Combining-after-Decoding Turbo Hybrid ARQ by Utilizing Doped-Accumulator

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***Abstract*—This letter proposes a doped-accumulator assisted packet-combining-after-decoding (ACC-CAD) technique with different doping rates per transmission phases for hybrid automatic repeat request (ARQ). Horizontal iteration (*HI*) performed for decoding the serially concatenated codes (SCC) is followed by Vertical iteration (*VI*) to exchange extrinsic log-likelihood ratio (LLR) of the uncoded (systematic) bits, and the *HI*/*VI* chain is repeated. The doped-accumulator enables the two extrinsic information transfer (EXIT) curves of the SCC to match very well and the convergence tunnel to open until a point very close to the (1.0,1.0) mutual information point. Excellent performance of our technique is verified through EXIT analysis as well as frame-error-rate (FER) and throughput simulations.**

***Index Terms*—Hybrid ARQ, turbo principle, doped-accumulator.**

I. INTRODUCTION

EXCHANGING soft information in an iterative (turbo) fashion for packet combining in automatic repeat request (ARQ) systems has been intensively investigated in the last few years. Reference [1] first introduced the utilization of the Turbo encoder structure in hybrid ARQ (HARQ). The HARQ system forms parallel concatenation of multiple recursive convolutional codes, which is corresponding to multiple transmissions of the same information, if random interleaving is performed in the parallel branches of the encoder. At the receiver side, the packet combining-after-decoding (CAD)[1] is performed to exploit the soft information expressed in the form of *a priori* log-likelihood ratio (LLR) of the uncoded (systematic) bits obtained in the last transmission for decoding the current transmission. It is shown in [1] that a considerable frame-error-rate (FER) performance gain can be achieved over conventional ARQ without packet combining.

The CAD concept has been adopted in various forms that can be found in the literatures. Reference [2] exploits the structure of [1] with systematic recursive convolutional code as the encoder instead of turbo code. In the technique shown in [2], only either systematic or parity bits are retransmitted, and hence a considerable throughput improvement can be achieved. Reference [3] considers rate compatible punctured turbo coded HARQ to provide higher throughput. A similar technique is proposed in [4], where the parity spreading interleaver (PSI) is employed to re-order the parity bits as a scheme for flexible puncturing.

An alternative way to packet combining in the framework of ARQ in multipath propagation environment is combining-before-decoding (CBD). The CBD technique combines all path energies of the transmitted signals to achieve diversity gain at the receiver.[2] CBD can be performed by joint soft equalization and packet combining using a maximum *a posteriori* (MAP) equalizer, as introduced in [6]. The algorithm is an extension of the well-known turbo equalization to a single-input multiple-output (SIMO) channel where signal samples, received in each retransmission phase, are stored and combined by the turbo equalizer; this concept is referred to as integrated equalization (IEQ) in [6]. The same idea for packet combining by using MAP equalization is also proposed in [7]. Although the MAP algorithm can achieve the optimal performance, its computational complexity increases exponentially with the channel memory length. Most recently, the technique presented in [8] employs frequency domain turbo equalization with soft combining ARQ at the equalizer in a full-duplex system.

Another form of CBD is combining the interleaved LLRs of equalizers outputs, known as turbo-diversity transmit (TDT) in [5] or bit-interleaving diversity (BID) in [9]. Reference [5] introduces an improved turbo-diversity scheme to IEQ by exploiting different interleaver in every retransmission to reduce the Euclidean distance dispersion. As for IEQ, retransmission of the same codeword with an identical interleaver for every retransmission achieves only a path diversity. It can easily be expected that in frequency-selective fading channels, the both CAD and CBD techniques can achieve a diversity order of multipath number k transmission(s). On the top of the diversity gain, both the techniques can achieve coding gain, where this letter shows that higher coding gain can be achieved by CAD. We show the capacities of the CAD and CBD techniques, and verify the achievability of their corresponding asymptotic performances via computer simulations. We then show that the introduction of doped-accumulator $DAcc$ [10] – [11] with different doping rates in each transmission can improve the CAD performance while reducing the computational complexity.

The rest of this paper is organized as follows. The proposed turbo HARQ strategy is presented in Section II. Results of the extrinsic information transfer (EXIT) chart analyses provided in Section III are used to verify the simulation results presented in Section IV. Finally, Section V concludes this letter.

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[1] The terminologies "before" and "after" are relative to the channel decoder.

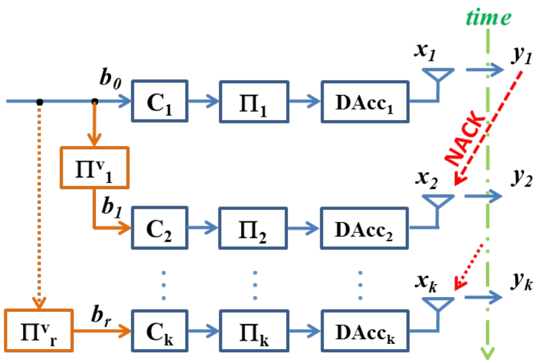

Fig. 1. Transmitter of doped-accumulator assisted turbo HARQ systems

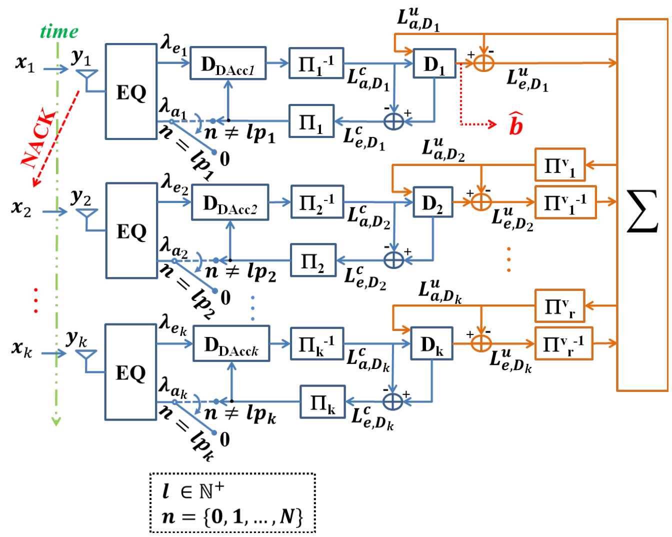

Fig. 2. Receiver of doped-accumulator assisted turbo HARQ systems

II. Proposed Turbo HARQ with Doped-Accumulator

The transmitter of the turbo HARQ system proposed in this letter is depicted in Fig. 1. The maximum times of transmission attempts is set at K (with $K-1$ retransmissions). The bit stream $\boldsymbol{b}_0$ of the first transmission attempt is encoded by a rate R non-systematic non-recursive convolutional codes (NSNRCC) C_1. The output of C_1 is then interleaved by a random interleaver Π_1. The output of Π_1 is fed into doped accumulator $DAcc$ with a generator G = $[3,2]_8$. $DAcc$ introduces no redundancy and its decoder requires only moderate additional complexity. The doped-accumulated bits are modulated by BPSK. Puncturing may be performed to achieve higher throughput as indicated in [3]. However, this paper leaves the issue of code optimization as a future study.

When the transmitter receives a positive acknowledgement (ACK), it indicates a successful decoding and thereby the transmitter moves on to the next frame transmission. If the transmitter receives a negative acknowledgement (NACK), it indicates an unsuccessful decoding at the receiver, and hence the transmitter retransmits the same information.

For the k-th attempt, $k = \{1, 2, \cdots, K\}$, $\boldsymbol{b}_0$ is interleaved every r-th retransmission $r = \{1, 2, \cdots, k-1 \,|\, k \neq 1\}$, yielding $\boldsymbol{b}_r = \Pi^v_r(\boldsymbol{b}_0)$,where $\Pi^v_r(\cdot)$ is the function performing vertical interleaving. The transmitter transmits a vectorized signal having N symbols, as $\mathbf{x}_k = [x_k(1), x_k(2), \cdots, x_k(N)]^T \in \mathbb{C}^{N\times 1}$.

At the receiver side, the received signal $\mathbf{y}_k$ of the k-th transmitted frame is given by $\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \nu_k \in \mathbb{C}^{N\times 1}$, where ν_k is a zero mean complex additive white Gaussian noise (AWGN) vector with variance σ^2, corresponding to the specified signal-to-noise power ratio (SNR). The channel is assumed to be static within a frame, and statistically independent varies frame-by-frame. The channel matrix $\mathbf{H}_k$ is a Toeplitz matrix with the dimensionality $(N+L-1)\times N$, where L is the channel memory length. With the cyclic prefix (CP) transmission, $\mathbf{H}_k$ becomes an $N \times N$ circulant matrix with the multipath channel response on its first column: $[h(0), h(1), \cdots, h(L-1), 0, \cdots, 0]^T$. Then we can utilize the characteristic of the circularity to obtain a diagonal matrix of frequency domain channel matrix $\mathbf{\Psi}_k = \mathbf{F}^H\mathbf{H}_k\mathbf{F}$. This property is well utilized in the frequency-domain soft cancellation and minimum mean squared error (FD/SC-MMSE) turbo equalization (EQ). However, since the algorithm of the FD/SC-MMSE equalization technique is already well known [12]–[14], this letter does not provide its details. The results of log-likelihood ratio (LLR) are exchanged via horizontal iteration (*HI*) between the equalizer+deaccumulator (EQ+D_{DAcc}) and the decoder (D). For the decoders D_{DAcc} and D, we use Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [15].

A. Packet Combining After Decoding

The *HI* takes place independently in different phases of transmission, as shown in Fig. 2. Therefore the channel matrix size stays the same for every *HI*. The *HI* is performed until no relevant improvement in mutual information (MI) is achieved, and then the obtained extrinsic LLR of the systematic information bits, $L^u_{e,D}$, is combined into L^u_s as

$$L^u_s = L^u_{e,D_1} + \sum_{i=2}^{k} \Pi^v_{i-1}(L^u_{e,D_i}). \quad (1)$$

L^u_s is fed back to soft-input soft-output (SISO) channel decoders as the *a priori* information, as depicted in Fig. 2, referred to as vertical iteration (*VI*). The (*VI*) can be seen as iterative decoding process of parallel concatenated code. Finally, by performing sufficient times of iterative *HI-VI-HI-VI* decoding, the final hard decision, $\hat{\boldsymbol{b}}$, is made on the *a posteriori* LLR of the information (systematic) bits.

B. Doped-Accumulator

$DAcc$ is composed of a memory-1 systematic recursive convolutional code with octal generator of ($[3,2]3_8$) followed by heavy puncturing of the parity bits. Every p-th[3]systematic

[2]The combining should not necessarily be in the path energy domain, but can be in their LLR domain, as indicated in [5].

[3]p without index indicates the doping rate, while p with the index k indicates the doping rate for the k-th transmission.

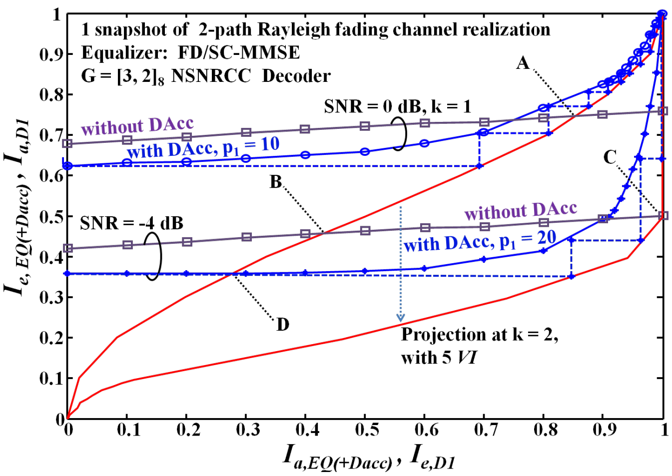

Fig. 3. EXIT chart of Equalizer (+ D_{DAcc}) for single snapshot of channel realization and NSNRCC decoder at k = {1,2}

bits are replaced by the parity bits. As shown in Fig. 1, $DAcc$ is performed after the its corresponding interleaver. At the receiver, D_{DAcc} follows the FD/SC-MMSE equalization. The *a priori* LLR to the equalizer is set to zero for every p-th bit, as shown in Fig. 2. Hence, it is important to note that when $p = 1$, DAcc is equivalent to a full accumulator; in this case, no LLR of the accumulated bits is fed back from the channel decoder to the equalizer.

III. EXIT ANALYSIS

The structure of the CAD technique enables the use of different doping rates of the $DAcc$ transmission-by-transmission to achieve better matching of the EXIT curves. The doping rate of the first transmission is determined by evaluating the EXIT curves of inner and outer code so that they are best matched among the all possible values of the doping rates, while keeping the convergence tunnel open. In such parameter settings, no retransmission is required if the received SNR is larger than the threshold at which the convergence tunnel opens.

Fig. 3 shows the EXIT curves of EQ and $EQ + D_{DAcc}$, with a generator $[3,2]_8$ NSNRCC's decoder, for a single snapshot channel realizations at the first transmission and the same channel for retransmission.[4] The figure also shows the trajectory with the maximum iteration of 350. We set the interleaver length to 20000 bits. For $k = 1$, we set the doping rate p_1 of $DAcc$ assisted CAD (ACC-CAD) to 10 and the average SNR to 0 dB. It shows that the EQ curve intersects the decoder curve at point "A", while $EQ+D_{DAcc}$ curve with $p_1 = 10$ makes the convergence tunnel open until a point very close to the (1.0,1.0) MI point.

At SNR = -4 dB, which is relatively low, the EQ curve intersects the decoder curve at point "B" whereas $EQ+D_{DAcc}$ curve with the doping rate $p_1 = 20$ at first intersects at the point "D" at which the MI is lower than at the point "B". Hence, with $p_1 = 20$, the performance of the system with $DAcc$ for the first transmission is worst than the system without $DAcc$. Furthermore, even though the MI at the intersection point can be raised by adjusting the doping rate, it is still difficult to reach a point close enough to the (1.0,1.0) MI point at the low SNR. CAD solve this problem with the help of *VI* that pushes down the decoder curve, resulting in better matching between $EQ + D_{DAcc}$ and decoder curves. Moreover, the gap between the two curves can further be reduced by adjusting the doping rate.

IV. SIMULATION RESULTS

To evaluate the performances, computer simulations has been conducted in multipath Rayleigh fading channels. Randomly generated binary sequence with a length of 512 information bits per frame were used as $\boldsymbol{b}_0$. Then, for ACC-CAD, $\boldsymbol{b}_0$ and $\boldsymbol{b}_r$ are independently coded by the same memory-1 rate 1/2 NSNRCC encoder with generator polynomial $([3\ 2])_8$. For comparison, we also tested memory-2 rate 1/2 NSNRCC encoder with generator polynomial $([7\ 5])_8$ for both CAD and CBD without $DAcc$. Here, the total memory that used at the receiver for all the evaluated systems are the same. However, both memory-1 code NSNRCC and $DAcc$ with the proposed system, having only two states in their trellis diagram, and hence decoding complexity is lower than the other cases used for comparison.

We set 2-path independent and identically distributed (i.i.d.) block fading channel with equal average path gains. We assume that the receiver has perfect knowledge about the channel, including frame and symbol timings. We assume stop-and-wait protocol with error-free feedback channel. We also assume perfect packet error detection.

First, we evaluate the FER performances of the ACC-CAD with doping rates $p_1 = 10$, $p_2 = 20$, $p_3 = 50$, and $p_4 = 100$. Then for comparison, we evaluate CAD without $DAcc$ and refer TDT in [5] as the CBD. We also evaluate the outage probability of CBD and CAD in 2-path Rayleigh fading channels via a series of Monte Carlo simulations with respect to the fading variations, based on the *static* capacity with the Chase combining and incremental redundancy HARQ schemes, respectively [16].[5] It is found that the decay of FER curve follows asymptotically that of the outage curve.

We perform simulations with maximum retransmission to 3 ($K = 4$). For fair comparison, we set the maximum iteration times at 350 for every (re)transmission for all the tested systems.[6] With CAD, for $k = 2$, we perform first one *HI* in each branch then followed by five *VI*s; the *HI* and *VI* process are repeated 50 times, referred as activation ordering pattern 50(H1V5). The activation ordering pattern for $k = 3$ and $k = 4$ are 43(H1V5) and 38(H1V5), respectively.

For the first transmission (without combining), the both CAD and CBD systems schemes achieve the same outage

[4]The channel of the second transmission is generally different from the first, but in this figure we assume the same because the purpose of the EXIT analysis shown in this Section is to demonstrate how the EXIT curve change by the *VI*.

[5]For fair comparison, the total signal energy is normalized to per-transmission.

[6]The iteration is stopped when no relevant increase in the mutual information is obtained after five consecutive iterations.

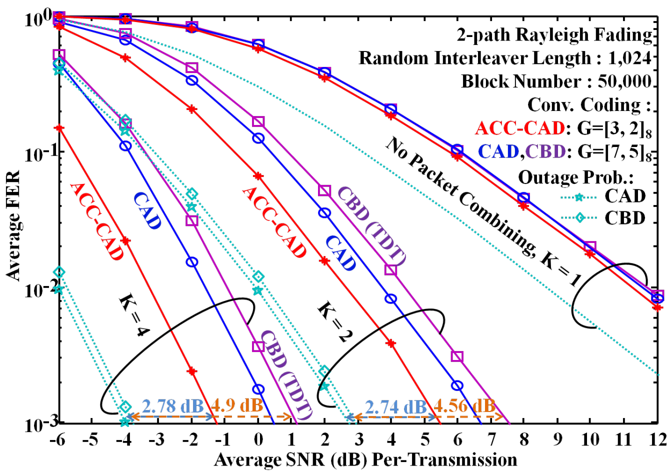

Fig. 4. FER performance

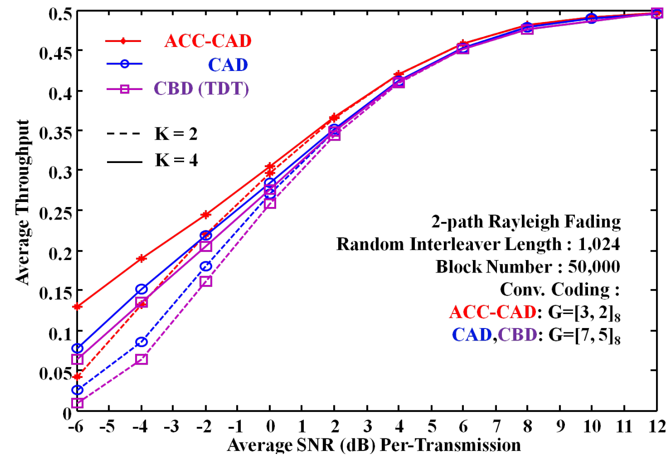

Fig. 5. Throughput performance

probability. In this case, the system performance is determined by the path diversity and coding gains that can be achieved by the *HI* only. It is found from Fig. 4 that the proposed CAD outperforms the systems without $DAcc$, as predicted from the EXIT analysis.[7] With $k > 1$, the outage probability of CAD is lower than CBD; hence it is found that CAD clearly outperforms CBD. With ACC-CAD, the gap between FER and the outage probability, at FER = 10^{-3} for $K = 2$, is 2.74 dB whereas with the CBD the gap is 4.56 dB. Similarly, for $K = 4$, the gap between the FER curve and the outage probability with the ACC-CAD is 2.78 dB whereas the CBD is 4.90 dB.

The excellent performance of the proposed systems is also shown in terms of throughput efficiency at relatively low SNR in Fig. 5. With the ACC-CAD technique, as the throughput curves with every K value converge at SNR around 4 dB, we only need one retransmission, when SNR > 4 dB, to achieve the highest throughput. Fig. 5 also shows that ACC-CAD with only one retransmission outperforms the CBD with three retransmissions over the SNR value range of -2 dB $\leq$ SNR $\leq$ 8 dB in average.

V. Conclusion

This letter has proposed CAD HARQ technique using doped-accumulator ($DAcc$) and vertical iteration (*VI*) decoding. It has been shown that the proposed system outperforms the HARQ systems without $DAcc$. With the help of EXIT chart analysis, it has been shown that a better matching between $EQ + D_{DAcc}$ and NRNSCC decoder's EXIT curves, concatenated with *VI* decoding, can be achieved with the proposed technique, which results in excellent FER and throughput performances. The superiority of CAD over CBD has also been verified via computer simulations in multipath Rayleigh fading channels.

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[7] Result for $K = 3$ is not shown to simplify the figure.