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Author(s)	Sano, Katsuhiko; Tojo, Satoshi
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Dynamic Epistemic Logic for Channel-Based Agent Communication

Katsuhiko Sano[1] and Satoshi Tojo[1]

School of Information Science, Japan Advanced Institute of Science and Technology
{v-sano,tojo}@jaist.ac.jp

Abstract. This paper studies channel-based agent communication in terms of dynamic epistemic logic. First, we set up two sorted syntax which can deal with not only each agent's belief but also agents and channels between them. Second, we propose a context-sensitive 'inform'-action operator whose effectivity always assumes the existence of channel between agents. Its context-sensitivity can be achieved by downarrow binder from hybrid logic. Third, we provide complete Hilbert-style axiomatizations for both static and dynamic parts of our logic.

1 Introduction

It has been long since the notion of *agent*, by which we mean an independent inference engine within a computer, became the prevalent idea to represent artificial intelligence. Since the communication is the most distinguished feature of the presence of intelligence, its logical formalization in multiple rational agents has commonly been accepted as an important research goal. For example, based on the mobile agents platform by FIPA/ACL [1], [2] has added *communication channel* in multi-agent interaction to represent communicability between agents. Recently, [3] proposed a research program to investigate how knowledge, belief, and preferences are influenced by social relationship, and set up *Facebook Logic* for an analysis of knowledge in a social network.

In the above history of formalization of agent communication, we raise the following three requirements for our logical study of agent-communication.

(i) An informing action is basically initiated locally; thus, when information is correctly transferred, a sender agent should have a communication channel to the recipient agent.
(ii) An existence of channel may vary through a given state.
(iii) An effect of informing action at a state should be valid only on the state.

In this paper, we propose two-dimensional semantics satisfying (ii) and the informing action operator possessing three indices to implement the context-sensitivity, together with a sender and a recipient agent ((i) and (iii)). A semantic core of our paper shared with [3] can be summrized as in the following diagram:

$$w \models B_a p \rightsquigarrow (w, a) \models Bp.$$

We incorporate the information 'a' of agents into the ordinary Kripke semantics of $B_a p$ (the agent a believes that p) and regard $B\varphi$ as a property of the agent a, i.e., '- believes that he/she has a property p'.

We proceed as follows. Section 2 introduces our static syntax and its two-dimensional semantics, which is the same one as in [3]. Unlike [3], however, we also add a machinery of hybrid logic (nominals, satisfaction operators, and downarrow binder) to the dimension of possible worlds. Section 3 introduces running examples of this paper. Section 4 introduces a dynamic informing action operator and Section 5 investigates its semantic consequences. Section 6.1 gives a complete axiomatization of two-dimensional hybrid logic with frame axioms and global assumptions on models (Theorem 1). As far as the authors know, this is an unknown result of a hybrid expansion of Facebook Logic. Section 6.2 employs reduction axioms for our dynamic operator to give a complete axiomatization of our dynamic logic (Theorem 2). Section 7 concludes this paper.

2 Two-dimensional Semantics for Agent Beliefs via Channels

Our syntax consists of the set $\mathsf{N}_1 = \{ i, j, ... \}$ of state nominals, the set $\mathsf{N}_2 = \{ n, m, ... \}$ of agent nominals, the set $\mathsf{P} = \{ p, q, ... \}$ of unary *properties* of agents (or, *concept names* in description logics [4]), the belief operator B for agents, the channel operator C, the boolean connectives $\neg$, $\wedge$, the satisfaction operator $@$, and the downarrow binder $\downarrow$. The set $\mathcal{F}$ of all *formulas* of our syntax is defined inductively as follows:

$$\varphi ::= i \mid n \mid p \mid \neg \varphi \mid \varphi \wedge \psi \mid B\varphi \mid C\varphi \mid @_i\varphi \mid @_n\varphi \mid \downarrow i.\, \varphi \mid \downarrow n.\, \varphi,$$

where $i \in \mathsf{N}_1$, $n \in \mathsf{N}_2$ and $p \in \mathsf{P}$. We define $\langle C \rangle \varphi := \neg C \neg \varphi$ and $\langle B \rangle \varphi := \neg B \neg \varphi$. We also introduce the Boolean connectives as ordinary abbreviations. We can read the following formulas intuitively as:

p	'the current agent has a property p'.
Cp	'all the agents accessible via channels from the current agent satisfy p'.
$@_n \langle C \rangle m$	'there is a channel relation from n to m'.
$\langle C \rangle B @_n p$	'some agents accessible via channels from the current agent believe that the agent n satisfies p'.

For the above property (or concept name) p in P, the readers can take Father, Mother, $\mathsf{Parents}$, etc. More examples can be found in [4,3].

Let us move to the semantics. Roughly speaking, we need to incorporate channel structures between agents into Kripke frames of logic of belief. It is also natural to assume that channel structures may vary through worlds from a given Kripke frame. We also reflect this aspect into our semantics. A *social Kripke frame* (*s-frame*, in short) $\mathfrak{F} = (W, A, R, \asymp)$ consists of a non-empty set W of possible worlds, a non-empty set A of agents, A-indexed family $R = (R_a)_{a \in A}$ of binary relations on W, and W-indexed family $\asymp = (\asymp_w)_{w \in W}$ of binary relations on A. R_a is the same concept as an accessibility relation for the agent a in Kripke semantics for logic of belief, while $\asymp_w \subseteq A \times A$ reflect the idea of channel structures varying through worlds. Define $R_a(w) := \{ w' \in W \mid w R_a w' \}$, i.e., all the R_a-accessible worlds from w. A *social Kripke model* (*s-model*, in short) $\mathfrak{M} = (\mathfrak{F}, V)$ is a pair of s-frame $\mathfrak{F}$ and a valuation $V : \mathsf{N}_1 \cup \mathsf{N}_2 \cup \mathsf{P} \to \mathcal{P}(W \times A)$ satisfying $V(i) = \{ w \} \times A$ for some $w \in W$ ($i \in \mathsf{N}_1$), and $V(n) = W \times \{ a \}$ for some $a \in A$ ($n \in \mathsf{N}_2$). If we regard $W \times A$ as a two-dimensional space and W and A as x-axis and y-axis respectively, then the denotation

$V(i)$ is a vertical line and the denotation $V(a)$ is a horizontal line over $W \times A$. When $V(i) = \{ w \} \times A$, we usually write $\underline{i}$ to mean w, and so, $V(i) = \{ \underline{i} \} \times A$. Similarly, we use the notation $\underline{n}$ as $V(n) = W \times \{ \underline{n} \}$. Given any s-frame $\mathfrak{F} = (W, A, R, \asymp)$ and a valuation V on $\mathfrak{F}$, we define a *satisfaction relation* $\models$ as follows:

$$
\begin{array}{lll}
\mathfrak{M}, (w,a) \models i & \text{iff} & \underline{i} = w, \\
\mathfrak{M}, (w,a) \models n & \text{iff} & \underline{n} = a, \\
\mathfrak{M}, (w,a) \models p & \text{iff} & (w,a) \in V(p), \\
\mathfrak{M}, (w,a) \models \neg\varphi & \text{iff} & \mathfrak{M}, (w,a) \not\models \varphi, \\
\mathfrak{M}, (w,a) \models \varphi \wedge \psi & \text{iff} & \mathfrak{M}, (w,a) \models \varphi \text{ and } \mathfrak{M}, (w,a) \models \psi, \\
\mathfrak{M}, (w,a) \models B\varphi & \text{iff} & wR_a w' \text{ implies } \mathfrak{M}, (w',a) \models \varphi, \text{ for all } w' \in W, \\
\mathfrak{M}, (w,a) \models C\varphi & \text{iff} & a \asymp_w a' \text{ implies } \mathfrak{M}, (w,a') \models \varphi, \text{ for all } a' \in A, \\
\mathfrak{M}, (w,a) \models @_i\varphi & \text{iff} & \mathfrak{M}, (\underline{i},a) \models \varphi, \\
\mathfrak{M}, (w,a) \models @_n\varphi & \text{iff} & \mathfrak{M}, (w,\underline{n}) \models \varphi, \\
\mathfrak{M}, (w,a) \models \downarrow i.\, \varphi & \text{iff} & (\mathfrak{F}, V[i := w]), (w,a) \models \varphi, \\
\mathfrak{M}, (w,a) \models \downarrow n.\, \varphi & \text{iff} & (\mathfrak{F}, V[n := a]), (w,a) \models \varphi,
\end{array}
$$

where $V[i := w]$ (or $V[n := a]$) is the same valuation as V except $V(i) = \{ w \} \times A$ (or $V(n) = W \times \{ a \}$, respectively). $\downarrow i.$ and $\downarrow n.$ allow us to 'bookmark' the current world and agent with the labels i and n, respectively. In order to avoid complication of notations, we keep using nominals for bound variables of downarrow binders.

In the literatures of logic of belief, it is common to use the belief operator $B_n p$ (read: 'the agent n believes that φ'). In our setting, we can express the same content by $@_n B\varphi$ whose semantics is calculated as

$$\mathfrak{M}, (w,a) \models @_n B\varphi \;\text{ iff }\; wR_{\underline{n}}w' \text{ implies } \mathfrak{M}, (w', \underline{n}) \models \varphi, \text{ for all } w' \in W.$$

Note that $B@_n\varphi$ is different from $@_n B\varphi$, because the former tells the belief of the current agent but the latter is concerned with the belief of the agent n. We read $B@_n\varphi$ as 'the current agent believes that the agent n satisfies φ'.

Given any s-model $\mathfrak{M}$ and any set Γ of formulas, $\mathfrak{M}, (w,a) \models \Gamma$ means that $\mathfrak{M}, (w,a) \models \varphi$ for all $\varphi \in \Gamma$. Γ *valid* on $\mathfrak{M}$ (written: $\mathfrak{M} \models \Gamma$) if $\mathfrak{M}, (w,a) \models \Gamma$ for all (w,a) of $\mathfrak{M}$. Γ is *valid* on $\mathfrak{F}$ if Γ is valid on $(\mathfrak{F}, V)$ for all valuations V on $\mathfrak{F}$.

3 Running Examples of This Paper

Let us consider the following scenario: Ann just signed up Facebook and has no friend yet. She is very interested in a new mobile (say, iPhone5) but does not decide to buy it. She wants to get more friends in Facebook to listen to opinions from the others. Assume that $\mathsf{P} = \{ p \}$, where 'p' means '- will buy a mobile'.

Definition 1. *Define an* ordinary Kripke model $(S, \mathcal{R}, v)$ *where* $S := \{ s_{\mathbf{u}}, s_{\mathbf{t}}, s_{\mathbf{f}} \}$, $\mathcal{R} := \{ (s_{\mathbf{u}}, s_{\mathbf{t}}), (s_{\mathbf{u}}, s_{\mathbf{f}}) \} \cup \{ (x,x) \mid x \in S \}$ *and* $v(p) := \{ s_{\mathbf{t}} \}$.

We can regard $(S, \mathcal{R}, v)$ as a *s-model* for a single agent, say Ann, as follows. Let $A = \{ a \}$ (a means 'Ann') and define $W := S$, $R_a := \mathcal{R}$, $\asymp_x := \emptyset$ for all $x \in S$, and $V(p) = \{ (s_{\mathbf{t}}, a) \}$. Then, one can easily verify that Ann does not believe at $s_{\mathbf{u}}$ that she will buy a mobile and that she will not buy it (i.e., neither Bp nor $B\neg p$ is true at $(s_{\mathbf{u}}, a)$), while she believes at $s_{\mathbf{t}}$ (or $s_{\mathbf{f}}$) that she will buy a mobile (or will not buy it, respectively). This is a reason why we employ the indices $\mathbf{u}$, $\mathbf{t}$, and $\mathbf{f}$ in the elements of S.

Suppose that Ann now got a friend, whose name is Bea. Bea and Cate, another user, are friends, but Ann and Cate are not friends yet. In syntactic side, let us set up $\mathsf{N}_2 = \{\,\mathsf{AN}, \mathsf{BE}, \mathsf{CA}\,\}$. How can we construct s-model from the Kripke model $(S, \mathcal{R}, v)$ above? We regarded $(S, \mathcal{R}, v)$ as modeling a single agent. In order to model a community of three agents, it is natural to prepare three copies of $(S, \mathcal{R}, v)$.

Definition 2. *Define $\mathfrak{M}_1 = (W, A, R, \asymp, V)$ as follows. Let $W = S \times S \times S$ and $A = \{\,a, b, c\,\}$. When $(x_a, x_b, x_c) \in W$, we assume that x_a, x_b, and x_c represent the current state of Ann (a), Bea (b), and Cate (c), respectively. As for R, we define R_a by $(x_a, x_b, x_c)R_a(x_a, x_b, x_c)$ iff $x_a \mathcal{R} y_a$, R_b by $(x_a, x_b, x_c)R_b(x_a, x_b, x_c)$ iff $x_b \mathcal{R} y_b$, and similarly for R_c. Define $\asymp_{(x_a,x_b,x_c)} = \{\,(a, b), (b, a), (b, c), (c, b)\,\}$ for all $(x_a, x_b, x_c) \in W$. Finally, define a valuation V so as $V(\mathsf{AN}) = W \times \{\,a\,\}$, $V(\mathsf{BE}) = W \times \{\,b\,\}$, $V(\mathsf{CA}) = W \times \{\,c\,\}$ and $((x_a, x_b, x_c), a) \in V(p)$ iff $x_a = s_{\mathbf{t}}$, $((x_a, x_b, x_c), b) \in V(p)$ iff $x_b = s_{\mathbf{t}}$, and $((x_a, x_b, x_c), c) \in V(p)$ iff $x_c = s_{\mathbf{t}}$. (remark that we assume $\mathsf{P} = \{\,p\,\}$, and an arbitrary valuation suffices for any $i \in \mathsf{N}_1$.)*

An underlying idea of, e.g., R_a is that Ann cannot guess how Bea and Cate can imagine their possible states from the current state.

Example 1. Suppose that all the agents except Cate will not buy a mobile, i.e., $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}) \in W$ is a current tuple of states.

(i) Ann and Bea can see the state $s_{\mathbf{f}}$ from $s_{\mathbf{u}}$, while Cate cannot do that. Then, each of Ann and Bea does not believe that she will buy a mobile, but Cate believes so. In $\mathfrak{M}_1$, Bea is a friend of Cate, and so, $\mathfrak{M}_1, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models \langle C \rangle\, Bp$ (Bea has a friend who believes that she will buy a mobile).

(ii) Let us also check an example of iterated belief: it is true that Bea does not believe that Ann believes that she will buy a mobile at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_1$. Let us see why. Since Ann's belief state is $s_{\mathbf{u}}$, we obtain $\mathfrak{M}_1, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), a) \models \neg Bp$, which implies $\mathfrak{M}_1, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models \neg @_{\mathsf{AN}} Bp$. Since $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}) R_b (s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ holds, we finally obtain $\mathfrak{M}_1, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models \neg B @_{\mathsf{AN}} Bp$. At $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_1$, we can also verify that Cate does not believe that Ann believes that she will buy a mobile: $\mathfrak{M}_1, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), c) \models \neg B @_{\mathsf{AN}} Bp$. ■

Consider the following modifications to $\mathfrak{M}_1$: Later Bea and Cate are no longer friends, but Ann and Bea are still friends. This gives us another s-model $\mathfrak{M}_2$.

Definition 3. *Define s-model $\mathfrak{M}_2$ as the same models as $\mathfrak{M}_1$ except that we replace $\asymp$ of $\mathfrak{M}_1$ with $\approx_{(x_a,x_b,x_c)} = \{\,(a, b), (b, a)\,\}$ for all $(x_a, x_b, x_c) \in W$.*

Example 2. Now, in $\mathfrak{M}_2$, Bea can no longer access Cate, and so, $\mathfrak{M}_2, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models \neg \langle C \rangle\, Bp$ (Bea does not have a friend who believes that she will buy a mobile). As for the iterated beliefs above, we can still say that Bea and Cate do not believe that Ann believes that she will buy a mobile at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_2$, because the truth of them is independent of channel structures. ■

Note that both of $\asymp$ of $\mathfrak{M}_1$ and $\approx$ of $\mathfrak{M}_2$ are constant or rigid, i.e., $\asymp_{(x_a,x_b,x_c)}$ is always the same for all $(x_a, x_b, x_c) \in W$ and similarly for $\approx$ (we will consider a channel relation depending on an element of W later in *Example* 3).

4 Dynamic Semantics for Context-Sensitive Agent Communication

When an agent informs one of the other agents of something, our basic assumption is that we need a (context-dependent) channel between those agents. The notion of channel was formalized in terms of $\asymp$-relation in our s-model.

When the agents cooperate to achieve one goal, they need to communicate with each other. Moreover, we assume that it is important to specify *when* agents communicate, since each agent's surroundings are ever changing. Even if a message to an agent a from an agent b is useful to a at an instance t, it may become useless to a at an instant $t + 1$.

For this aim, what we want to do is to introduce the action operator $[\varphi!_m]$, whose meaning is 'after the *current agent* informs the agent m of "the current agent satisfies φ" in the *current state*.' If there is a channel from the current agent to m, this action $[\varphi!_m]$ will change m's belief only at the current state. Otherwise, the action $[\varphi!_m]$ will not change m's belief. If φ is $@_n\psi$, then $[(@_n\psi)!_m]$ means 'after the current agent informs, at the current state, the agent m of "the agent n satisfies φ".'

There is a technical problem to introduce $[\varphi!_m]$ into our static syntax. We cannot reduce the occurrences of $[\varphi!_m]$ when our syntax has two kinds of satisfaction operators $@_i$ and $@_n$. That is, $[\varphi!_m]@_i\psi \leftrightarrow @_i[\varphi!_m]\psi$ and $[\varphi!_m]@_n\psi \leftrightarrow @_n[\varphi!_m]\psi$ do not hold in general. Let us concentrate on the first one. Since an inform-action $[\varphi!_m]$ occurs at the world $\underline{i}$ in $@_i[\varphi!_m]\psi$, but it occurs at the current world in $[\varphi!_m]@_i\psi$, the effects of two actions should be different in terms of worlds.

In order to define $[\varphi!_m]$, we borrow the idea of [3, pp.184-6] to define an indexical public announcement operator into this context. That is, we first introduce $[\varphi!^i_{(n,m)}]$ ('after the agent n informs, in the state i, the agent m of "n is φ", ψ') for context-sensitive agent communication, and then define our intended operator $[\varphi!_m]$ with the help of two kinds of downarrow binders.

Definition 4. *Let us expand our static syntax with a new dynamic operator* $[\varphi!^i_{(n,m)}]$ *and denote the set of all formulas of this new syntax by* $\mathcal{F}^+$. *Given any* s*-models* $\mathfrak{M} = (W, A, R, \asymp, V)$, *we can provide the semantic clause for* $[\varphi!^i_{(n,m)}]\psi$ *as follows.*

$$\mathfrak{M}, (w, a) \models [\varphi!^i_{(n,m)}]\psi \ \text{ iff } \ \mathfrak{M}^{\varphi!^i_{(n,m)}}, (w, a) \models \psi,$$

where $\mathfrak{M}^{\varphi!^i_{(n,m)}} = (W, A, R^{\varphi!^i_{(n,m)}}, \asymp, V)$ *and* $R_a^{\varphi!^i_{(n,m)}}$ *is defined by*

$$R_a^{\varphi!^i_{(n,m)}}(w) = \begin{cases} R_{\underline{m}}(w) \cap [\![\varphi]\!]_{\underline{n}} & \text{if } a = \underline{m} \text{ and } \underline{n} \asymp_w \underline{m} \text{ and } w = \underline{i}; \\ R_a(w) & \text{o.w.} \end{cases}$$

where $[\![\varphi]\!]_a = \{ w \in W \mid \mathfrak{M}, (w, a) \models \varphi \}$ *for all* $a \in A$.

Similarly to the static syntax, let us define the notion of validity for $\mathcal{F}^+$. Now, we can define the following operators for context sensitive agent communication.

– $[\varphi!_{(n,m)}]\psi := \downarrow i.\, [\varphi!^i_{(n,m)}]\psi$.
 (‘after the agent n informs m of “n satisfies φ” in the current state, ψ’).
– $[\varphi!_m]\psi := \downarrow n.\, \downarrow i.\, [\varphi!^i_{(n,m)}]\psi$.
 (‘after the current agent informs m of “I satisfy φ” in the current state, ψ’).

We can also provide a set of reduction axioms for $[\varphi!^i_{(n,m)}]$ as in Table 1.

Proposition 1. *All the axioms in Table 1 are valid on all s-frames.*

Proof. The validity of the reduction axiom for $[\varphi!^i_{(n,m)}]B\psi$ just reflects the definition of $R^{\varphi!^i_{(n,m)}}$. For the proof, we need to use the equivalence $[\![\varphi]\!]_{\underline{n}} = [\![@_n\varphi]\!]_a$. □

Table 1. Reduction Axioms for $[\varphi!^i_{(n,m)}]$

$[\varphi!^i_{(n,m)}]\psi$	$\leftrightarrow \psi \quad (\psi \in \mathsf{P} \cup \mathsf{N}_1 \cup \mathsf{N}_2)$
$[\varphi!^i_{(n,m)}]\neg\psi$	$\leftrightarrow \neg[\varphi!^i_{(n,m)}]\psi$
$[\varphi!^i_{(n,m)}]\psi \wedge \theta$	$\leftrightarrow [\varphi!^i_{(n,m)}]\psi \wedge [\varphi!^i_{(n,m)}]\theta$
$[\varphi!^i_{(n,m)}]C\psi$	$\leftrightarrow C[\varphi!^i_{(n,m)}]\psi$
$[\varphi!^i_{(n,m)}]B\psi$	$\leftrightarrow ((m \wedge @_n \langle C\rangle\, m \wedge i) \rightarrow B(@_n\varphi \rightarrow [\varphi!^i_{(n,m)}]\psi))\wedge$
	$(\neg(m \wedge @_n \langle C\rangle\, m \wedge i) \rightarrow B[\varphi!^i_{(n,m)}]\psi)$
$[\varphi!^i_{(n,m)}]@_j\psi$	$\leftrightarrow @_j[\varphi!^i_{(n,m)}]\psi \quad (j \in \mathsf{N}_1)$
$[\varphi!^i_{(n,m)}]@_l\psi$	$\leftrightarrow @_l[\varphi!^i_{(n,m)}]\psi \quad (l \in \mathsf{N}_2)$
$[\varphi!^i_{(n,m)}]\downarrow j.\,\psi$	$\leftrightarrow \downarrow j.\,[\varphi!^i_{(n,m)}]\psi$ ($j \in \mathsf{N}_1$ is fresh in φ)
$[\varphi!^i_{(n,m)}]\downarrow l.\,\psi$	$\leftrightarrow \downarrow l.\,[\varphi!^i_{(n,m)}]\psi$ ($l \in \mathsf{N}_2$ is fresh in n, m, and φ)
$[\varphi!^i_{(n,m)}][\psi!^j_{(l,e)}]\theta$	$\leftrightarrow ((m \wedge @_n \langle C\rangle\, m \wedge i \wedge e \wedge @_l \langle C\rangle\, e \wedge j) \rightarrow [(\varphi \wedge @_l[\varphi!^i_{(n,m)}]\psi)!^i_{(n,m)}]\theta)\wedge$
	$(\neg(m \wedge @_n \langle C\rangle\, m \wedge i) \wedge e \wedge @_l \langle C\rangle\, e \wedge j) \rightarrow [(@_l[\varphi!^i_{(n,m)}]\psi)!^i_{(n,m)}]\theta)\wedge$
	$(\neg(e \wedge @_l \langle C\rangle\, e \wedge j) \rightarrow [\varphi!^i_{(n,m)}]\theta) \quad (n, m, l, e \in \mathsf{N}_2)$

Proposition 2. *The following are valid on all s-frames.*

(i) $@_i @_n \langle C\rangle\, m \rightarrow @_i([\varphi!^i_{(n,m)}]@_m B\psi \leftrightarrow @_m B(@_n\varphi \rightarrow [\varphi!^i_{(n,m)}]\psi))$.
(ii) $@_i \neg @_n \langle C\rangle\, m \rightarrow @_i([\varphi!^i_{(n,m)}]@_m B\psi \leftrightarrow @_m B[\varphi!^i_{(n,m)}]\psi)$.
(iii) $\neg @_l m \rightarrow @_i([\varphi!^i_{(n,m)}]@_l B\psi \leftrightarrow @_l B[\varphi!^i_{(n,m)}]\psi)$.

Proposition 2 says that, if there is a channel from n to m in the state i, then the informing action $[\varphi!^i_{(n,m)}]$ will change the agent m’s belief, but otherwise, the informing action will not change m’s belief.

5 Running Examples in Dynamic Context

In order to demonstrate that the action $[\varphi!_{(n,m)}]$ captures our motivation, let us consider the following three successive inform-actions in Example 1 of section 3. Suppose that the current world is $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_1$.

(i) Bea informs Ann that Ann will buy a mobile: $[(@_{\mathsf{AN}}p)!_{(\mathsf{BE},\mathsf{AN})}]$

(ii) Ann informs Bea that Ann believes that she will buy a mobile: $[(Bp)!_{(\mathsf{AN},\mathsf{BE})}]$

(iii) Bea informs Cate that Ann believes that she will buy a mobile: $[(@_{\mathsf{AN}}Bp)!_{(\mathsf{BE},\mathsf{CA})}]$

Recall from Example 1 that, at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_1$, Ann does not believe that she will buy a mobile ($\neg\, @_{\mathsf{AN}}Bp$). Recall also that Bea and Cate do not believe that Ann believes that she will buy a mobile ($\neg\, @_{\mathsf{BE}}B@_{\mathsf{AN}}Bp$ and $\neg\, @_{\mathsf{CA}}B@_{\mathsf{AN}}Bp$). Let us see each effect of the inform-actions above one by one.

After the first inform action $[(@_{\mathsf{AN}}p)!_{(\mathsf{BE},\mathsf{AN})}]$ (this succeeds, since there is a channel from Bea to Ann), Ann's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ becomes $\{ s_{\mathbf{t}} \} \times S \times S$ = $R_a((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})) \cap [\![@_{\mathsf{AN}}p]\!]_b$ [1]. Therefore, after the first action, Ann changes her belief, i.e., she now believes that she will buy a mobile ($@_{\mathsf{AN}}Bp$).

Since there is a channel from Ann to Bea in $\mathfrak{M}_1$, the second action $[(Bp)!_{(\mathsf{AN},\mathsf{BE})}]$ changes Bea's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ into $\{ s_{\mathbf{t}}, s_{\mathbf{u}} \} \times S \times S$ (note that the first action does not change Bea's accessibility relation). After the second inform-action, Bea changes her belief on Ann , i.e., Bea now *believes* that Ann believes that she will buy a mobile ($@_{\mathsf{BE}}B@_{\mathsf{AN}}Bp$) at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$.

Because there is a channel from Bea to Cate in $\mathfrak{M}_1$, the third action $[(@_{\mathsf{AN}}Bp)!_{(\mathsf{BE},\mathsf{CA})}]$ also succeeds in changing Cate's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ into $\{ s_{\mathbf{t}}, s_{\mathbf{u}} \} \times S \times S$. Then, after the above successive inform-actions, Cate changes her belief on Ann, i.e., Cate *believes* that Ann believes that she will buy a mobile ($@_{\mathsf{CA}}B@_{\mathsf{AN}}Bp$) at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$. This example demonstrates that, even if there is no direct channel between Ann and Cate, message passing via channels can change Cate's belief on Ann.

For comparison, consider the effect of the successive actions above at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_2$ from Example 2 of section 3, where there is no channel from Bea to Cate. At this world of $\mathfrak{M}_2$, recall from Example 2 that Cate still does not believe that Ann believes that she will buy a mobile ($\neg\, @_{\mathsf{CA}}B@_{\mathsf{AN}}Bp$). Unlike the case of $\mathfrak{M}_1$, the third action does not succeed in changing Cate's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$. Therefore, Cate does not change her belief on Ann, i.e., Cate still does not believe that Ann believes that she will buy a mobile ($\neg\, @_{\mathsf{CA}}B@_{\mathsf{AN}}Bp$) at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$.

Example 3 (Informing Channels). In our running example, channel relations of $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are *rigid*, i.e., channel relations are invariant through all elements of W = $S \times S \times S$. Let us consider non-rigid channels in this example and see an effect of informing a channel itself between agents. Let us take the following requirement on a relationship on Bea and Cate: Bea and Cate are friends in Facebook only when they have the same opinion for deciding to buy a mobile. Following this requirement, define a new channel relation $\sim$ by: $\sim_{(x_a,x_b,x_c)} = \{ (a,b), (b,a), (b,c), (c,b) \}$ (if $x_b = x_c$) and $\sim_{(x_a,x_b,x_c)} = \{ (a,b), (b,a) \}$ (if $x_b \neq x_c$). We define $\mathfrak{M}_3$ as the same s-models except we use $\sim$ instead of $\asymp$. Note that channels between Ann and Bea are still rigid. Throughout this example, we always assume that our current state is $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$. Then, we can say at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_3$ that Bea *does not* believe that she has a friend who will buy a mobile:

$$\mathfrak{M}_3, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models \neg\, B\, \langle C \rangle\, p.$$

[1] Note that $[\![@_{\mathsf{AN}}p]\!]_b = [\![p]\!]_a = \{ s_{\mathbf{t}} \} \times S \times S$ in $\mathfrak{M}_1$ and $R_a((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})) = S \times S \times S$.

This is because $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}) R_b (s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ and Bea does not have a friend who will buy a mobile at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_3$ (note that Bea's belief state $s_{\mathbf{u}}$ is different from Cate's belief state $s_{\mathbf{t}}$).

Suppose that Ann and Cate are not friends in Facebook, but they are so in real life. Cate told Ann that she will buy a mobile and that she wants to be a friend of Bea in Facebook. After chatting with Cate, Ann made the following successive inform-actions in Facebook:

(i) Ann informs of Bea that Cate will buy a mobile: $[(@_{\mathsf{CA}} p)!_{(\mathsf{AN},\mathsf{BE})}]$.
(ii) Ann informs of Bea that Cate is a friend of Bea: $[(@_{\mathsf{CA}} \langle C \rangle \mathsf{BE})!_{(\mathsf{AN},\mathsf{BE})}]$.

After the first action at $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ of $\mathfrak{M}_3$ (note that there is always a channel between Ann and Bea), Bea's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ become $S \times S \times \{ s_{\mathbf{t}} \}$. Furthermore, the second action will change Bea's accessible worlds from $(s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}})$ into $S \times \{ s_{\mathbf{t}} \} \times \{ s_{\mathbf{t}} \}$. After these two actions, Bea can only accesses to the tuple of states where both Bea and Cate will buy a mobile, i.e., Bea and Cate are friends by our definition of $\sim$. Therefore, after the above successive inform-action, Bea now believes that she has a friend who will buy a mobile. That is,

$$\mathfrak{M}_3, ((s_{\mathbf{u}}, s_{\mathbf{u}}, s_{\mathbf{t}}), b) \models [(@_{\mathsf{CA}} p)!_{(\mathsf{AN},\mathsf{BE})}][(@_{\mathsf{CA}} \langle C \rangle \mathsf{BE})!_{(\mathsf{AN},\mathsf{BE})}] B \langle C \rangle p.$$

In this way, an action of informing a channel itself can also change agents' belief. ■

6 Complete Axiomatizations of Static and Dynamic Logics

6.1 Hilbert-style Axiomatization of Static Logic with Global Assumptions

This section give a complete axiomatization of our intended logic in the *static* syntax.

If concept names $\mathsf{Mother}, \mathsf{Father}, \mathsf{Parents}$ are in P, it is natural to assume the equivalence $(\mathsf{Mother} \lor \mathsf{Father}) \leftrightarrow \mathsf{Parents}$ (regarded as 'TBox' in description logic [4]). We want to validate this particular equivalence at all agents and worlds in a given s-model. In this sense, we call it a *global assumption*. A global assumption could be any formula of $\mathcal{F}$ but it should be regarded as axioms in the level of s-model but not in the level of s-frame. In what follows, we will give a semantic consequence relation and a deducibility relation of our static syntax under the existence of global assumptions.

Definition 5. *Given a set Φ of global assumptions and a class F of s-frames, φ is a* local consequence of Ψ under global assumptions Φ for F (*notation: $\Phi; \Psi \models_{\mathsf{F}} \varphi$*) *if, for all $\mathfrak{F} = (W, A, R, \asymp) \in \mathsf{F}$ and all valuations V on $\mathfrak{F}$ such that $(\mathfrak{F}, V) \models \Phi$ holds, $(\mathfrak{F}, V), (w, a) \models \Psi$ implies $(\mathfrak{F}, V), (w, a) \models \varphi$ for all $(w, a) \in W \times A$.*

Note that we restrict our attention to the set of valuations V on $\mathfrak{F}$ such that Φ is valid on s-model $(\mathfrak{F}, V)$ in this definition.

Let us move to the corresponding proof-theoretic derivability relation to $\Phi; \Psi \models_{\mathsf{F}} \varphi$. First of all, we do not allow the following *uniform substitutions* to global assumptions.

Definition 6. *σ is* a uniform substitution *if it is the inductive extension of a mapping sending $p \in \mathsf{P}$ to a formula and a nominal of N_u to a nominal of N_u ($u = 1, 2$).*

Table 2. Axioms and Rules of Two-dimensional Hybrid Logic for Agent Beliefs via Channels

	Modal Axioms
CT	all classical tautologies
K	$\Box(p \to q) \to (\Box p \to \Box q) \quad (\Box \in \{ B, C \})$.
	Hybrid Axioms for Nominals and Satisfaction Operators
K@	$@_\alpha(p \to q) \to (@_\alpha p \to @_\alpha q)$, where $\alpha = i$ or n.
Dual	$\neg @_\alpha p \leftrightarrow @_\alpha \neg p$, where $\alpha = i$ or n.
Ref	$@_\alpha \alpha$, where $\alpha = i$ or a.
Intro	$\alpha \wedge p \to @_\alpha p$, where $\alpha = i$ or n.
Agree	$@_\alpha @_\beta p \to @_\beta p$, where $(\alpha, \beta) = (i, j)$ or (n, m).
$\mathbf{Back}_B$	$@_i p \to B @_i p$.
$\mathbf{Back}_C$	$@_n p \to C @_n p$.
	Hybrid Axioms for Downarrow Binders
$\mathbf{DA}_1$	$@_j(\downarrow j.\, \varphi \leftrightarrow \varphi[i/j])$
$\mathbf{DA}_2$	$@_n(\downarrow m.\, \varphi \leftrightarrow \varphi[n/m])$
	Interaction Axioms
Com@	$@_n @_i p \leftrightarrow @_i @_n p$
$\mathbf{Red@}_1$	$@_i a \leftrightarrow a$
$\mathbf{Red@}_2$	$@_n i \leftrightarrow i$
$\mathbf{Dcom}B\mathbf{@}_2$	$@_n B p \leftrightarrow @_n B @_n p$
$\mathbf{Dcom}C\mathbf{@}_1$	$@_i C p \leftrightarrow @_i C @_i p$
	Rules
MP	$\varphi \to \psi, \varphi / \psi$
Nec□	$\varphi / \Box\varphi (\Box \in \{ B, C \})$.
Nec@	$\varphi / @_\alpha \varphi (\alpha \in \mathsf{N}_1 \cup \mathsf{N}_2)$.
Name	$\alpha \to \varphi / \varphi$, where $\alpha \in \mathsf{N}_1 \cup \mathsf{N}_2$ does not occur in φ.
$\mathbf{BG}_B$	$@_i \langle B \rangle j \to @_j \varphi / @_i B\varphi$, where $i, j \in \mathsf{N}_1$ and $j \neq i$ does not appear in φ.
$\mathbf{BG}_C$	$@_n \langle C \rangle m \to @_n \varphi / @_n C\varphi$, where $n, m \in \mathsf{N}_2$ and $m \neq n$ does not appear in φ.

If we allows global assumptions to be closed under uniform substitutions, we can derive from $(\mathsf{Mother} \vee \mathsf{Father}) \leftrightarrow \mathsf{Parents}$ that $(\mathsf{Woman} \vee \mathsf{Man}) \leftrightarrow \mathsf{Parents}$, which is undesirable. On the other hand, we want to allow uniform substitutions to logical axioms such as tautologies, basic axioms of modal logic. Therefore, in order to incorporate global assumptions to a deducibility relation, we need to restrict the use of uniform substitutions carefully. First, we define the theoremhood under frame axioms (to capture the information of F in $\Phi; \Psi \models_{\mathsf{F}} \varphi$) and global assumptions Φ, and then define our intended deducibility relation.

Definition 7. *Given any set $\mathcal{A}$ of formulas, regarded as the* frame axioms*, we write $\Phi \vdash_{\mathcal{A}} \varphi$ if φ in the smallest set of formulas that contains Φ and all the* substitution instances *of both $\mathcal{A}$ and all the axioms listed in Table 2 and is closed under all the rules of Table 2. We say that φ* is derivable from Ψ under global assumptions Φ and frame axioms $\mathcal{A}$ *(written: $\Phi; \Psi \vdash_{\mathcal{A}} \varphi$) if there is a finite subset $\Psi' \subseteq \Psi$ such that $\Phi \vdash_{\mathcal{A}} \bigwedge \Psi' \to \varphi$, where $\bigwedge \Psi'$ is the conjunction of all finite elements of Ψ' (if $\Psi' = \emptyset$, we define $\bigwedge \Psi' := \top$).*

Remark that we do *not* require global assumptions Φ to be closed under uniform substitutions in this definition, while we require frame axioms $\mathcal{A}$ and the axioms in Table 2 to be closed under uniform substitutions. Therefore, $B(\varphi \to \psi) \to (B\varphi \to B\psi)$ is derivable (from $\emptyset$) under any global assumptions and any frame axioms, but $(\mathsf{Woman} \vee \mathsf{Man}) \leftrightarrow \mathsf{Parents}$ is *not derivable* (from $\emptyset$) under a global assumption $(\mathsf{Mother} \vee \mathsf{Father}) \leftrightarrow \mathsf{Parents}$ and no frame axioms.

Definition 8. *We say that a set Γ of formulas* defines *a class* F *of* s*-frames if, for all* $\mathfrak{F} \in \mathsf{F}$*,* Γ *is valid on* $\mathfrak{F}$ *iff* $\mathfrak{F} \in \mathsf{F}$.

In what follows in this paper, we denote the class of all s-frames by $\mathsf{F}_{\rm all}$.

Proposition 3 (Soundness). *Let* $\mathcal{A}, \Phi, \Psi \cup \{ \varphi \} \subseteq \mathcal{F}$ *and* $\mathcal{A}$ *define* F*. Then,* $\Phi; \Psi \vdash_{\mathcal{A}} \varphi$ *implies* $\Phi; \Psi \models_{\mathsf{F}} \varphi$*. In particular,* $\Phi; \Psi \vdash_{\emptyset} \varphi$ *implies* $\Phi; \Psi \models_{\mathsf{F}_{\rm all}} \varphi$*.*

Proof. Let us only check the validity of $@_n Bp \leftrightarrow @_n B @_n p$. Fix any s-model $\mathfrak{M}$ and any (w, a) of $\mathfrak{M}$. Then, $\mathfrak{M}, (w, a) \models @_n Bp$ iff $\mathfrak{M}, (w, \underline{n}) \models Bp$ iff $wR_{\underline{n}}w'$ implies $\mathfrak{M}, (w', \underline{n}) \models p$ for all $w' \in W$ iff $wR_{\underline{n}}w'$ implies $\mathfrak{M}, (w', \underline{n}) \models @_n p$ for all $w' \in W$ iff $\mathfrak{M}, (w, \underline{n}) \models B @_n p$ iff $\mathfrak{M}, (w, a) \models @_n B @_n p$, as required. □

Let us say that $\varphi \in \mathcal{F}$ is a *pure formula* if it does not contain any symbol from P.

Theorem 1 (Strong Completeness). *Let* $\mathcal{A}$ *be a set of pure formulas and* $\mathcal{A}$ *define a class* F *of* s*-frames. Given any sets* $\Phi, \Psi \cup \{ \varphi \} \subseteq \mathcal{F}$*,* $\Phi; \Psi \models_{\mathsf{F}} \varphi$ *implies* $\Phi; \Psi \vdash_{\mathcal{A}} \varphi$*. In particular,* $\Phi; \Psi \models_{\mathsf{F}_{\rm all}} \varphi$ *implies* $\Phi; \Psi \vdash_{\emptyset} \varphi$*.*

Proof (Sketch). A basic idea of the proof is a combination of completeness arguments in [5] (to deal with global assumptions) and [6] (to handle two-dimensionality of our static syntax). We show the contrapositive implication. Let us say that Ψ is $(\mathcal{A}, \Phi)$-*consistent* if $\Phi; \Psi \nvdash_{\mathcal{A}} \bot$. Suppose $\Phi; \Psi \nvdash_{\mathcal{A}} \varphi$, i.e., $\Psi \cup \{ \neg \varphi \}$ is $(\mathcal{A}, \Phi)$-consistent. A key idea for global assumptions here is to employ the following 'doubly' $@$-prefixed formulas: Given any set $\Sigma \subseteq \mathcal{F}$, we define $@\Sigma := \{ @_i @_n \varphi \mid \varphi \in \Sigma$ and $(i, n) \in \mathsf{N}_1 \times \mathsf{N}_2 \}$. A subset of $@\mathcal{F}$ is called an *ABox* (we followed the terminology of [5]). A *maximally* $(\mathcal{A}, \Phi)$*-consistent ABox* is a $\subseteq$-maximal element among $(\mathcal{A}, \Phi)$-consistent ABoxes. By Lindenbaum construction, we use fresh nominals as if Henkin-constants in FOL and construct a maximally $(\mathcal{A}, \Phi)$-consistent ABox Σ such that $@_i @_n \Psi \cup \{ \neg \varphi \} \subseteq \Sigma$ for some nominals (i, n). Then, we define the Henkin-style canonical model $\mathfrak{M}^{\Sigma} = (W^{\Sigma}, A^{\Sigma}, R^{\Sigma}, \asymp^{\Sigma}, V^{\Sigma})$ consisting of:

- $W^{\Sigma} := \{ |i| \mid i \in \mathsf{N}_1 \}$, where $|i| := \{ j \mid @_i @_n j \in \Sigma$ for some $n \in \mathsf{N}_2 \}$.
- $A^{\Sigma} := \{ [n] \mid n \in \mathsf{N}_2 \}$, where $[n] := \{ m \mid @_i @_m j \in \Sigma$ for some $i \in \mathsf{N}_1 \}$.
- $|i| R^{\Sigma}_{[n]} |j|$ iff $@_i @_n \langle B \rangle j \in \Sigma$
- $[n] \asymp^{\Sigma}_{|i|} [m]$ iff $@_i @_n \langle C \rangle m \in \Sigma$.
- $(|i|, [n]) \in V(\varphi)$ iff $@_i @_n \varphi \in \Sigma$ ($\varphi \in \mathsf{P} \cup \mathsf{N}_1 \cup \mathsf{N}_2$).

By $@_i @_n \Psi \cup \{ \neg \varphi \} \subseteq \Sigma$, we can show $\mathfrak{M}^{\Sigma}, (|i|, [n]) \models \Psi$ but $\mathfrak{M}^{\Sigma}, (|i|, [n]) \not\models \varphi$ (here we need interaction axioms of Table 2). By construction, we can assure that $\mathfrak{M}^{\Sigma} \models \Phi$. Moreover, $(W^{\Sigma}, A^{\Sigma}, R^{\Sigma}, \asymp^{\Sigma})$ is in F, since $\mathcal{A}$ defines F and $\mathcal{A}$ is a set of pure formulas and all points of W^{Σ} and A^{Σ} are named by some nominals. Therefore, $\Phi; \Psi \not\models_{\mathsf{F}} \varphi$, as required. □

Example 4. (i) $\mathcal{A}_1 = \{ @_n \neg \langle C \rangle n, @_n \langle C \rangle m \to @_m \langle C \rangle n \}$ defines irreflexivity and symmetry of $\asymp_w$ and $\mathcal{A}_2 = \{ @_i \langle B \rangle i, @_i \langle B \rangle j \to @_j \langle B \rangle i, (@_i \langle B \rangle j \wedge @_j \langle B \rangle k) \to @_i \langle B \rangle k \}$ defines that R_a is an equivalence relation. By Theorem 1, the union of those *pure* axioms provides a complete axiomatization of a hybrid expansion of Facebook Logic [3].

(ii) Global axioms Φ_1 = { (Mother $\vee$ Father) $\leftrightarrow$ Parents } assure us that concept name Parents has an intended definition in the level of s-model. Φ_2 = { $@_{\mathsf{AN}} \langle C \rangle$ BE, $@_{\mathsf{BE}} \langle C \rangle$ AN } assure us that we can restrict our attension to the s-models where there are two-way channels between Ann and Bea. We can augment our logic with global assumptions $\Phi_1 \cup \Phi_2$ and frame axioms $\mathcal{A}_1 \cup \mathcal{A}_2$ without losing our completeness result. ■

6.2 Complete Axiomatization of Dynamic Logic via Reduction Axioms

Similarly to the static syntax, we define the notions of definability, semantic consequence relation $\Phi; \Psi \models_{\mathsf{F}} \varphi$, etc. also for the set $\mathcal{F}^+$ of all formulas in the static syntax with $[\varphi!^i_{(n,m)}]$. For simplicity, this section does not consider any frame axioms of Section 6.1. Given any $\Phi, \Psi \subseteq \mathcal{F}^+$, let us define $\Phi; \Psi \vdash^+ \varphi$ if there exists some finite subset Ψ' such that $\bigwedge \Psi' \to \varphi$ is in the smallest set of $\mathcal{F}^+$ such that it contains Φ, all reduction axioms of Table 1 and all the substitution instances of axioms of Table 2 and that it is closed under all the rules of Table 2. Note that we do not require that global assumptions Φ and the reduction axioms are closed under uniform substitutions.

Theorem 2 (Strong Completeness). *Let $\Phi \subseteq \mathcal{F}$ be any global assumptions containing no occurrence of B. Then, for any $\Psi \cup \{ \varphi \} \subseteq \mathcal{F}^+$, $\Phi; \Psi \vdash^+ \varphi$ iff $\Phi; \Psi \models_{\mathsf{F}_{\mathrm{all}}} \varphi$.*

Proof. Here we only establish the right-to-left direction (completeness), since soundeness follows Proposition 1. By reduction axioms of Table 1, let us fix a translation $\tau : \mathcal{F}^+ \to \mathcal{F}$ such that $\varphi \leftrightarrow \tau(\varphi)$ is valid on $\mathsf{F}_{\mathrm{all}}$ for all $\varphi \in \mathcal{F}^+$. For our goal, let us assume that $\Phi; \Psi \models_{\mathsf{F}_{\mathrm{all}}} \varphi$. We can show $\Phi; t[\Psi] \models_{\mathsf{F}_{\mathrm{all}}} t(\varphi)$ in the syntax of $\mathcal{F}$ as follows. Take any s-frame $\mathfrak{F} \in \mathsf{F}_{\mathrm{all}}$ and any valuation V such that $\mathfrak{M} \models \Phi$, where $\mathfrak{M} = (\mathfrak{F}, V)$. Moreover, assume that $\mathfrak{M}, (w, a) \models t[\Psi]$. We need to establish $\mathfrak{M}, (w, a) \models t(\varphi)$. Then, also in the syntax of $\mathcal{F}^+$, we obtain $\mathfrak{M} \models \Phi$ and $\mathfrak{M}, (w, a) \models t[\Psi]$, which implies $\mathfrak{M}, (w, a) \models \Psi$ by definition of τ. By assumption, $\mathfrak{M}, (w, a) \models \varphi$ hence $\mathfrak{M}, (w, a) \models t(\varphi)$, as desired. Then, we can proceed as follows: $\Phi; t[\Psi] \models_{\mathsf{F}_{\mathrm{all}}} t(\varphi)$ iff $\Phi; t[\Psi] \vdash_{\emptyset} t(\varphi)$ by Proposition 3 and Theorem 1. By definition of $\vdash^+$ in $\mathcal{F}^+$, $\Phi; t[\Psi] \vdash^+ t(\varphi)$. By the translation τ by reduction axioms, this is equivalent with $\Phi; \Psi \vdash^+ \varphi$, as required. □

7 Conclusion

In connection with our three requirements: (i), (ii), and (iii) in the introduction, our contribution can be summarized as follows. (i) First, we employed the notion of local announcement, contrary to the *public* announcement operator [7], assuming the existence of channels between agents for the individual announcement. (ii) Next, we proposed that agents' communicability should depend on agents' belief situation. As preceding works, [8,9] assumed that the social network relations were context-independent. However, we regarded that communicability might change dependent on environments in which the agent is embedded. (iii) Finally, we contended that an effect of informing action at a given state should be valid only on the state. The act of commanding by Yamada [10] at a given state w required us to change the agent's other accessible states besides w. We, however, shared the idea of [11], where *time-dependent* command was proposed.

In this paper, we have specified agent communication in rather strict formalisms. For example, our information transfer may be considered as commanding or forcing, and no room for alternative belief for each agent. In addition, even though an agent changed his/her belief, other agents cannot know such belief changes unless there exists an explicit informing action. Furthermore, a belief change minimally propagates, dependent on the exact state of the sender agent's informing action. We admit these settings reflect only an aspect of agent communication; we need to consider the feasibility of our logic (or its possible extended version, cf. [12]), and the comparison to the other options is our future work.

References

1. Foundation for Intelligent Physical Agents (FIPA): Communicative act library specification (2000) `http://www.fipa.org`.
2. van Eijk, R.M., de Boer, F.S., van der Hoek, W., Meyer, J.J.C.: Process algebra for agent communication: A general semantic approach. In Huget, M.P., ed.: Communication in Multiagent Systems. Volume 2650 of LNCS., Springer (2003) 113–128
3. Seligman, J., Liu, F., Girard, P.: Logic in the community. In: Proceedings of the 4th Indian conference on Logic and its applications. Volume 6521 of Lecture Notes in Computer Science. (2011) 178–188
4. Baader, F., Lutz, C.: Description logic. In Blackburn, P., van Benthem, J., Wolter, F., eds.: Handbook of Modal Logic. Elsevier (2007) 757–820
5. Schröder, L., Pattinson, D.: Named models in coalgebraic hybrid logic. In Marion, J.Y., Schwentick, T., eds.: Proceedings of STACS 2010. Leibniz International Proceedings in Informatics (2010) 645–656
6. Sano, K.: Axiomatizing hybrid products: How can we reason many-dimensionally in hybrid logic? Journal of Applied Logic **8**(4) (December 2010) 459–474
7. Plaza, J.A.: Logics of public communications. In Emrich, M.L., Pfeifer, M.S., Hadzikadic, M., Ras, Z.W., eds.: Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems. (1989) 201–216
8. Pacuit, E., Parikh, R.: Reasoning about communication graphs. In van Benthem, J., Löwe, B., Gabbay, D., eds.: Interactive Logic: Games and Social Software. Texts in Logics and Games. Amsterdam University Press (2008)
9. Ruan, J., Thielscher, M.: A logic for knowledge flow in social networks. In Wang, D., Reynolds, M., eds.: AI 2011. Volume 7106 of LNCS. (2011) 511–520
10. Yamada, T.: Acts of commanding and changing obligations. In: Proceedings of the Seventh International Workshop on Computational Logic in Multi-Agent Systems, Springer-Verlag (2006) 1–19
11. Lewis, D.: A problem about permission. In Saarinen, E., Hilpinen, R., Niiniluoto, I., Hintikka, M., eds.: Essays in Honour of Jaakko Hintikka. Reidel, Dordrecht (1979) 163–175.
12. Sano, K.: Generalized product of coalgebraic hybrid logics. In Corradini, A., Klin, B., Cîrstea, C., eds.: Algebra and Coalgebra in Computer Science - 4th International Conference, CALCO 2011. Volume 6859 of LNCS. (2011) 329–343