

Title	Lyapunov stability analysis of energy constraint for intelligent home energy management system
Author(s)	Umer, Saher; Tan, Yasuo; Lim, Azman Osman
Citation	IEICE Technical Report on Ubiquitous and Sensor Networks (USN), 112(242): 131-135
Issue Date	2012-10-17
Type	Journal Article
Text version	publisher
URL	http://hdl.handle.net/10119/11585
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Description	

インテリジェント宅内エネルギー管理システムのためのエネルギー制約下のリャプノフ安定解析

ウマル サハル 丹 康雄 リム アズマンオスマン

北陸先端科学技術大学院 情報科学研究科 〒923-1292 石川県能美市旭台 1-1

E-mail: {saher, ytan, aolim}@jaist.ac.jp

あらまし 宅内エネルギー管理システム(HEMS)は、家電機器作動の監視および管理を担っており、一連の要求に応じたエネルギーの負荷配分を行う。家電機器の増加に伴い、宅内電力消費の逼迫による停電リスクが増し、その結果、HEMSは不安定なものとなる。本論文では、リャプノフアプローチを用いたエネルギー消費のHEMSによる安定性をモデル化し、さらに、エネルギー制約下におけるHEMSによる安定性についての検討を行った。

キーワード 宅内エネルギー管理システム, リャプノフ安定, エネルギープロファイル, 電力消費

Lyapunov Stability Analysis of Energy Constraint for Intelligent Home Energy Management System

UMER Saher TAN Yasuo LIM Azman Osman

Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan

E-mail: {saher, ytan, aolim}@jaist.ac.jp

Abstract Home energy management system (HEMS) is responsible for monitoring and managing the operation of appliances, and providing load shifting according to a set of requirements. As more and more in-home appliances are increased, the energy consumption in home leads an increase in the risk of power blackout. Consequently, the HEMS becomes an system. In this paper, we model the stability of HEMS of energy consumption using Lyapunov approach. We also study the stability of our HEMS in the presence of the energy constraint.

Keyword home energy management system, Lyapunov stability, energy profile, energy consumption

1. INTRODUCTION

Homes in today's world tend to include more and more electrically powered devices. Much effort is put on improving energy consumption, but their integration towards an intelligent home often remains unconsidered and power consumption in home area (i) tends to grow and (ii) leads an increase in the risk of power blackout. Therefore, balancing the energy use in home is a very challenging issue when the power supply is limited. The technology to manage and balance the home energy use is well-known as a home energy management system (HEMS) [1]. The HEMS is a networked system which is responsible for monitoring and managing the operation of in-home appliances, and helps residences in order to reduce power consumption according to the specific set of requirements [2]. The deployment of HEMS technology offers several attractive objectives, among them most prominently is stability of home energy when energy increased by adding home appliances to improve residential comfort and peace of mind for the home owner. As households are undoubtedly one of the main energy consumers today, also a positive effect for the

environment can be expected if power consumption is reduced. In the last decade, intelligent home is populated by multitude of devices (actuators and sensors) that cooperate in an intelligent way to control different domains of the home such as lighting/shading, heating/ventilation/air-conditioning, but also home appliances and consumer electronics [3].

This paper established a global stability criterion by introducing an optimized scheme for home and system optimum is achieved at the equilibrium between the energy flow of each appliance and price proportional to that energy flow. Based on the proposed explicitly structure of HEMS, we solve system optimization based stability analysis for efficient utilization and real-time monitoring in the presence of limited energy capacity constraint. This research result essentially shows that stability is related to utility (desire of the user to turn ON the appliance) in a fundamental way. In particular, it is possible to design stable user utility functions such that the corresponding system converges to the optimal yet stable energy flow.

As the consumed energy approaches the provision

limited capacity, the overall energy of HEMS shrinks, causing an increased risk of power outage. The user's side system should be stable with utility in price because the price is a good indicator to reflect the consumed energy. The reason is that even a small reduction in the peak consumed energy can result in a vast change in price, even eliminating price spikes. Although user's satisfaction is also an important factor in the successful implementation the HEMS. In this paper, our main aim is to model and study the stability of the HEMS by using the Lyapunov stability approach. The proposed stability approach tries to maximize the user comfort along with price under limited capacity of the HEMS. Furthermore, the Lyapunov stability analysis enables the residential customer to smartly control their energy consumed as a reaction to changes in the price of energy. In our HEMS model we observed the energy consumption due to air conditioner, IH (induction heater), light, outlets, window actuators, and TV booster. The aggregation impact of energy consumed by HEMS is evaluated using price, energy consumed, and energy supply limit for given HEMS.

The rest of this paper is organized as follows. Research background and motivation that are related to this paper is summarized in Section 2. In Section 3, we describe the model stability of the HEMS with energy constraint and their mathematical representations. Simulation study and discussion are presented in Section 4. Finally, we conclude our research and future works in Section 5.

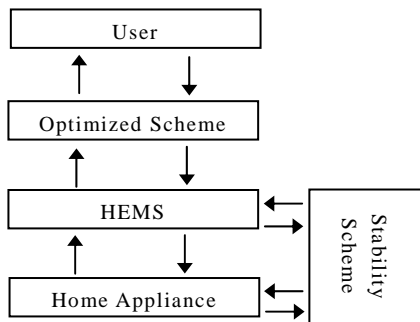


Fig. 1: Stability of the HEMS

2. RESEARCH BACKGROUND AND MOTIVATION

2.1 Home Energy Management System (HEMS)

As stated in [4], HEMS consists of a group of functions that facilitate remote monitoring, controlling, planning and repairing of operations and provide information on the status of installed devices and the network. In particular, the HEMS has following four functions:

- *Auto configuration*: auto-configuration is the most

important function for user of home network services because many homes have a wrong configuration.

- *Easy monitoring*: comfort and easy access to real-time information in energy consumption help the user in paying attention to energy saving.
- *Remote controlling*: online access to a customer's usage pattern and device status enables appliances to be controlled remotely.
- *Smart planning*: automatic peak load management provides smart planning for reducing energy consumption.

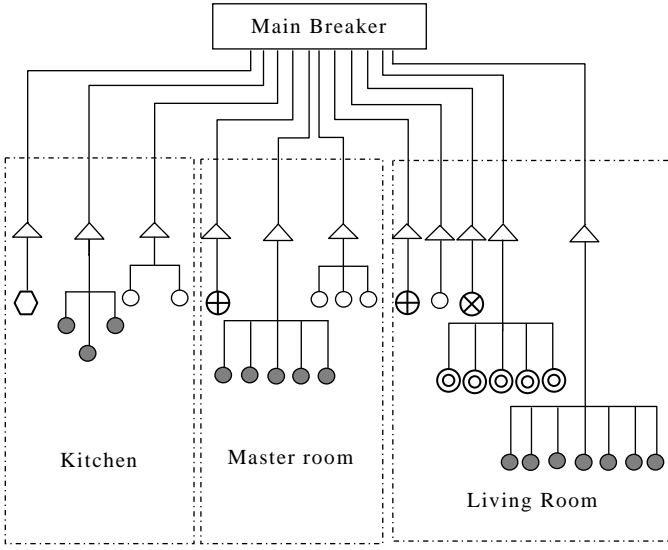
2.2 Research Motivation

The need for electricity has been growing rapidly in recent years as a result of addition of home appliance at home to make life comfortable for the user. These technological advances allow the shortage of electricity. Additionally, the electricity consumed varies significantly over time. HEMS often encounter unexpectedly high energy consumed level due to various reasons. The HEMS need to be continuously balance the energy available for HEMS and the consumed energy for reliable energy delivery across the appliances. As the consumed energy increased than the available energy limit for HEMS also decreased, which makes the HEMS susceptible to outage [5]. The purpose of this research is to correctly model and in-depth study the stability scheme of the HEMS based on the spot prices and energy constraint as depicted in Fig. 1. It underlines the following tasks: (i) development of a price-responsive optimized scheme, (ii) development of a model for home appliance use in the presence of energy constraint, and (iii) application of Lyapunov functions to ensure the stability of HEMS.

3. STABILITY ANALYSIS

3.1 HEMS Model

HEMS plays an essential role to not only optimize the operation of home appliances, but also make sure the stability is always guaranteed in a home. For the sake of simplicity, we consider a home structure consists of three domain areas, e.g., kitchen, master room and living room as shown in Fig. 2. Each divided domain area has home appliances for the comfort of the user having a connection with main breaker through power lines. The main breaker has a fixed capacity of energy limit is C_{MB}^{max} . For instance, less than 60 Ampere.



△	Hub connection
⬡	IH (induction heater)
●	Light
○	Outlet
⊕	AC (air-conditioner)
⊗	TV booster
⊙	Window actuator

Fig. 2: HEMS model

3.2 Mathematical Representation

We rather look at the whole home appliance in a home. We simply take a kitchen as a representative model for explanation purpose.

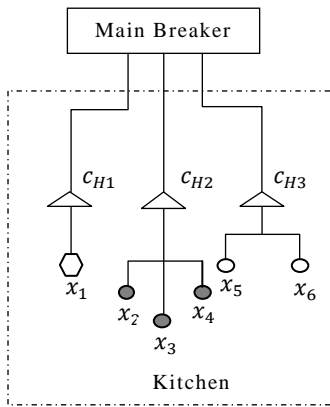


Fig. 3: A representative model of kitchen

c_{H1}	Maximum capacity for hub breaker1 $c_{H1} \leq 30$
c_{H2}	Maximum capacity for hub breaker2 $c_{H2} \leq 20$
c_{H3}	Maximum capacity for hub breaker3 $c_{H3} \leq 20$
x_1	IH connected with c_{H1}

x_2	Light connected with c_{H2}
x_3	Light connected with c_{H2}
x_4	Light connected with c_{H2}
x_5	Outlet with c_{H3}
x_6	Outlet with c_{H3}
x_i	Energy flow of home appliance i
A	Represent appliance status ON(1), OFF(0)
M	Total number of home appliances

Consider a set M of home appliances, and let x_i be the energy flow of a home appliance i from hub, $i \in M$. Let C_j be a finite capacity of hub j , for $j \in J$. Let i and j be the non-empty subset of M and J , respectively. Therefore, the status of home appliance can be expressed as

$$A = \begin{cases} A_{ij} = 1 & \text{if } j \in i, \text{energy flow } i \text{ from hub } j \\ A_{ij} = 0 & \text{otherwise} \end{cases} \quad (1)$$

This defines a 0-1 matrix $A = (A_{ij}, i \in M, j \in J)$. Associate an energy flow to home appliance i with the desire of the user that the user wants to turn ON. Suppose that if energy flow x_i is allocated to home appliance i then this has utility $U_i(x_i)$ to the user. Assume that the utility $U_i(x_i)$ is an increasing, strictly concave and continuously differentiable function of x_i over the range $x_i \geq 0$ [6]. Assume further that utilities are additive, so that the aggregate utility of energy flows $x = (x_i, i \in M)$ is $\sum_{i \in M} U_i(x_i)$. Let $U = (U_i, i \in M)$ and $C = (C_j, j \in J)$. Under this model the system optimization energy flows solve the following issues.

SYSTEM (U, A, C):

$$\begin{aligned} & \text{maximize } \sum_{i \in M} U_i(x_i) \\ & \text{subject to } Ax \leq C \\ & \text{over } x \geq 0 \end{aligned} \quad (2)$$

While this system optimization problem is mathematically fairly traceable (with a strictly concave objective function and a convex feasible region), it involves utilities U that are unlikely to be known by the network. We are thus led to consider two simpler problems.

Suppose energy flow x_i will be charged an amount of price per unit time, w_i and home appliance i receives in return the energy flow x_i proportional to w_i . Say $x_i = w_i/\lambda_i$, where λ_i could be regarded as a charge per unit flow for i . Then the utility maximization problem for energy flow x_i is as follows.

APPLIANCE ($U_i; \lambda_i$):

$$\begin{aligned} & \text{maximize } U_i\left(\frac{w_i}{\lambda_i}\right) - w_i \\ & w_i \geq 0 \end{aligned} \quad (3)$$

Suppose next that the network knows the vector $w = (w_i, i \in M)$, and attempts to maximize the function $\sum_{i \in M} w_i \log x_i$. The network's optimization problem is then as follows.

NETWORK ($A, C; w$):

$$\begin{aligned} & \text{maximize } \sum_{i \in M} w_i \log x_i \\ & \text{subject to } Ax \leq C \\ & \text{over } x \geq 0. \end{aligned} \quad (4)$$

It is known [7] that there always exist vectors $\lambda = (\lambda_i, i \in M)$, $w = (w_i, i \in M)$ and $x = (x_i, i \in M)$, satisfying $w_i = \lambda_i x_i$ for $i \in M$, such that w_i solves **APPLIANCE**($U_i; \lambda_i$) for $i \in M$ and x solves **NETWORK** ($A, C; w$), further, the vector x is then the unique solution to **SYSTEM** (U, A, C).

The Lagrangian [8] for the problem of **NETWORK** ($A, C; w$) can be written as

$$L(x; \mu) = \sum_{i \in M} w_i \log x_i + \mu^T (C - Ax) \quad (5)$$

In the optimization problem, the method of Lagrangian provides a strategy for finding the local maxima and minima of a function subject to equality constraint. Where μ is a Lagrangian multiplier (or charge per unit flow for i), by differentiating equation (5), we will obtain

$$\frac{\partial L}{\partial x_i} = \frac{w_i}{x_i} - \sum_{j \in i} \mu_j \quad (6)$$

From the above equation we get, the optimum to the optimization problem is given by

$$x_i = \frac{w_i}{\sum_{j \in i} \mu_j} \quad (7)$$

where $(x_i, i \in M)$, $(\mu_j, j \in J)$ solve

$$\mu \geq 0, Ax \leq C, \mu^T (C - Ax) = 0 \quad (8)$$

At each time t , we denote the energy flow by $x_i(t)$ that can be represented by the differential equation

$$\frac{d}{dt} x_i(t) = \kappa \left(w_i - x_i(t) \sum_{j \in i} \mu_j(t) \right) \quad (9)$$

Where

$$\mu_j(t) = p_j \left(\sum_{s: j \in s} x_s(t) \right) \quad (10)$$

Consider the system of differential equations (9) and (10), suppose that $p_j(y)$ is a price charged by home appliance i , per unit flow of x_i , when the total flow of energy through home appliance i is y . Then by adjusting the flow on hub j , $x_i(t)$, in accordance with equations (9) and (10), the system attempts to equalize the aggregate cost of this flow, $x_i(t) \sum_{j \in i} \mu_j(t)$, with target value w_i , for every $i \in M$.

3.3 Lyapunov Stability

Suppose that $p_j(y)$ is a continuous function to calculate the energy consumption with respect to price when the total energy flow through home appliance is y . Under mild regularity conditions on the function p_j , $j \in J$, the expression

$$V(x) = \sum_{i \in M} w_i \log x_i - \sum_{j \in J} \int_0^{\sum_{s: j \in s} x_s} p_j(y) dy \quad (11)$$

provides a Lyapunov function for the optimized system of differential equations (9) and (10), and we deduce the vector x maximizing $V(x)$ is a stable point of the system, to which all trajectories converge. The function p_j , $j \in J$, may be chosen so that the maximization of the Lyapunov function $V(x)$ arbitrarily closely approximates the optimization problem **NETWORK** ($A, C; w$), and, in this sense, is a relaxation of the HEMS system. The Lyapunov function equation (11) thus provides an enlightening analysis of the global stability of the system (9) and (10), and of the relationship between this system and the **NETWORK** ($A, C; w$).

3.4 Global Stability

Let the function $V(x)$ be defined by equation (11) where $w_i > 0, i \in M$, and suppose that, $j \in J$, the function $p_j(y)$, $y \geq 0$, is a non-negative, continuous, increasing function of y , not identically zero.

Theorem

The strictly concave function $V(x)$ is a Lyapunov function for the system of differential equations (9) and (10). The unique value x maximizing $V(x)$ is a stable point of the system, to which all trajectories converge.

Proof

The assumption on $w_i > 0, i \in M$, and $p_j, j \in J$, ensure that $V(x)$ is strictly concave on $x \geq 0$ with an interior maximum; the maximizing value of x is thus unique. Observe that

$$\frac{\partial}{\partial x_i} V(x) = \frac{w_i}{x_i} - \sum_{j \in i} p_j \left(\sum_{s: j \in s} x_s \right); \quad (12)$$

setting these derivatives to zero identifies the maximum. Further

$$\begin{aligned} \frac{d}{dt}V(x(t)) &= \sum_{i \in M} \frac{\partial V}{\partial x_i} \cdot \frac{d}{dt}x_i(t) \\ &= k \sum_{i \in M} \frac{1}{x_i(t)} \left(w_i - x_i(t) \sum_{j \in i} p_j \left(\sum_{s: j \in s} x_s(t) \right) \right)^2 \end{aligned} \quad (13)$$

establishing that $V(x(t))$ is strictly increasing with t , unless $x(t) = x$, the unique x maximizing $V(x)$. The function $V(x)$ is thus a Lyapunov function for the system (9) and (10), and theorem follows (see [9], Chapter 5). \square

4. SIMULATION STUDY AND DISCUSSION

In this section, we study the mathematical representation of the stability of the HEMS model in the previous section. Consider the kitchen architecture in Fig. 3, where $J = 3$ and $M = 6$. Let $w_i = 0.01692$, $i \in M$, suppose these two home appliances (e.g., oven and IH rice cooker) are connected to the kitchen's outlet. The average currents of oven (x_5) and IH rice cooker (x_6) are 13.2 A and 11.0 A, respectively. When the oven is turned on, the result values of Lagrangian and Lyapunov are 0.169 and 12.773. The positive integer of Lyapunov indicates the system status is stable all the times. When the IH rice cooker is turned on while the oven is on, the value of Lyapunov is -7.648, in which it shows the system status is unstable when both are turned on at the same time.

It can summarize that the Lyapunov stability approach can be used as a tool to monitor the system stability of HEMS in real-time. Moreover, this principle of Lyapunov stability is also applicable to monitor the stability of main breaker. Further research work on the stability issue of the HEMS with delay constraints is also considerable and interesting.

5. CONCLUDING REMARKS

In this paper we have addressed the stability issue of the HEMS in the presence of limited energy supply. An optimization framework for the HEMS leads to an overall stability of home, in which the user chooses a charge per unit time that the user is willing to pay, for the proportional energy flow. Stability is established by showing an appropriate formulation of the overall optimization problem; the network's implicit objective function provides a Lyapunov function for the HEMS by the optimum flow. The network's optimization scheme leads naturally to the energy flow based on prices. Future research work will be investigating the stability of the HEMS when the human daily activity is considered.

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