for $\frac{u}{U_0}$. It is evident that the following velocity profiles satisfy all of the boundary conditions.

$$\frac{u}{U_0} =(1 - \frac{y}{\delta})^q, \tag{2}$$

in the range $0 \leq \frac{y}{\delta} \leq 1$, where q is a positive real number parameter. Equation (2) is considered as the approximate solution on the flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U_0, where each solution takes a unique value of q. The value of q must be determined by the boundary conditions and the Reynolds number Re=$U_0 \cdot \frac{\delta}{v}$, where v is the kinematic viscosity of the fluid.

It is known that the transition from laminar to turbulent flow in the boundary layer is governed by the Reynolds number $Re = U_\infty \cdot \frac{d}{v}$, where U_∞ is the free stream velocity, and the boundary layer thickness. The critical Reynolds number ($Re_{crit.,}$) at which the transition is initiated, is of 2,800 approximately (e.g.[9], [10])

In the case of the present flow, as shown in Fig.1, at 1 atmospheric pressure and temperature at 20°C, water has the kinematic viscosity 1.004×10^{-2} cm^2/s. When water is chosen as the fluid, and the constant velocity U_0 = 10 cm/s and the gap between the two walls δ=10 cm are set, we obtain the Reynolds number $Re \simeq 10^4$. The result of this calculation clearly illustrates how the flow is liable to be turbulent under an ordinary situation. The solution (2) is smooth analytical functions and thus this is only valid for laminar flow.

The fundamental equations for fluid mechanics are the Navier-Stokes equation. This inherently nonlinear set of partial differential equations has no general solution, only several exact solutions, which are trivial in practice, and have been found [11]. All of these exact solutions are for laminar flows, and no turbulent flow solution is available yet. However, it is considered that each of the laminar solutions in (2) represents an approximate turbulent solution. In this regard, we consider that the solutions (2) are applicable for laminar flow as well as turbulent flow to some extent. However, it should be noted that the applicability of the present solutions to turbulent flow is severely limited.

(c) Let us examine whether this solution is game information or not. The non-dimensional velocity $\frac{u}{U_0}$ varies from 1 to 0 with increasing non-dimensional distance $\frac{y}{\delta}$ in many ways with changing the parameter q. It can be considered that $\frac{u}{U_0}$ represents the uncertainty of game outcome. This is why uncertainty of game outcome takes the value of 1 at the start, and it decreases with increasing the game length and becomes the value of 0 at the end of the game.

(d) Visualize the assumed flow with some means. Imagine that the assumed flow is visualized with neutral buoyant particles. Motion of the visualized particles is detected by the eye almost instantaneously through light and is mapped on our retina [1], so that during these processes, the motion of the fluid particles is transformed into that of the information particles by light carrying the images of fluid particles. This is why the motion of the fluid particles is intact in the physical space, but only the reflected lights, or electromagnetic waves consisting of photons can reach the retina. Photons are then converted to electrochemical particles and are passed along the visual cortex for further processing in parts of the cerebral cortex [1]. Photons and/or electrochemical particles are considered to be information particles. It is, therefore, natural to expect that the flow in the physical world is faithfully transformed to that in the information world, or brain including eye, which is referred to as ”informatical world” hereafter. During this transformation, the flow solution in the physical world changes into the information in the informatical world.

TABLE I: Correspondences between flow and game information

Physical world(flow)	Informatical world(game)
u: flow velocity	I: current uncertainty of game outcome
U_0: plate velocity	I_0: initial uncertainty of game outcome
y: vertical distance	L: current game length
δ: gap between two walls	L_0: total game length

(e) Proposed are correspondences between the flow and game information, which are listed in Table 1.

(f) Obtain the mathematical expression of the information dynamic model. Considering the correspondences in Table 1, (2) can be rewritten as

$$\frac{I}{I_0} = \left(1 - \frac{L}{L_0}\right)^q \tag{3}$$

Introducing the following non-dimensional variables in (3),

$$\xi=\frac{I}{I_0} \text{ and } \eta=\frac{L}{L_0},$$

we finally obtain the mathematical expression of the uncertainty of game outcome ξ as

$$\xi = (1 - \eta)^q \text{ for } 0 \leq \eta \leq 1, \tag{4}$$

where η is the non-dimensional current game length, and q the positive real number parameter. Fig.2 illustrates how the uncertainty of game outcome ξ in (4) varies with the non-dimensional game lenght η at each value of parameter q. We expect that the greater the value of q is, the greater the strength difference between the two teams (or players) in a game is, and vice versa.

III. Verification of Model

A. Congkak

History of Congkak: Congkak [12], [13], [14] is short for Main Congkak, which is Indonesian for cowrie shell, but some people believe that actually the name of the game originated from the word congak, which in old Malay language means mental calculation without writing it down. Congkak is a popular mancala game in Malaysia, Brunei, Singapore and Indonesia. Many Indonesians believe that the game originated in Malacca Kingdom where it became very popular and spread to the South-East Asia region. This spread was due to the many travelers who visited the kingdom because it was a trading city. In the early days, Congkak was mostly played by the royal family and palace

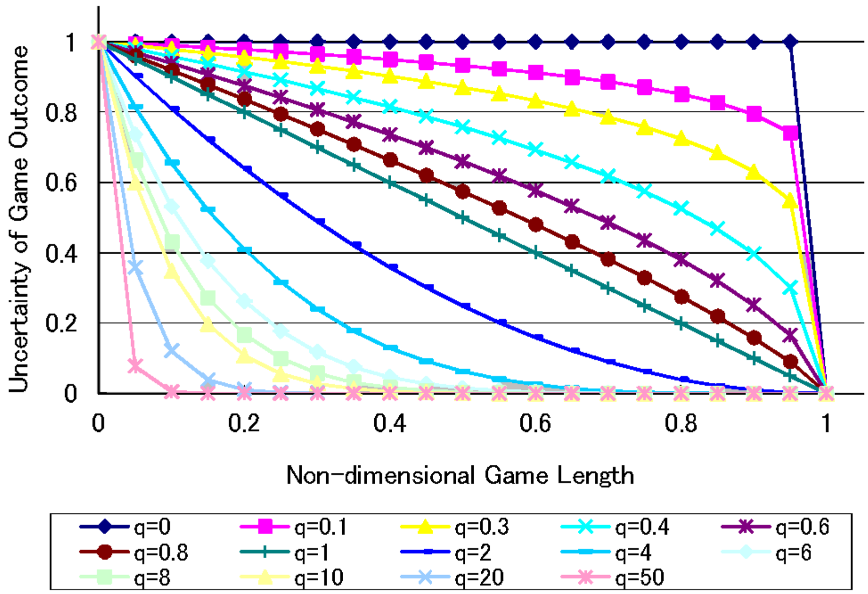

Fig. 2: Uncertainty of game outcome ξ against non-dimensional game length η

residents, however later it spread to the general population of the kingdom and today it is usually played by girls and women. As the Congkak board is often shaped like a boat it is believed that it is based on the legend of a fisherman unable to go to sea during rainy season who lost his income during this time. To prevent boredom they created this game which is similar to their boat.

Today many Congkak tournaments are organized for children in Malaysia, e.g. in Kuala Lumpur, Kuala Terengganu, Pekan and Seremban. Several hotels in southern Borneo offer Congkak courses to tourists. Since 2004, the Malaysian Embassy and the Malaysian Association in France sponsor each year a Congkak tournament to spread Malaysian culture in Europe. Another tournament is held in Wales during the Cardiff European Games, an annual meeting of Malaysians from all across Europe. In Brunei, Congkak is also played during the night of royal ceremonials such as the Istiadat Malam Berjaga-jaga at the palace or nobility's residence.

Congkak consists of: Congkak uses an oblong game board called papan congkak, which has two rows each one with five to ten playing pits. These pits are called lubang kampong ("village") or lubang anak ("child") in Malaysia. Most widespread boards have 2×7 playing pits. In addition, there is at either end a larger hole to store the captured counters. The store is called lubang rumah ("house") in Malaysia. Each player owns the store to their left. Each of the small pits contains at the beginning of the game as many counters (usually cowrie shells or tamarind seeds called anak-anak buah in Malaysia) as each row counts small pits.

How to play Congkak: 2 players sit opposite each other. Each player owns the row of houses directly in front of them houses and the storehouse on their left.

(1) Players play simultaneously beginning with any one of their houses and dropping seeds clockwise into each house until the player is finished with all the seeds in their hand. On their round, a seed is placed in a player's storehouse but not their opponent's.

(2) On ending a round, the player takes all the seeds of the house that they dropped the last seed in and the process is repeated until the last seed is dropped into an empty house.

(3) If the last seed falls in a house that is part of a player's village, they can pick all the seeds from the opponent's house that lies opposite it and put them in their storehouse.

(4) If the last seed drops in their storehouse, they can continue the game, picking a house of their choice from their side.

(5) When the last seed drops in an empty house, they are considered mati ("dead") and ends their turn. The opponent continues until they similarly end their turn.

Data analyses: Mardhiah plays Congkak against Husna under the rules mentioned above. The non-dimensional advantage $\alpha(\eta)$ is defined as

$$\alpha(\eta)= \frac{[S_M(\eta)-S_H(\eta)]}{S_T} \text{ for } 0 \leq \eta \leq 1,$$

where $S_M(\eta)$ is Mardhiah's current score, $S_H(\eta)$ Husna's current score, S_T the total score for the two players in the game, and η the non-dimensional game length. The sign of non-dimensional advantage is positive when Mardhiah gets advantage, while it is negative when Husna gets advantage. The uncertainty of game outcome ξ is derived by

$$\xi = \begin{cases} 1-|\alpha(\eta)| & \text{for } 0 \leq \eta < 1 \\ 0 & \text{for } \eta = 1 \end{cases}$$

Fig.3 shows how the non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ depend on the non-dimensional game length η. Mardhiah leads the game until $\eta \simeq 0.369$, but after this point Husna gets advantage and keeps it until the end. However, uncertainty of game outcome ξ is kept within 0.8 and 1 until the very end of the game, so this game is considered to be quited tight one. In this figure the best fit model curve ξ=(1-η)$^{0.15}$ has been plotted.

B. Othello

History of Othello: Othello is a board game involving abstract strategy and played by two players on a board with 8 rows and 8 columns and a set of distinct pieces for each side [15], [16]. Pieces typically are disks with a light and a dark face, each side belonging to one player. The player's goal is to have a majority of their pieces showing at the end of the game, turning over as many of their opponent's pieces as possible. The modern rule set used on the international tournament stage originated in Mito, Japan.

How to play: Word, "outflank" means to place a disc on the board, so that your opponent's row (or rows) of disc(s) is bordered at each end by a disc of your color. A "row" may be made up of one or more discs. Othello rules are summarized as follows.

(a) Black always moves first.

(b) If on your turn you cannot outflank and flip at least one opposing disc, your turn is forfeited and your opponent moves again. However, if a move is available to you, you may not forfeit your turn.

(c) A disc may outflank any number of discs in one or more rows in any number of directions at the same time-horizontally, vertically or diagonally. A row is defined as one or more discs in a continuous straight line.

(d) You may not skip over your own color disc to outflank an opposing disc.

(e) Disc(s) may only be outflanked as a direct result of a move and must fall in the direct line of the disc placed down.

(f) All disc(s) outflanked in any one move must be flipped, even if it is to the player's advantage not to flip them all.

(g) A player who flips a disc which should not have been turned, may correct the mistake as long as the opponent has not made a subsequent move. If the opponent has already moved, it is too late for change and the disc(s) remain as is.

(h) Once a disc is placed on a square, it can never be moved to another square later in the game.

(i) If a player runs out of discs, but still has an opportunity to outflank an opposing disc on her or his turn, the opponent must give the player a disc to use. This can happen as many times as the player needs and can use a disc.

(j) When it is no longer possible for either player to move, the game is over. Discs are counted and the player with the majority of their color discs on the board is the winner. Note that it is possible for a game to end before all 64 squares are filled.

Othello has fast become one of the most popular and most often played games in our history, spawning contests, and tournaments on regional, national and even worldwide levels. And the rules of Othello explained as above, are very simple and the final destination is clear enough, but what exactly you are supposed to be trying to do in the early and middle stages of the game is unclear.

Data analyses: The present Othello game is played by Huy, who acts as both black and white players. The non-dimensional advantage $\alpha(\eta)$ is defined as follows,

$$\alpha(\eta) = \frac{Ad(\eta)}{ACT(1)} \text{ for } 0 \leq \eta \leq 1,$$

where $Ad(\eta)$ is the advantage or evaluation function scores, $ACT(1)$ the total advantage change at the end of the game. $ACT(\eta)$ is expressed by

$$ACT(\eta) = ACT(\frac{m}{N}) = \sum_{1 \leq i \leq m} |Ad(i) - Ad(i-1)|,$$

where m is the current move, N the total moves at the end of the game, and i a positive integer. And, $\eta = \frac{m}{N}$ the non-dimensional game length. Uncertainty of game outcome ξ is expressed by

$$\xi = \begin{cases} 1-|\alpha(\eta)| & \text{for } 0 \leq \eta < 1 \\ 0 & \text{for } \eta = 1 \end{cases}$$

Fig.4 shows how non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ depend on the non-dimensional

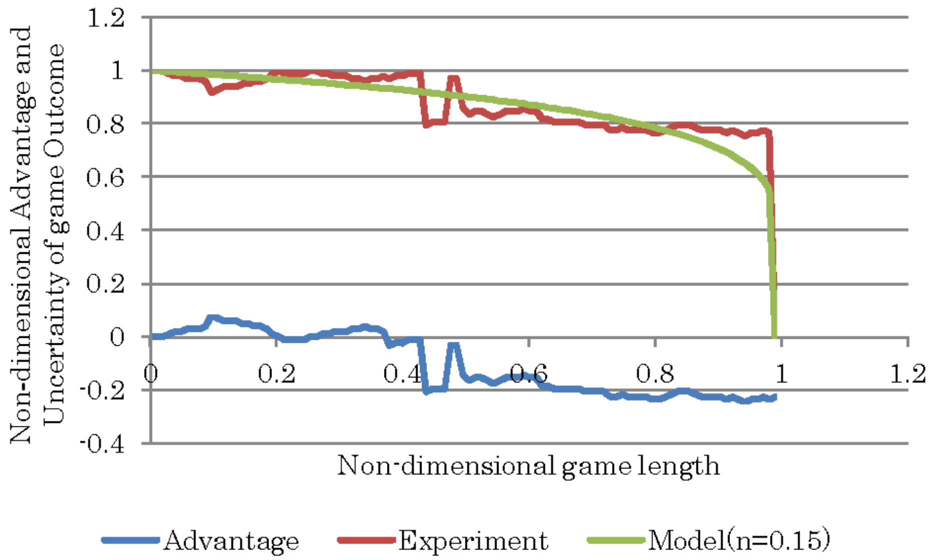

Fig. 3: Non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ against non-dimensional game length η for Congkak.

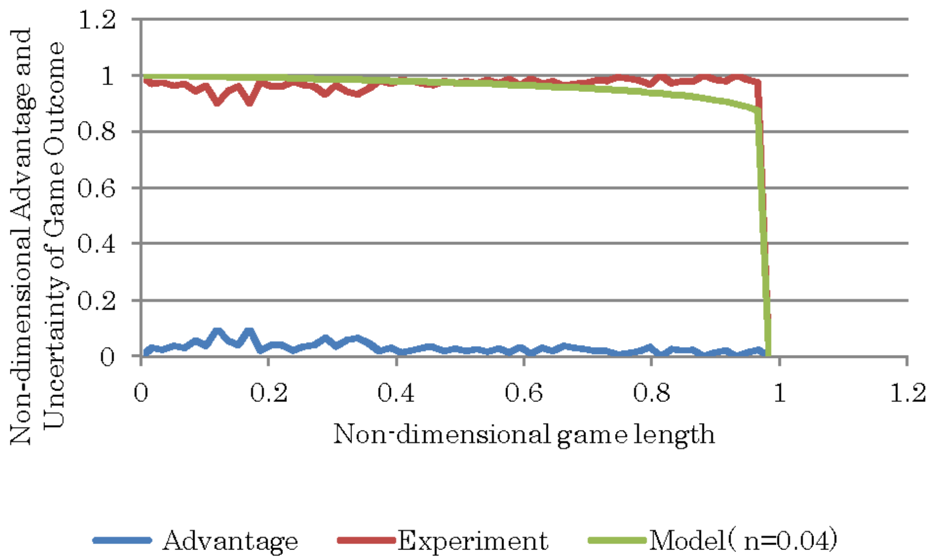

Fig. 4: non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ against non-dimensional game length η for Othello.

game length η.

It may be evident in Fig.4 that non-dimensional advantage $\alpha(\eta)$ is always positive, so that Black keeps advantage through the game, though it is smaller than 0.1. In this figure, the best fit model curve ξ=(1 - η)$^{0.04}$ has been plotted.

IV. Discussion

This section describes how uncertainty of Soccer game outcome changes with increasing the game length, where the goal scores of 2010 FIFA World Cup 3rd Place (Germany vs. Uruguay) are used for illustration. Germany wins the game against Uruguay by the score 3 to 2. This game is full of

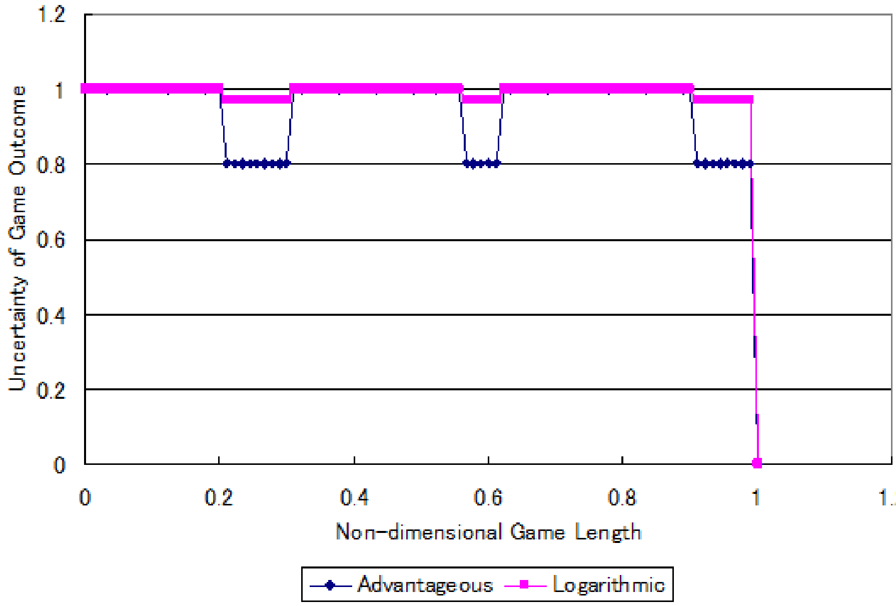

Fig. 5: Uncertainty ξ_u of game outcome against non-dimensional game length η for 2010 FIFA World Cup, 3rd Place.

thrill, with alternating changes from offense to defense, or from defense to offense many times. The game is balanced at the start, and then Germany gets the first goal. Uruguay makes the game balanced by taking the second goal, and then reversed by the third goal. The game is made balanced again by Germany's fourth goal. Finally, Germany gets the fifth goal near the end and keeps the lead until the end of the game.

To begin with, the advantage $\alpha(\eta)$ is defined by

$$\alpha(\eta)= \frac{[S_1(\eta)-S_2(\eta)]}{S_t} \text{ for } 0 \leq \eta \leq 1,$$

where $S_1(\eta)$ is the current score sum for team 1, $S_2(\eta)$ the current score sum for team 2, S_t the total score(s) for the game, and η the normalized game length. The sign of advantage is defined in such a way that it is positive when team 1 keeps advantage, while it is negative when team 2 takes advantage.

It may be worth noting the remarkable similarity between logarithmic uncertainty of game outcome $\xi_{lu}(\eta)$:

$$\xi_{lu}(\eta) = \begin{cases} -\sum_{i=1}^{2} p_i(\eta)\log_2 p_i(\eta) & \text{for } 0 \leq \eta < 1 \\ 0 & \text{for } \eta = 1 \end{cases}$$

where $p_1(\eta)$ and $p_2(\eta)$ are winning rates for teams 1 and 2, respectively, and the entropy $H(X)$ defined by [2]: Information theory has been used to study the properties of random variables. If a random variable X can assume the state x, and $P(X = x)$ is the probability for X to assume the specific state x, we can define a measure $H(X)$ called entropy as

$$H(X) = -\sum_{x}[P(X = x)]\log[P(X = x)],$$

This is often described as the uncertainty about the outcome of X gained if one is to observe the state of x, without having prior knowledge about X. Note that in the expression of $\xi_{lu}(\eta)$ when the base of the logarithm is 2, the unit of $\xi_{lu}(\eta)$ is "bit", when the base is Euler's number η, the unit is "nat", and when the base is 10, the unit is "digit". We choose the value of 2 as the base, for it is unnecessary to normalize $\xi_{lu}(\eta)$ in this case. It may be evident in Fig. 5 that $\xi_{au}(\eta)$:

$$\xi_{au}(\eta) = \begin{cases} 1 - |\alpha(\eta)| & \text{for } 0 \leq \eta < 1 \\ 0 & \text{for } \eta = 1 \end{cases}$$

is always smaller than the logarithmic uncertainty of game outcome $\xi_{lu}(\eta)$. It may be evident that the logarithmic uncertainty of game outcome $\xi_{lu}(\eta)$ obscures the uncertainty of game outcome by introducing the logarithmic value of winning rate $p_i(\eta)$. Thus, it is here suggested that the logarithmic uncertainty of game outcome $\xi_{lu}(\eta)$ or Shannon's entropy provides only a measure of uncertainty of game outcome, but not itself. The uncertainty of game outcome is considered to be given by the present proposed advantageous uncertainty of game outcome $\xi_{au}(\eta)$.

V. Conclusion

New knowledge and insights obtained through the present study have been discussed and summarized as follows.

(a) Uncertainty of game outcome ξ for the present game record of Congkak is approximated with the model curve:

$$\xi = (1 - \eta)^{0.15},$$

while that of Othello with the model curve:

$$\xi = (1 - \eta)^{0.04},$$

where η is the non-dimensional game length. This means that Othello is more balanced than Congkak in the games shown in this study, and thus it is considered that the former is more exciting than the latter. However, it must be noted that this conjecture is neither universal nor objective, because the results are highly dependent on individual feeling or emotion of game players. According to the classification by [17], Othello can be classified as ”one-sided game”, while Congkak as ”seesaw game”.

(b) It is inferred that the logarithmic uncertainty of game outcome or Shannon’s entropy [2] provides only an order of uncertainty of game outcome, but not itself. It is considered that the value required is given by the present proposed advantageous uncertainty of game outcome.

(c) An information dynamic model representing the uncertainty of game outcome has been derived based on fluid mechanics. Its usefulness has been confirmed by game experiments using Congkak and Othello.

(d) Congkak has been recently introduced into Japan for the first time, as far as the present authors are aware, and analyzed in order to explore improvement of its entertainment. As a result, it is realized that Congkak is a unique regional game in South-East Asia, having a high possibility to spread out widely.

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