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Description	

Outage Probability Analysis for Correlated Sources Transmission over Rician Fading Channels

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***Abstract*—In this paper, the outage probability of a Slepian-Wolf transmission system is analysed theoretically. We consider two correlated binary information sequences where bit-$flipping$ model is adopted, where the the information bits of the second source are the $flipped$ version of the information bits of the first one, with a probability P_f. Here, we extend our previous work of the correlated sources transmission over block Rayleigh fading channels to such that the channel between one source and the destination is assumed to suffer from block Rayleigh fading, whereas the channel between the other source and the destination is assumed to suffer from block Rician fading. The outage probability is analysed theoretically. The results show that when $P_f = 0$, the outage curves versus signal-to-noise power ratio (SNR) exhibits sharper decay than that with the $2nd$ order diversity, as the line-of-sight (LOS) component power of Rician channel increases. It is also found that the outage curves asymptotically converge in to that with the $1st$ order diversity when $P_f \neq 0$. Furthermore, this paper also determines the optimal power allocation under the condition that total transmit power of the system is kept constant. It can be concluded that increasing the ratio of LOS component power of Rician channels can not always improve the outage performance, so far as $P_f \neq 0$.**

I. Introduction

Currently, cooperative relay protocols, such as amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) techniques, have drawn significant attention and been widely studied. Among those, the AF protocol suffers from noise accumulation where the noise is amplified at the relay node. The DF protocol has a drawback that the data block is discard if error(s) is/are found in the information part at relay. Recently, cooperative communication system using DF that allows intra-link (source-relay link) errors have been shown in [1] to provide good performance. The Slepian-Wolf coding theorem [2] is used to utilize the correlation of source and relay, hence it is referred to as Slepian-Wolf relaying system. The outage probability of the relaying systems has been evaluated in [3], where some asymptotic properties also have been revealed. The results show that the $2nd$ order diversity can always be achieved if the information sequences

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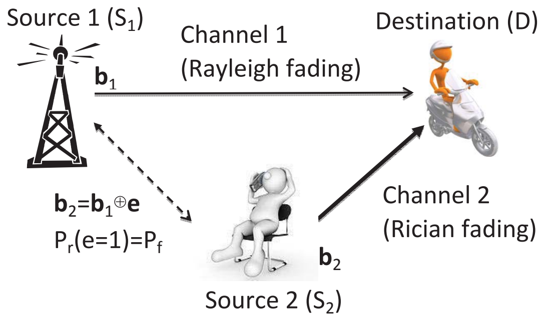

Fig. 1. System model for Slepian-Wolf transmission system: different fading channel scenarios.

transmitted from the source and relay are fully correlated, and the asymptotic tendency exhibited in the outage curve gradually converges in to that of the $1st$ order diversity if the sequences are not fully correlated. Furthermore, the optimal relay locations are analysed in [4]. It is found that when the relay moves towards to the destination, the decay of the outage curve tends to be equivalent to without diversity. References [3] and [4] assume that the both source-destination (SD) and relay-destination (RD) links are suffering from block Rayleigh fading. In practice, the SD and RD links are often found to suffer from different fading. Hence, it is quite reasonable to assume that the RD channel has line-of-sight (LOS) component, resulting in the channel being Rician-distributed. Hence, it is very interesting to identify the impact of the LOS component in the RD channel on the outage probability.

Since the primary purpose of this paper is to analyse the outage probability of two correlated binary sources transmission, one via block Rayleigh fading channel, and the other via block Rician fading channel, we assume a very simple transmission system, as shown in Fig. 1. This paper does not assume any specific structure of relaying system, and only focus on the problem of two correlated binary sources transmission where the correlation between the two sources can be expressed by the bit-$flipping$ model [5]. One of the important works of this paper is to identify the effect of Rician factor K on the outage probability, where K denotes the ratio of the LOS component power-to-non-LOS components average power. It has to be noted that the separation of source-channel coding holds, according to Shannon's separation theorem [6]. Hence the only difference in the channel property from [3] and [4] is the probability density functions (PDFs) of their instantaneous signal-to-noise power ratios (SNRs).

Interestingly, although the asymptotic behaviour of the outage probability found in this work is the same as that shown in [3], the convergence to the asymptotic performance is different, depending on the K factor. Furthermore, this paper also investigates the optimal power allocation for minimizing the outage probability with the condition that the total transmit power of two channels is kept constant. The optimal power allocation largely depends on the K factor when two sources are fully correlated. Surprisingly, if two sources are not fully correlated, increasing the K factor does not always achieve lower outage probability.

The remainder of the paper is organized as follows: the system model used to analyse the outage probability is described in Section II. The outage probability derivation is presented in Section III. Asymptotic tendency is then provided in Section IV based on the analytical results. Section V shows numerical results and performance analysis. Finally, this paper is concluded in Section VI with some concluding remarks.

II. System Model

As shown in Fig. 1. $\mathbf{b}_1$ and $\mathbf{b}_2$ are the information bit sequence to be sent from source 1 (S_1) and source 2 (S_2) through Channel 1 and Channel 2, respectively, to the destination D. $\mathbf{b}_1$ and $\mathbf{b}_2$ are correlated, and the correlation is modelled by $\mathbf{b}_1 = \mathbf{b}_1 \oplus \mathbf{e}$ with $\Pr(e = 1) = P_f$ [5]. Channel 1 is suffering from block Rayleigh fading whereas Channel 2 is suffering from block Rician fading. If the pair of rates R_1 and R_2 falls into the inadmissible region as shown in Fig. 2, the receiver can not recover transmitted data losslessly, according to the Slepian-Wolf theorem. More explicitly, R_1 and R_2 must satisfy the three inequalities [2]:

$$R_1 \geq H(\mathbf{b}_1|\mathbf{b}_2), \tag{1}$$

$$R_2 \geq H(\mathbf{b}_2|\mathbf{b}_1), \tag{2}$$

$$R_1 + R_2 \geq H(\mathbf{b}_1, \mathbf{b}_2), \tag{3}$$

where $H(\cdot|\cdot)$ and $H(\cdot, \cdot)$ denote the conditional and the joint entropies, respectively. With the assumption that Channel 1 suffers from block Rayleigh fading, the probability density function (PDF) of instantaneous SNR γ_1 is given by

$$p(\gamma_1) = \frac{1}{\overline{\gamma}_1} \exp(-\frac{\gamma_1}{\overline{\gamma}_1}), \tag{4}$$

where $\overline{\gamma}_1$ represents the average SNR of Channel 1. As noted above, Channel 2 is assumed to suffer from block Rician fading channel, of which variation is independent to Channel 1. The PDF of instantaneous SNR γ_2 is given by [7]

$$p\left(\gamma_2\right) = \left(\frac{\left(1+K\right)e^{-K}}{\overline{\gamma}_2}\right) \exp\left(-\frac{(1+K)\gamma_2}{\overline{\gamma}_2}\right) \cdot I_0\left(2\sqrt{\frac{K\left(1+K\right)\gamma_2}{\overline{\gamma}_2}}\right), \tag{5}$$

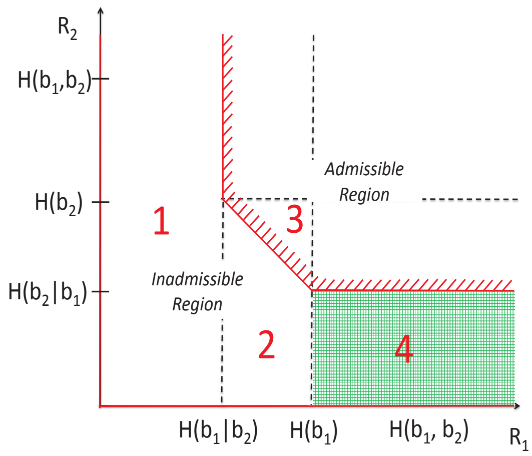

Fig. 2. Admissible rate region of Slepian-Wolf theorem.

where $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind and $\overline{\gamma}_2$ is the average SNR of Channel 2. The Rician factor K in (5) is the ratio of the power in the LOS component to the power in the non-LOS multipath components [8]. K represents the severity of fading. With $K = \infty$, the channel is equivalent to a static additive white Gaussian noise (AWGN) channel, and with $K = 0$ the channel reduces to Rayleigh fading channel.

III. Outage Derivation

The set of rate pair (R_1,R_2) is composed of the admissible and inadmissible regions, as shown in Fig. 2. If rate pair (R_1,R_2) falls into the inadmissible region, the transmission outage event occurs and the decoder can not guarantee the reconstruction of original information with arbitrarily small error probability. Furthermore, the inadmissible region can be divided into three areas 1, 2, and 4 as shown in Fig. 2.

In this paper, we consider the case where only the information of S_1 needs to be recovered, same with the scenario of two Rayleigh fading channels in [3], [4]. Hence, the area 4 is also included as the admissible rate region. Therefore, the outage probability of the system model we consider can be expressed as

$$P_{\text{outage}} = P_1 + P_2, \tag{6}$$

where P_1 and P_2 denote the probabilities that the rates R_1 and R_2 fall into the inadmissible area 1 and area 2, respectively. Hence, outage analysis falls into the probability calculation that R_1 and R_2 fall into the unbounded rectangular area 1 and the trapezoidal area 2 in Fig. 2. The relationship between the threshold instantaneous SNR γ_i and its corresponding source coding rate R_i is given by

$$R_i = \frac{1}{R_{ci}} \log_2(1 + \gamma_i), \tag{7}$$

where R_{ci} denotes the system efficiency of the transmission chain representing the coding rate and multiplicity of the modulation for i-th ($i = 1, 2$) channel [5]. Since, we assumed that the Source-Channel separation [6] holds, R_{ci} can be independent of R_i. Thus, P_1 can then be mathematically expressed as

$$\begin{aligned} P_1 &= \Pr\left[0 < R_1 < H(\mathbf{b}_1|\mathbf{b}_2), R_2 > 0\right] \\ &= \int_{\gamma_1=0}^{2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1} p(\gamma_1)d\gamma_1 \int_{\gamma_2=0}^{\infty} p(\gamma_2)d\gamma_2, \\ &= 1-\exp\left(-\frac{2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}{\overline{\gamma}_1}\right), \end{aligned} \tag{8}$$

according to the assumption that Channel 1 and Channel 2 are statistically independent. P_2 can be calculated in the same way as P_1, as

$$\begin{aligned} P_2 =& \Pr\left[H(\mathbf{b}_1|\mathbf{b}_2) < R_1 < H(\mathbf{b}_1), R_1+R_2 < H(\mathbf{b}_1,\mathbf{b}_2)\right] \\ =& \int_{\gamma_1=2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}^{2^{R_{c1}H(\mathbf{b}_1)}-1} p(\gamma_1) \\ &\cdot \int_{\gamma_2=0}^{2^{\left[R_{c2}H(\mathbf{b}_1,\mathbf{b}_2)-\frac{R_{c2}}{R_{c1}}\log_2(1+\gamma_1)\right]}-1} p(\gamma_2)d\gamma_2 d\gamma_1. \end{aligned} \tag{9}$$

By introducing the cumulative density function (CDF) of Rician fading [7], P_2 can be reduced to

$$\begin{aligned} P_2 = \int_{\gamma_1=2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}^{2^{R_{c1}H(\mathbf{b}_1)}-1} \frac{1}{\overline{\gamma}_1}\exp\left(-\frac{\gamma_1}{\overline{\gamma}_1}\right)\left[1-Q_1\left(\sqrt{2K},\right.\right. \\ \left.\left.\sqrt{2(1+K)\,\frac{2^{\left[R_{c2}H(\mathbf{b}_1,\mathbf{b}_2)-\frac{R_{c2}}{R_{c1}}\log_2(1+\gamma_1)\right]}-1}{\overline{\gamma}_2}}\right)\right] d\gamma_1, \end{aligned} \tag{10}$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-Function. The recursive adaptive Simpson quadrature algorithm [9] can be used to approximate the integral of (10), with accurate calculation error control.

IV. Asymptotic Tendency Analysis

A. Fully Independent Case

When $\mathbf{b}_1$ and $\mathbf{b}_2$ are fully independent ($P_f = 0.5$), $H(\mathbf{b}_1|\mathbf{b}_2) = H(\mathbf{b}_1)$. Hence, the probability P_2 equals to 0 according to (10). Consequently, the outage probability is entirely dominated by P_1. Moreover, P_1 is just the same as the outage probability of single Rayleigh channel's case, as

$$P_{\text{out,Rayleigh}} = 1-\exp\left(-\frac{2^{R_{c1}}-1}{\overline{\gamma}_1}\right), \tag{11}$$

which corresponds to the $1st$ order diversity.

B. Large Average SNR Case

When $\overline{\gamma}_1 \to \infty$ and $\overline{\gamma}_2 \to \infty$, the probability $P_2 \to 0$ because the Marcum Q-Function in (10) approaches 1. Furthermore P_1 can also be rewritten by Maclaurin expansion, as

$$P_1 = \sum_{n=0}^{\infty} \frac{(-1)^{(n)}}{n+1!}\left(\frac{2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}{\overline{\gamma}_1}\right)^{n+1}. \tag{12}$$

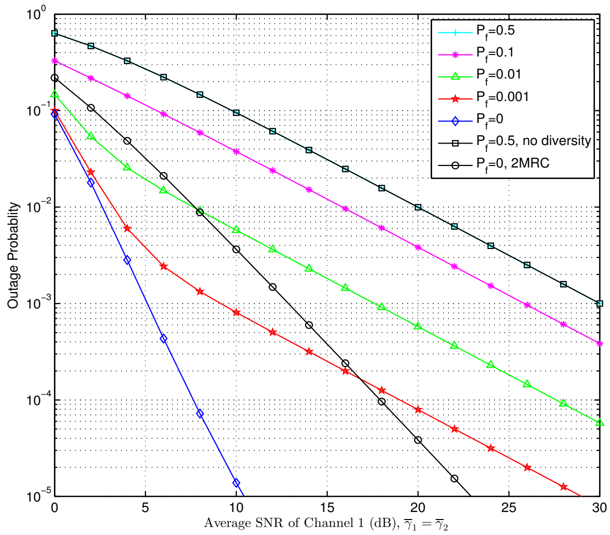

Fig. 3. Outage probability with different P_f for $K = 10$.

However, the first term of (12) dominates P_1 when $\overline{\gamma}_1$ is large enough. Thus, P_1 can be approximated, as

$$P_1 \approx \frac{2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}{\overline{\gamma}_1}, \tag{13}$$

which follows the tendency of the $1st$ order diversity. This observation indicates that outage performance is determined by non-LOS components when the average SNRs $\overline{\gamma}_1$ and $\overline{\gamma}_2$ become large.

C. Fully Correlated Case

When $\mathbf{b}_1$ and $\mathbf{b}_2$ are fully correlated ($P_f = 0$), the conditional entropy $H(\mathbf{b}_1|\mathbf{b}_2) = 0$. Therefore, the probability P_1 equals to 0 according to (8). The outage probability is completely dominated by P_2. It is found from the numerical integration, of which results are shown in Fig. 3, that the outage curves decay is sharper than the $2nd$ order diversity when the K factor becomes large. However, strict mathematical proof remains as an open problem.

V. Numerical Results

A. Same Average SNR of Rayleigh and Rician Channels

In Fig. 3, the theoretical outage probability versus average SNR are drawn with a parameter P_f varying from 0 to 0.5, assuming that average SNR of two channels are equivalent. It should be noticed that with the Rician factor $K = 10$, fading variation of Channel 2 is not as severe as that of Channel 1. The outage probability curves of the Slepian-Wolf transmission system, assuming that two channels are both Rayleigh fading, maximum-ratio-combining (MRC) curve with $P_f = 0$ and no diversity curve with $P_f = 0.5$, are also presented for comparison. We can see that no diversity gain can be achieved when $P_f = 0.5$, and the outage curve is exactly the same as that in Rayleigh fading without diversity. This is also consistent with the asymptotic tendency analysis result provided in subsection IV-A. It should be noticed that, the

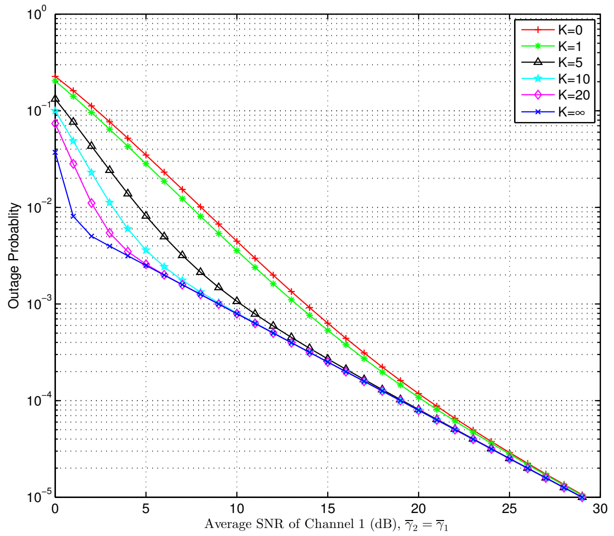

Fig. 4. Outage probability with different K for $\overline{\gamma}_1 = \overline{\gamma}_2$.

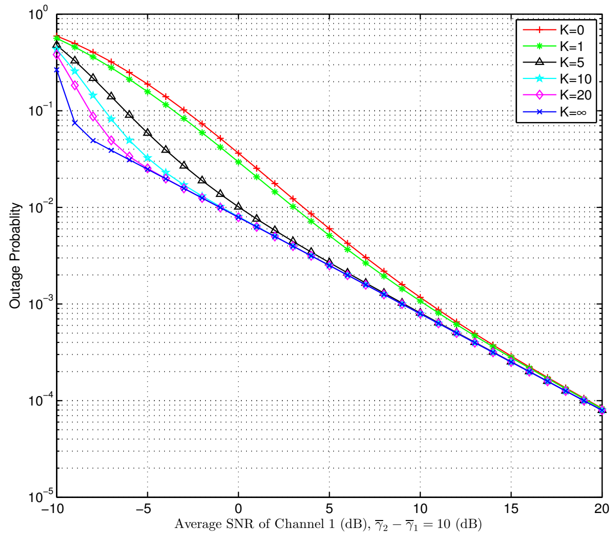

Fig. 5. Outage probability with different K for $\overline{\gamma}_1 \neq \overline{\gamma}_2$.

lower the P_f value, the smaller the outage probability. The outage probability can achieve sharper decay than with the $2nd$ order diversity if $\mathbf{b}_1$ and $\mathbf{b}_2$ are entirely correlated ($P_f = 0$). In this case, only P_2 dominates the outage probability.

Fig. 4 shows the outage probability curves with the factor K as a parameter, where the $bit\text{-}flipping$ probability is fixed at $P_f = 0.001$, indicating that $\mathbf{b}_1$ and $\mathbf{b}_2$ are highly correlated, but $\mathbf{b}_2$ still contain some errors. It is found that when the average SNR $\overline{\gamma}_1$ of Channel 1 is small, the outage probability reduce quickly, however, the decay of the outage curves always asymptotically converge in to the $1st$ order diversity when the $\overline{\gamma}_1$ increases. This observation is also exactly consistent with the conclusion provided in the asymptotic tendency analysis of subsection IV-B. Although the asymptotic behaviour is the same as that shown in [3] that assumes the both channels suffer from Rayleigh fading, the convergence to the asymptotic performance is different, depending on the LOS component of Channel 2. So far as $P_f \neq 0$, the outage probability curves always show the tendency described above.

B. Impact of Different Average SNR

In this subsection, the outage probabilities are shown with the factor K as a parameter. The difference from the previous subsection is that the average SNRs of the two channels are assumed to be not identical. In Fig. 5, the outage probability curves are depicted, where $\overline{\gamma}_2 - \overline{\gamma}_1 = 10$ dB, and $P_f = 0.001$. P_1 is independent of the K factor, according to (8), whereas P_2 is affected by K. Hence, the decay of outage curves asymptotic converge in to the $1st$ order diversity, if $P_f \neq 0$. When the average SNR is lower than the value at which the outage starts to converge to such asymptotic tendency, P_2 is dominated by the K factor. Hence, the turning point depends on difference between $\overline{\gamma}_1$ and $\overline{\gamma}_2$.

C. Optimal Power Allocations

In this subsection, the total transmit power of the system is assumed to be fixed, and the noises variance ${\sigma_n}^2$ of the both channel are normalized to the unity, as

$$\begin{cases} E_1 + E_2 = E_T \\ E_1/E_T = \alpha \\ E_2/E_T = 1-\alpha \\ {\sigma_n}^2 = 1, \end{cases} \tag{14}$$

where the E_1 and E_2 are the average transmit powers over the two channels, respectively, and E_T represents the transmit power totalling over the two channels. α indicates the ratio of the transmit power allocated to S_1 which is transmitted via Channel 1, and the value is kept constant within a block duration, because of the block fading assumption. The goal of this subsection is to find the optimal α that minimizes the outage probability with the transmit SNR $E_T/{\sigma_n}^2$ and the $bit\text{-}flipping$ probability P_f as parameters. Based on the assumption described above, the outage probabilities can be expressed by

$$P_1 = 1 - \exp\left(-\frac{2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)} - 1}{\alpha E_T}\right) \tag{15}$$

and

$$\begin{aligned} P_2 = &\int_{\gamma_1 = 2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)}-1}^{2^{R_{c1}H(\mathbf{b}_1)}-1} \frac{1}{\alpha E_T} \\ &\cdot \exp\left(-\frac{\gamma_1}{\alpha E_T}\right)\left[1 - Q_1\left(\sqrt{2K},\right.\right. \\ &\left.\left.\sqrt{2(1+K)\frac{2^{\left[R_{c2}H(\mathbf{b}_1,\mathbf{b}_2) - \frac{R_{c2}}{R_{c1}}\log_2(1+\gamma_1)\right]} - 1}{(1-\alpha)E_T}}\right)\right] d\gamma_1. \end{aligned} \tag{16}$$

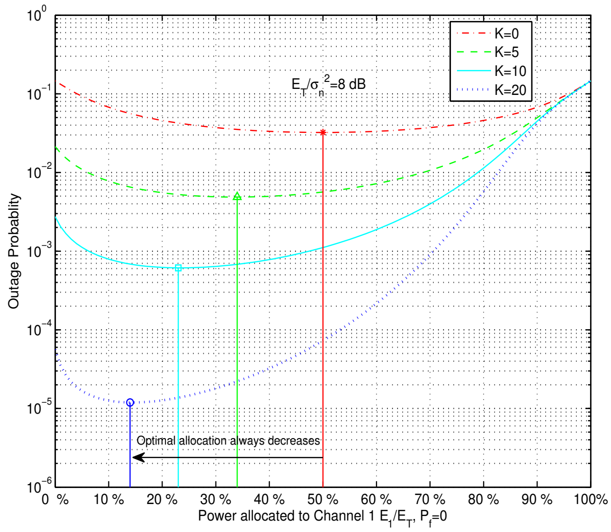

Fig. 6. Optimal power allocation with different K for $P_f = 0$.

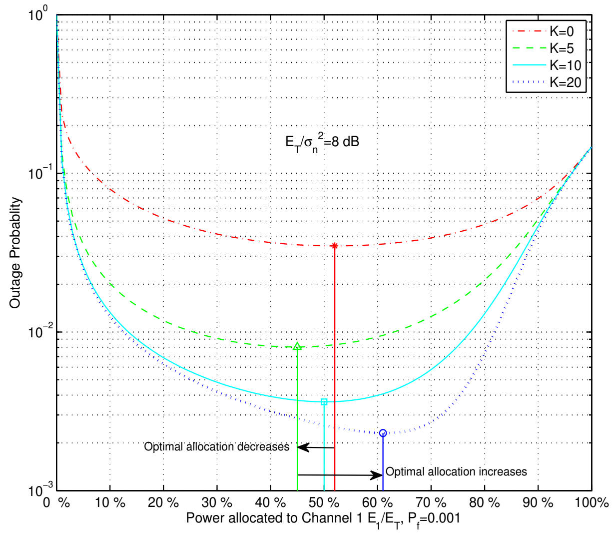

Fig. 7. Optimal power allocation with different K for $P_f = 0.001$.

Fig. 6 illustrates the outage probability curves with different Rician factor K values for $P_f = 0$ and $E_T/\sigma_n{}^2 = 8$ dB. The x-axis represents the ratio α of transmit power allocated to S_1. The 100% point indicates that transmit power is totally allocated to S_1, and 50% point represents the transmit power is allocated to both sources equally. It is found from Fig. 6 that if $\mathbf{b}_1$ and $\mathbf{b}_2$ are fully correlated, the lowest outage probability can be achieved when transmit power is equally allocated to the both channels, when $K = 0$. It can also be found that, the optimal power ratio α that achieves the minimum outage probability becomes small as the K factor becomes larger. This is reasonable because when the fading variation with Channel 2 is reduced, we should allocate more power to the S_2 which is transmitted via stable Rician Channel 2 than S_1 via Channel 1 suffering from Rayleigh fading.

Fig. 7 shows the power allocation ratio α versus the outage probability with the K factor as a parameter for $P_f = 0.001$ and $E_T/\sigma_n{}^2 = 8$ dB. It should be noted that the optimal α value, that achieves the smallest outage probability, is first decreased, and than increased over 50%, as the K value is increased. This indicates that although the larger the ratio of LOS component in Channel 2, the less the outage probability. However, the optimal allocation ratio differs, depending on the K factor value. This is because since $\mathbf{b}_1$ and $\mathbf{b}_2$ are not fully correlated, $\mathbf{b}_2$ is transmitted successfully over Channel 2 having strong LOS component even though $\mathbf{b}_2 \neq \mathbf{b}_1$. This indicates that allocating more power to S_2 which is transmitted via Channel 2 having strong LOS component can not help improve the outage performance. On the contrary, it is more effective to allocate more power to S_1 than to S_2 in such scenario.

VI. Conclusion

In this paper, we have derived the outage probability of a Slepian-Wolf transmission system with two correlated binary sources over block fading channels, one suffering from block Rayleigh fading, and the other from block Rician fading. The source correlation is assumed to be represented by a *bit-flipping* model. It has been found that when two sources are fully correlated ($P_f = 0$), the outage curve can achieve sharper decay than the $2nd$ order diversity, as the LOS component power of the Rician channel increases. In the case when $0 < P_f < 0.5$, the outage probability curves asymptotically plateau to the $1st$ order diversity, and the turning point depends on difference between $\overline{\gamma}_1$ and $\overline{\gamma}_2$.

Furthermore, the optimal power allocation is analysed under the condition that the total transmit power is fixed. The analytical results show that lower outage probability can not always be achieved through increasing ratio of LOS component power of Rician channel, so far as $P_f \neq 0$.

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