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## Synthesis method of gain scheduling control using spline-type Lyapunov functions

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Recently, required performance levels of control systems are getting higher and higher. An example of this can be seen in control of airplanes since 1950's. As high improvement of capability of airplanes, they are demanded to fly in wide range of speeds, altitudes and poses. Traditional feedback control with a fixed gain no longer attains desired performances or even stability of airplanes. To overcome this problem, controllers of an airplane are *scheduled* corresponding to flight states of the airplane. As in this example, gain scheduling control is a control strategy to adjust the controller corresponding to on-line observation of changes of dynamics of a plant. For highly demanded stability and performances under various working conditions, it is desired to verify stability and performances of gain scheduling control systems theoretically.

These backgrounds motivate study of gain scheduling control from the viewpoint of control theory. As an expression of gain scheduling control systems, linear parameter-varying (LPV) systems have been used. LPV system is a state space equation whose coefficients have scheduling parameters which indicates changes of dynamics of a plant. When we describes a plant as an LPV system, it is a big merit that the expression of LPV systems allow to analyze and synthesize control systems using a wealth of results of linear control theory.

For gain scheduling control based on the LPV description, some design methods are proposed. In the method called frozen parameter method, controllers are designed to guarantee stability and demanded performances of each of linear time invariant systems with the scheduling parameters fixed at some points, and controllers

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are scheduled corresponding to parameters on-line observed. However, in the case that scheduling parameters vary quickly, this method does not guarantee even stability. On the other hand, there have been proposed methods to guarantee stability at any parameters varying speed via a quadratic Lyapunov function constant with respect to the gain scheduling parameters. However, by this method control is conservative if parameters are only slowly varying. To overcome these deficiencies of previous methods, an approach using parameter-dependent Lyapunov function is proposed. This method enables to synthesize control systems taking account of parameters varying speed. On the contrary to the frozen parameter method, it guarantees stability and performances of a control system, and is less conservative in design than using those parameter-independent quadratic Lyapunov functions.

Some results give existence conditions of a controller which guarantees stability and  $L_2$ -gain performance as a specification of control system design. These conditions are described in terms of on linear matrix inequalities (LMIs) on parameter-dependent Lyapunov functions, which are solved by effective computation algorisms. In this approach, existence conditions of a controller are given as scheduling parameterdependent LMI conditions. Then, though it is necessary to find solutions of continuously parameter-dependent LMI conditions, in order to find just solutions we have to solve infinite LMI conditions corresponding to each of fixed parameters However, computers cannot actually judge infinite LMI conditions. To this problem, Some researches show that it is sufficient to solve certain newly constructed finite LMI conditions, instead of computing the infinite LMI conditions. In these researches, the newly constructed finite LMI conditions satisfies the sufficiency for scheduling parameter-dependent LMI conditions to hold, but, the necessity to the original conditions is not proved. For this reason, even if solutions of parameter-dependent LMI conditions exist, the solutions are not always found by means of previous methods. Therefore, in order to make a less conservative analysis and design, it is important to construct LMI conditions necessary and sufficient to parameter-dependent LMI conditions.

In this paper, we propose a method to construct LMI conditions equivalent to parameter-dependent LMI conditions. This new method is as follows : We consider piecewise continuous spline-type functions as solutions of parameter-dependent LMI conditions. Finite LMI conditions sufficient to the original one are constructed. The most important point of our result is that, if each continuous section of spline functions is sufficiently small, it is proved that the renew constructed LMI conditions in this research are also necessary conditions to original LMI conditions.

First, the new method proposed in this research is applied to parameter-dependent LMI conditions which estimate stability and  $L_2$ -gain performance of LPV systems, and we get the new LMI conditions equivalent to the original one. By numerical example we verify that the renew finite LMI conditions is necessary and sufficient to original LMI conditions. Second, we extend this result to the case of LPV systems with state feedback. The feedback gain is formulated with spline functions obtained

from solutions of the constructed finite LMI conditions.