On the Duality of Source and Channel Correlations: Slepian-Wolf Relaying Viewpoint

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***Abstract*—In this paper, we derive the theoretical outage probability of a transmission system in the presence of source and channel correlations in the block Rayleigh fading channels, based on the Slepian-Wolf theorem. Two transmitters and one common receiver are assumed, where the correlation knowledge between the two source information streams can be expressed as a bit-flipping model. The information bits at each transmitter are separately encoded and sent to a common decoder. In addition, we also assume the channels suffering from independent or correlated Rayleigh fading. It is known that the outage event happens when the instantaneous signal noise ratio (SNR) is lower than the given threshold ratio. This paper shows that the outage probability of the system model described above can be expressed by double integrals of the admissible rate region according to the Slepian-Wolf theorem, with respect to the $joint$ probability density function (pdf) of the corresponding instantaneous signal amplitudes (or the equivalent SNRs) of the channels. The results show that the second order diversity of the theoretical outage curves can be achieved if and only if the two information streams are fully correlated, regardless of the channels being independent or not. On the contrary, the channel correlation makes opposite influence on the decay of the outage probability. However, if the two streams are not fully correlated, this influence gradually disappears as the average SNRs increases. In this sense, the source and channel correlation problems are dual with each other.**

I. INTRODUCTION

According to the remarkable contribution by Slepian and Wolf in [1], it has been proven that the distributed source coding scheme can achieve the same compression rate as the optimum joint encoding approach using one single encoder, by best exploiting the correlation knowledge of the source information streams. This theorem can be utilized as a supporting base of many applications, such as the relay system which comprises three basic components, a source, a relay and a destination nodes. Specifically, the source broadcasts the original information signal to both the relay and the destination nodes. In some of the relay strategies such as the Decode-and-Forward (DF) or Extract-and-Forward (EF) [2] schemes, relay aims to recover the original information before re-encoding and/or forwarding it to the destination. Due to the noise happening in the source-relay (SR) channel, the recovered information may contain some errors, but they are still correlated with the original data. The common destination node receives two correlated signal streams sent from the source and the relay via the source-destination (SD) and relay-destination (RD) channels, respectively. The joint decoding takes place at the receiver utilizing the source correlation knowledge.

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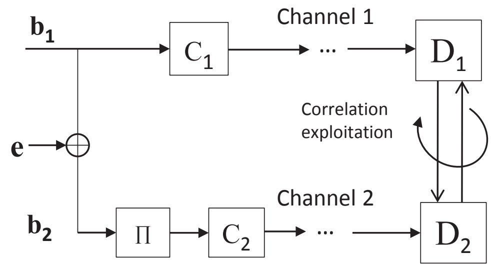

Fig. 1. System model of correlated source-channel transmission

For simplicity, in this paper the two correlated information streams, represented by $\mathbf{b_1}$ and $\mathbf{b_2}$ as shown in Fig. 1, are generated by a bit-flipping model satisfying the equations: $\mathbf{b_2} = \mathbf{b_1} \oplus \mathbf{e}$ and $P(\mathbf{e} = 1) = p_e$, where p_e denotes the flipping probability [3]. Obviously, $p_e = 0$ indicates the extreme situation of full correlation while $p_e = 0.5$ implies the completely independent case. The Source-Channel separation [4] is assumed.

The two channels shown in Fig. 1 are assumed to suffer from block Rayleigh fading, where the channel realization changes frame by frame. Moreover, we consider Channel 1 and 2 being either independent or correlated. The instantaneous channel gains of either one of the channels or both may be faded below the transmission requirement that depends on channel coding and modulation schemes. The admissible Slepian-Wolf rate region of the correlated source transmission is defined in [1], and it can be converted into the signal amplitude (or equivalently the SNRs) region. In this case, the outage capacity is dominated by the instantaneous channel realizations. Hence, it is straightforward to derive the outage probability by a double integral over the achievable regions with respect to the $joint$ probability density function (pdf) of the instantaneous signal amplitudes of the both channels.

This paper is organized as follows. First of all, the Slepian-Wolf theorem and the bit-flipping model are discussed in Section II. In Section III, the outage probability is defined and derived based on the Slepian-Wolf theorem in the case when Channel 1 and 2 are either independent or correlated. Moreover, the asymptotic analysis of the outage performance is also presented in this section. Finally, the conclusions are given in Section IV with some remarks.

II. System Model

The system model of the correlated source-channel transmission is shown in Fig. 1, where $\mathbf{b_1}$ and $\mathbf{b_2}$ denote the source bit streams transmitted from the first and second transmitters, respectively. The two information streams are correlated and $\mathbf{b_2}$ is a flipped version of $\mathbf{b_1}$ with a flipping probability p_e. The source correlation value can be further utilized at the joint decoder in order to enhance the decoding performance, based on the Slepian-Wolf theorem. Let $\mathbf{s_1}$ and $\mathbf{s_2}$ denote the transmitted symbols. The received signals $\mathbf{y_1}$ and $\mathbf{y_2}$ from the first and the second time slots, respectively, can be expressed as:

$$\mathbf{y_1} = \mathbf{h_1}\mathbf{s_1} + \mathbf{n_1}, \tag{1}$$

$$\mathbf{y_2} = \mathbf{h_2}\mathbf{s_2} + \mathbf{n_2}, \tag{2}$$

where $\mathbf{n_1}$ and $\mathbf{n_2}$ are the zero-mean additive white Gaussian noise (AWGN) components, both having the same variance σ_n^2 per dimension. $\mathbf{h_1}$ and $\mathbf{h_2}$ represent the complex Rayleigh fading envelops of the two channels, and the both are kept constant within a frame duration due to the block fading assumption. The instantaneous SNR of the $i\text{-}th$ $(i = 1, 2)$ channel $\gamma_i = |h_i|^2 E_i/N_0$, where E_i represents the per-symbol signal power which is normalized to 1, and $N_0 = 2\sigma_n^2$ which denotes the noise power spectral density. By assuming the independent block Rayleigh fading for both channels, the *pdf* of the instantaneous amplitude R_i of the i-th channel can be expressed as [5]

$$p\left(R_i\right) = \frac{2R_i}{P_{ri}} \exp(-\frac{R_i^2}{P_{ri}}), \tag{3}$$

where $P_{ri} = \left\langle |h_i|^2 E_i \right\rangle$, denoting the average received signal power of the $i\text{-}th$ channel. Therefore, the average SNR of the $i\text{-}th$ channel is $\Gamma_i = P_{ri}/N_0$. In this paper, we also examine the impact of the correlation $\rho = \langle h_1 h_2^* \rangle$. The *joint pdf* of instantaneous amplitudes R_1 and R_2 is then given by [5]

$$\begin{aligned} p(R_1, R_2) = &\frac{4R_1R_2}{P_{r1}P_{r2}(1-|\rho|^2)} I_0\left[\frac{2|\rho| R_1R_2}{\sqrt{P_{r1}P_{r2}}(1-|\rho|^2)}\right] \\ &\exp\left[-\frac{1}{1-|\rho|^2}\left(\frac{R_1^2}{P_{r1}} + \frac{R_2^2}{P_{r2}}\right)\right], \end{aligned} \tag{4}$$

where $I_0(\cdot)$ is the zero-th order modified Bessel's function of the first kind. According to [1], the admissible rate region is constituted as an unbounded polygon, represented by Part 3 as shown in Fig. 2. The original bits can be recovered if and only

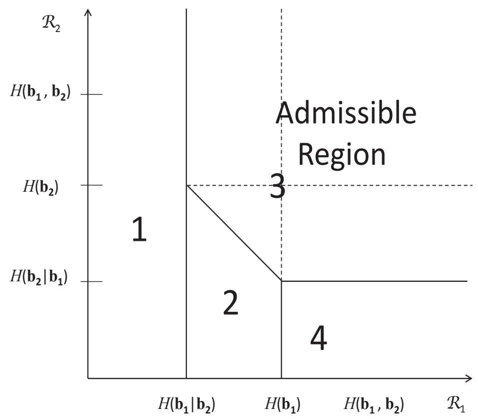

Fig. 2. Achievable rate region of Slepian-Wolf theorem

if the transmitting rates are within this area. For instance, if $\mathbf{b_1}$ is transmitted at the rate $\mathcal{R}_1$ which is equal to its entropy $H(\mathbf{b_1})$, then $\mathbf{b_2}$ can be transmitted at the rate $\mathcal{R}_2$ which is less than $H(\mathbf{b_2})$, but must be larger than their conditional entropy $H(\mathbf{b_2} \mid \mathbf{b_1})$. In other words, $\mathcal{R}_1$ and $\mathcal{R}_2$ should satisfy three equations [1]:

$$\mathcal{R}_1 \geqslant H(\mathbf{b_1} \mid \mathbf{b_2}), \tag{5}$$

$$\mathcal{R}_2 \geqslant H(\mathbf{b_2} \mid \mathbf{b_1}), \tag{6}$$

$$\mathcal{R}_1 + \mathcal{R}_2 \geqslant H(\mathbf{b_1}, \mathbf{b_2}). \tag{7}$$

where $H(\mathbf{b_1}, \mathbf{b_2})$ denotes the joint entropy of the correlated source information streams. Since we assume the binary symmetric source model $(P(1) = P(0) = 0.5)$, $H(\mathbf{b_1}) = H(\mathbf{b_2}) = 1$, $H(\mathbf{b_1} \mid \mathbf{b_2}) = H(\mathbf{b_2} \mid \mathbf{b_1}) = H(p_e)$, $H(\mathbf{b_1}, \mathbf{b_2}) = 1 + H(p_e)$ with $H(p_e) = -p_e \log_2(p_e) - (1 - p_e)\log_2(1 - p_e)$. The threshold amplitude is given by

$$R[H] = \sqrt{\left(2^{R_c H} - 1\right) N_0}, \tag{8}$$

where R_c represents the rate which takes into account of the channel coding and the modulation scheme [3]. However, the specific practical coding and modulation schemes are out of the scope of this paper. Equivalently, the inverse transform is defined as $\mathrm{H}[R] = \frac{1}{R_c}\log_2(1 + \frac{R^2}{N_0})$.

III. Outage Derivation

Besides the typical admissible region, the entire Slepian-Wolf rate region can be divided into 4 parts as shown in Fig. 2. In this paper, Part 4 should also be included as the admissible region, such as in the relay system, where $\mathbf{b_1}$ is the source information stream which we are interested in, while $\mathbf{b_2}$ can be seen as the recovered version of $\mathbf{b_1}$ at the relay. Although $\mathbf{b_2}$ may contains some errors due to the fading variation of the SR channel, it is still correlated with $\mathbf{b_1}$. By using Eq. (8), the Slepian-Wolf rate constraint can be transformed into the corresponding signal amplitude domain. It is known that the outage event happens when the instantaneous signal amplitudes of Channel 1 and 2 are out of the admissible region, and therefore the outage probability of our assumed model can be defined as [6]

$$P_{out} = P_1 + P_2, \tag{9}$$

where P_1 and P_2 denote the probabilities that the rates $\mathcal{R}_1$ and $\mathcal{R}_2$ fall into the inadmissible regions Part 1 and Part 2, as shown in Fig. 2. Therefore, the mathematical expressions of P_1 and P_2 are defined as follows:

$$P_1 = \int_{R_1=R[0]}^{R[H(\mathbf{b_1}|\mathbf{b_2})]} \int_{R_2=R[0]}^{R[\infty]} p\left(R_1, R_2\right) dR_1 dR_2, \tag{10}$$

$$P_2 = \int_{R_1=R[H(\mathbf{b_1}|\mathbf{b_2})]}^{R[H(\mathbf{b_1})]} \int_{R_2=R[0]}^{R[H(\mathbf{b_1},\mathbf{b_2})-\mathrm{H}(R_1)]} p\left(R_1, R_2\right) dR_1 dR_2. \tag{11}$$

The derivations of P_1 and P_2 are presented for different scenarios as follows and the numerical results are shown by assuming R_c equals to 1 and $\Gamma_1 = \Gamma_2$.

A. Independent Channels

If both Channel 1 and Channel 2 are statistically independent, the *joint pdf* of R_1 and R_2 can be expressed as $p(R_1, R_2) = p(R_1)p(R_2)$, and P_1 and P_2 can be further derived as

$$\begin{aligned} P_1 &= \int_{R_1=R[0]}^{R[H(\mathbf{b_1}|\mathbf{b_2})]} p(R_1)dR_1 \int_{R_2=R[0]}^{R[\infty]} p(R_2)dR_2 \\ &= \int_{R_1=R[0]}^{R[H(\mathbf{b_1}|\mathbf{b_2})]} \frac{2R_1}{P_{r1}} \exp(-\frac{R_1^2}{P_{r1}})dR_1 \\ &= 1 - \exp\left[-\frac{(2^{R_c H(p_e)} - 1)N_0}{P_{r1}}\right], \end{aligned} \tag{12}$$

$$\begin{aligned} P_2 &= \int_{R_1=R[H(\mathbf{b_1}|\mathbf{b_2})]}^{R[H(\mathbf{b_1})]} p\left(R_1\right) dR_1 \\ &\quad \cdot \left[-\exp\left(-\frac{R_2^2}{P_{r2}}\right)\right]_{R_2=0}^{\sqrt{(2^{[R_c H(\mathbf{b_2},\mathbf{b_1})-\log_2(1+\frac{R_1}{N_0})]}-1)N_0}} \\ &= \int_{R_1=R[H(\mathbf{b_1}|\mathbf{b_2})]}^{R[H(\mathbf{b_1})]} \frac{2R_1}{P_{r1}} \exp(-\frac{R_1^2}{P_{r1}}) \\ &\quad \cdot \left[1 - \exp\left(-\frac{(2^{[R_c H(\mathbf{b_2},\mathbf{b_1})-\log_2(1+\frac{R_1}{N_0})]} - 1)N_0}{P_{r2}}\right)\right] dR_1. \end{aligned} \tag{13}$$

Since no explicit solution is found for P_2, the numerical method may be used with sufficient accuracy. The theoretical outage curves of the system assumed are shown in Fig. 3, where the outage probability with maximum-ratio-combing (MRC) scheme [7] is also shown for comparison. Obviously, the second order diversity of the outage curve can be achieved only if $\mathbf{b_1}$ and $\mathbf{b_2}$ are fully correlated ($p_e = 0$), the mathematical proof of which is given in Appendix 1. It should be noted that the outage performance of the Slepian-Wolf transmission system is slightly better than that of the MRC scheme with diversity two. The mathematical proof of the

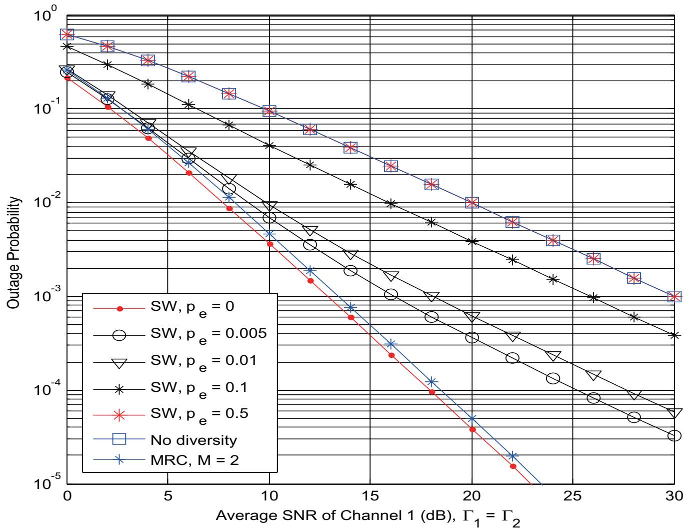

Fig. 3. Outage probabilities with source correlation, $|\rho| = 0$

asymptotic tendency is given in Appendix 2, but only for $\Gamma_1 = \Gamma_2$.

It is also found in Fig. 3 that with $p_e \neq 0$, the decay of the outage curve converges into the first order diversity, as Γ_1 and Γ_2 increases. The mathematical proof of this asymptotic tendency is shown in Appendix 3. Finally, with $p_e = 0.5$, the outage curve of our model is exactly the same as that with no-diversity.

B. Correlated Channels

With an assumption that Channel 1 and 2 are correlated, the signal amplitudes R_1 and R_2 follow the *joint pdf* $p(R_1, R_2)$ as shown in Eq. (4). Since the zero-*th* order modified Bessel function of the first kind $I_0(x)$ can be expanded as $I_0(x) = \sum_{n=0}^{\infty} \frac{(x/2)^{2n}}{(n!)^2}$, Eq. (4) can be re-written as:

$$\begin{aligned} p(R_1, R_2) =& \frac{4R_1R_2}{P_{r1}P_{r2}(1-|\rho|^2)} \exp\left(-\frac{R_1^2/P_{r1}}{1-|\rho|^2} - \frac{R_2^2/P_{r2}}{1-|\rho|^2}\right) \\ & \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \left(\frac{|\rho| R_1 R_2}{\sqrt{P_{r1}P_{r2}}(1-|\rho|^2)}\right)^{2n} \\ =& \sum_{n=0}^{\infty} q_1^{(n)} q_2^{(n)}, \end{aligned} \tag{14}$$

where $q_1^{(n)}$ and $q_2^{(n)}$ are expressed as

$$q_1^{(n)} = \frac{2R_1^{2n+1} |\rho|^n}{P_{r1}^{n+1}(1-|\rho|^2)^{n+1/2}} \exp\left(-\frac{R_1^2/P_{r1}}{1-|\rho|^2}\right) \left(\frac{1}{n!}\right), \tag{15}$$

$$q_2^{(n)} = \frac{2R_2^{2n+1} |\rho|^n}{P_{r2}^{n+1}(1-|\rho|^2)^{n+1/2}} \exp\left(-\frac{R_2^2/P_{r2}}{1-|\rho|^2}\right) \left(\frac{1}{n!}\right). \tag{16}$$

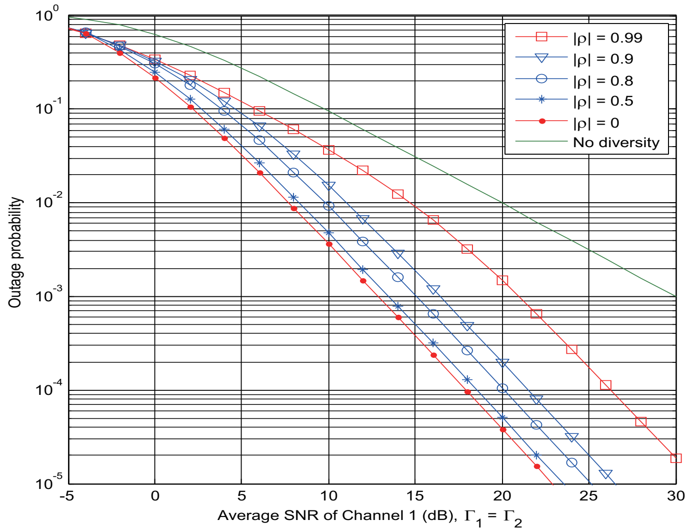

Fig. 4. Outage probabilities in the duality of source-channel correlation, $p_e = 0$

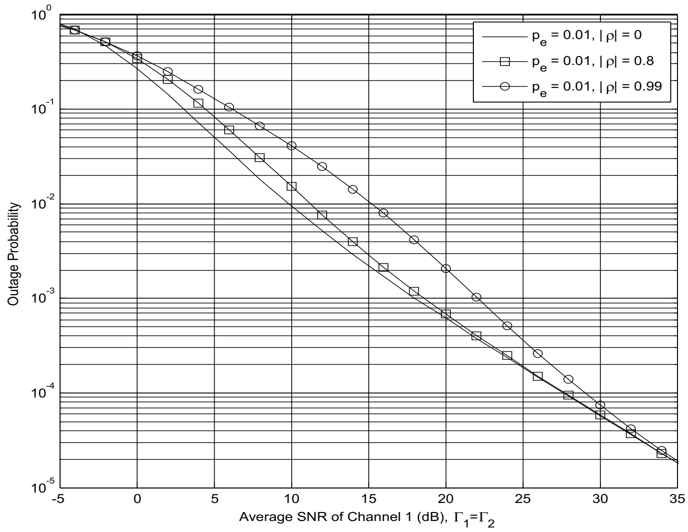

Fig. 5. Outage probabilities in the duality of source-channel correlation, $p_e \neq 0$

Now, given the fact that $p(R_1, R_2)$ are factored into a product of two independent terms, as shown by Eqs. (14)-(16), P_1 and P_2 can easily be calculated numerically. The results are shown in Fig. 4, where we assume the source streams are fully correlated ($p_e = 0$). Clearly, the larger the channel correlation, the larger the outage probability. However, the second order diversity can finally be achieved with arbitrary value of $|\rho| \neq 1$, when increasing the average SNRs. This asymptotic tendency is proven in Appendix 4.

Fig. 5 shows that when $p_e \neq 0$, the outage cavers change within a certain range of the average SNR values, giving different correlation factors. However, the outage curves can not achieve the second order diversity over the entire range of the average SNRs. See Appendix 4 for the proof of this tendency.

C. Duality Consideration

As observed before, when $\Gamma_1 \to \infty$, $\Gamma_2 \to \infty$ and $|\rho| = 0$, the outage probability yields the equivalent diversity order 1 asymptotically, as far as $p_e \neq 0$. On the other hand, when $p_e = 0$, the equivalent diversity order converges into two, so far as $|\rho| \neq 1$. This duality can easily be understood by considering that when $\Gamma_1 \to \infty$, $\Gamma_2 \to \infty$, only either the source bits transmitted from the two transmitters being different, or the complex fading envelops of the two channels having different values determines the diversity order.

IV. Conclusion

In this work, the outage probability of the correlated source transmission based on the Slepian-Wolf theorem has been derived, as well as the asymptotic tendency analysis, with the aim of its applications on DF or EF relay system. It has been shown mathematically that when the channel correlation $\rho = 0$, the second order diversity can always be achieved if $p_e = 0$. In the case when $0 < p_e < 0.5$, the diversity order gradually changes and finally converges into one, as the average SNRs become large. When sources are fully correlated, the second order diversity can always be achieved as long as the channels are not fully correlated. According to the observations described above, it can be concluded that the source and channel correlations are dual with each other.

Appendix 1

When $p_e = 0$, P_1 is always equal to 0, and therefore the outage probability is only dominated by the value of P_2. For the mathematical simplicity, the *pdf* of the instantaneous SNR $p(\gamma_i)$, instead of $p(R_i)$, is used to prove the asymptotic tendency of the outage curve, as $p(\gamma_i) = \frac{1}{\Gamma_i}\exp(-\frac{\gamma_i}{\Gamma_i})$. In the independent channels, by setting $H(\mathbf{b_1}, \mathbf{b_2}) = 1$ and $R_c = 1$

$$\begin{aligned} P_2 &= \int_{\gamma_1=0}^{1}\int_{\gamma_2=0}^{2^{[1-\log_2(1+\gamma_1)]}-1} p(\gamma_1)p(\gamma_2)d\gamma_1 d\gamma_2 \\ &= \int_0^1 p(\gamma_1)\cdot\left[-\exp\left(-\frac{\gamma_2}{\Gamma_2}\right)\right]_0^{2^{[1-\log_2(1+\gamma_1)]}-1} d\gamma_1 \\ &= \frac{1}{\Gamma_1}\int_0^1\left[\exp(-\frac{\gamma_1}{\Gamma_1}) - \exp\left(-\frac{\gamma_1}{\Gamma_1} - \frac{1-\gamma_1}{\Gamma_2(1+\gamma_1)}\right)\right]d\gamma_1. \end{aligned} \tag{17}$$

With the approximation that $\mathrm{e}^{-x} = \sum_{n=0}^{\infty}\frac{(-x)^n}{n!} \approx 1 - x$, Eq. (17) can be reduced to

$$\begin{aligned} P_2 &\approx \frac{1}{\Gamma_1}\int_0^1\left[1 - \frac{\gamma_1}{\Gamma_1} - \left(1 - \frac{\gamma_1}{\Gamma_1} - \frac{1-\gamma_1}{\Gamma_2(1+\gamma_1)}\right)\right]d\gamma_1 \\ &= \frac{1}{\Gamma_1}\int_0^1\left[\frac{1-\gamma_1}{\Gamma_2(1+\gamma_1)}\right]d\gamma_1 \\ &= \frac{1}{\Gamma_1}\left[\frac{2\ln(1+\gamma_1) - \gamma_1}{\Gamma_2}\right]_0^1 \\ &= \frac{2\ln 2 - 1}{\Gamma_1\Gamma_2}. \end{aligned} \tag{18}$$

The results shows that with $p_e = 0$ the outage curve follows the tendency of the second order diversity.

APPENDIX 2

Here, the proof of the advantage of the Slepian-Wolf relay system over MRC is presented. Assuming that $\Gamma_1 = \Gamma_2 (> 0)$ in both the schemes with $p_e = 0$ and $R_c = 1$, Eqs. (13) can be further reduced to

$$P_2 = \frac{1}{\Gamma_1} \int_0^1 \left\{ \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) - \exp\left[\frac{1}{\Gamma_1}\left(-\frac{1+\gamma_1^2}{1+\gamma_1}\right)\right] \right\} d\gamma_1. \tag{19}$$

According to [7], by setting the same threshold, the outage probability of the MRC scheme with the second order diversity can be expressed as

$$P_{out,mrc} = \frac{1}{\Gamma_1} \int_{\gamma_1=0}^1 \frac{\gamma_1}{\Gamma_1} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) d\gamma_1. \tag{20}$$

To prove that $P_{out,mrc} - P_2 > 0$, we define that $P_{gap} = P_{out,mrc} - P_2$ as

$$\begin{aligned} P_{gap} =& \frac{1}{\Gamma_1} \int_{\gamma_1=0}^1 \left\{ \exp\left[\frac{1}{\Gamma_1}\left(1 - \gamma_1 - \frac{2}{1+\gamma_1}\right)\right] \right. \\ & \left. + (\frac{\gamma_1}{\Gamma_1} - 1) \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \right\} d\gamma_1 \\ =& \frac{1}{\Gamma_1} \left\{ \int_{\gamma_1=0}^1 \exp\left[-\frac{1}{\Gamma_1}\left(\frac{1+\gamma_1^2}{1+\gamma_1}\right)\right] d\gamma_1 - \exp(-\frac{1}{\Gamma_1}) \right\}. \end{aligned} \tag{21}$$

Let $y_1(x) = \exp\left(-\frac{1+\gamma_1^2}{1+\gamma_1}\right)$. It is found that $y_1(x) \geqslant -1$ hold within the range of [0, 1] if $y_1(x)$ is concave, since $y_1(x) \geqslant min\{y_1(0), y_1(1)\} = -1$, according to the property of the concave function. $y_1(x)$ can be proven to be concave by showing that

$$\begin{aligned} y_1(x)'' =& \exp(-\frac{1+x^2}{1+x}) \left[\frac{2}{(1+x)^2} - 1\right]^2 \\ & - \frac{4\exp(-\frac{1+x^2}{1+x})}{(1+x)^3} < 0. \end{aligned} \tag{22}$$

By ignoring the common exponential terms in Eq. (22), because they are positive, it is found that giving a proof to Eq. (22) is equivalent to proving that $y_2(x) = \left[2 - (1+x)^2\right]^2 - 4(1+x) < 0$. Let $t = 1 + x$ $(t \in [1, 2])$. Then, $y_2(t) = \left(2 - t^2\right)^2 - 4t = t^4 - 4t^2 - 4t + 4$. The second order derivative of $y_2(t)$ can be expressed as

$$y_2(t)'' = 12t^2 - 8. \tag{23}$$

Obviously, $y_2(t)'' > 0$ within the range of $[1, 2]$. Therefore $y_2(t)$ is convex, and $y_2(t) < max\{y_2(1), y_2(2)\} = -3$. Hence, $y_2(t) < 0$, which is equivalent to $y_1(x)'' < 0$. Now $y_1(x)$ is proved to be concave, and consequently P_{gap} is proven to be positive.

APPENDIX 3

When $\mathbf{b_1}$ and $\mathbf{b_2}$ are not fully correlated $(p_e \neq 0)$, as $P_{r1} \to \infty$ and $P_{r2} \to \infty$, P_2 will approaches 0 as seen in Eq. (13) and only P_1 dominates the outage performance. Since $P_{r1} = \Gamma_1 N_0$ and $R_c = 1$ are assumed, Eq. (12) can be written as

$$P_1 = 1 - \exp\left(-\frac{2^{R_c H(p_e)} - 1}{\Gamma_1}\right) \approx \frac{2^{R_c H(p_e)} - 1}{\Gamma_1}. \tag{24}$$

Obviously, when the average SNR Γ_1 becomes large, the value of P_1 is inversely in proportion to Γ_1 and hence the diversity order converges into one.

APPENDIX 4

In the presence of the channel correlation, regardless of the source correlation, increasing the average SNRs Γ_1 and Γ_2, or equivalently increasing P_{r1} and P_{r2} yields:

$$\frac{2|\rho| R_1 R_2}{\sqrt{P_{r1} P_{r2}} (1 - |\rho|^2)} \approx 0 \quad (P_{r1} \to \infty,\ P_{r2} \to \infty) \tag{25}$$

Hence, with $I_0(0) = 1$, Eq. (4) can be approximated as

$$\begin{aligned} p(R_1, R_2) \approx& \frac{4R_1 R_2}{P_{r1} P_{r2} (1 - |\rho|^2)} \exp\left(-\frac{R_1^2/P_{r1}}{1 - |\rho|^2} - \frac{R_2^2/P_{r2}}{1 - |\rho|^2}\right) \\ =& \frac{2R_1}{P_{r1}\sqrt{1 - |\rho|^2}} \exp\left(-\frac{R_1^2}{P_{r1}(1 - |\rho|^2)}\right) \\ & \frac{2R_2}{P_{r2}\sqrt{1 - |\rho|^2}} \exp\left(-\frac{R_2^2}{P_{r2}(1 - |\rho|^2)}\right) \\ =& p({R_1}')p({R_2}'). \end{aligned} \tag{26}$$

where ${R_1}' = R_1\sqrt{1 - |\rho|^2}$ and ${R_2}' = R_2\sqrt{1 - |\rho|^2}$, with ${P_{r1}}' = \langle {R_1}' \rangle = P_{r1}(1 - |\rho|^2)$ and ${P_{r2}}' = \langle {R_2}' \rangle = P_{r2}(1 - |\rho|^2)$. Hence, with $P_{r1} \to \infty$ and $P_{r2} \to \infty$ (equivalently, ${P_{r1}}' \to \infty$ and ${P_{r2}}' \to \infty$), the asymptotic property of the outage probability exhibits the same tendency as in the case of independent channels, which indicates that the tendency of the diversity order only depends on the source correlation.

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