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Abstract

This paper presents basic knowledge of modal logics as a preliminary stage of studying epistemic logic. Modal logic is a logic based on classical logic. The most important characteristic of this logic is that it could deal with necessity ("it is necessary") and probability ("it is possible"), often represented by the symbol $\Box$ and $\Diamond$, respectively. A modern modal logic is created by Lewis(1932)[1]. In the early 1960s, Kripke introduce a semantics called "Kripke sematics" , which is widely used on modal logic today[2][3][4]. It is well-known that, when we denote "a person A knows φ" and "a person A believe φ" by "$K_A\varphi$" and "$B_A\varphi$" , respectively, there is a lot of common properties between K or B and $\Box$. The logic which deal with knowledge and belief is called epistemic logic, and there are lots of researches which aim to represent agent communication with this logic itself or some variations of this logic, where agent communication is the communication between multi-agent environment: an exchange of messages on Facebook is one of the example of agent communication, which is studied by K. Sano and S. Tojo(2013)[6]. Epistemic logic is based on modal logic, as mentioned above, thus it is important to understand basic knowledge of modal logic before studying agent communication.

In the first part of this thesis, we define the syntax of modal logic K: we denote the propositional variables by the lower-case letters, possibly with subscripts of superscripts; the lower-case Greek letters are reserved as formulas, and capital Greek letters are used for denoting sets of formulas. Syntactically, the only difference is that one unary symbol $\Box$ is added to Cl; a formula of the form $\Diamond\varphi$ is defined as the abbreviation of $\neg(\Box(\neg\varphi))$.

Second, we see Kripke semantics and its basic properties. One of the strong point of this semantics is that the model in this semantics could be drawn in graph, which could be intuitively understood without deep mathematical knowledge. A model of this semantics is a triple $\langle W, R, \mathfrak{V}\rangle$, where W is a non-empty set, R is an arbitrary binary relation on W, and $\mathfrak{V}$ is an arbitrary map from $\mathrm{Var}\mathcal{ML}$ to $\mathcal{P}(W)$. A pair $\langle W, R\rangle$ is called a frame and so a model of Kripke semantics could be also regarded as a pair of a frame and a valuation. After that, we see the truth-relation of this semantics, which is natural extension of that of classical logic, and some properties of modal symbols $/Box$ and $\Diamond$. We also see a few well-known operations for models which never change the truth-value of each formula. The operations we check are called generation and reduction: generation is defined by taking a generated submodel; and reduction is defined by taking a map which is called p-morphism. We could consider many classes of frames with their property, and the property of those classes are correspond to specific modal formula. We see some major classes and those corresponding formulas: the class of reflexive, transitive, symmetric and serial frames are correspond to the formula $\Box p \to p$, $\Box p \to \Box\Box p$, $p \to \Box\Diamond p$ and $\Box p \to \Diamond p$, respectively. There is a great tool, which is called Hintikka system, to construct a counter model. Hintikka system is a pair $\mathfrak{h} = \langle T, S\rangle$ where T is a set of a pair $t = (\Gamma, \Delta)$ of formulas, which is called tableau, with some restrictions and S a binary relation on T. With this system, we can also infer the decidability of this semantics.

Third, we introduce a Hilbert-style deduction system, which is called calculus K. The only difference between classical Hilbert-style system is that there is an additional axiom $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$ and inference rule RN: given a formula φ, we infer $\Box\varphi$. As calculs K is based on classical Hilbert-style system, a formula which is valid in classical logic is also valid in this system. There is a famous theorem in classical logic, which is called deduction theorem, i.e., $\Gamma \cup \varphi \vdash \psi \Rightarrow \Gamma \vdash \varphi \to \psi$. With some restrictions to RN, we can formulate deduction theorem for modal logic as it was formulated for classical and intuitionistic logic, however, the formulation of this theorem for modal logic is a bit different. Calculus K is sound and complete with respect to Kripke semantics of course. On proving the completeness, we use Hintikka system and a specific construction of tableaux. There could be many extensions of logic K, and soundness and completeness theorem alos holds for some of them. On provng these theorem, canonical models and filtration are useful; we briefly see these techniques.

Forth and the last, we briefly see another semantics which is called algebraic semantics. There are lots of semantics other than Kripke semantics, and algebraic semantics is one of them. As algebraic semantics is based on algebra and algebra is very abstract, this semantics could be possibly apply to many fields.

Almost all of the contents in this thesis are mainly based on A. Chagrov and M. Zakharyaschev(1997)[7].