Applying OWA operator to model group behaviors in uncertain QFD

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Abstract. It is a crucial step to derive the priority order of design requirements (DRs) from customer requirements (CRs) in quality function deployment (QFD). However, it is not straightforward to prioritize DRs due to two types of uncertainties: human subjective perception and user variability. This paper proposes an OWA based group decision-making approach to uncertain QFD with an application to a flexible manufacturing system design. The proposed model performs computations solely based on the order-based semantics of linguistic labels so as to eliminate the burden of quantifying qualitative concepts in QFD. Moreover, it incorporates the importance weights of users and the concept of fuzzy majority into aggregations of fuzzy preference relations of different DRs in order to model the group behaviors in QFD. Finally, based on a quantifier-guided net flow score procedure, the proposed model derives a priority ranking with a classification of DRs into important and unimportant ones so as to provide a better decision-support to the decision-maker.

1 Introduction

As an effective customer-driven quality management system, quality function deployment (QFD) incorporates the "voice of the customer" into appropriate company requirements at various product development stages, ranging from product planning and process design to manufacturing and delivery, to create higher customer satisfaction for the product. Among the four inter-linked stages of QFD [1], the first stage of QFD, usually called house of quality (HOQ), is of fundamental and strategic importance, since it is in this stage that the CRs for the product are identified and converted into appropriate DRs to fulfil customer satisfaction. In other words, HOQ links the "voice of the customer" to the "voice of the technician", through which the process and production plans can be developed in the other stages of the QFD system. The structures and analyzing methods of the other three QFD stages are essentially the same as the first one [8].

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Successful implementation of QFD often requires a significant number of subjective judgments from both customers and QFD design team [1,12]. Traditional QFD assumes that most of the input variables are precise and treated as crisp numerical data such as 1-3-9 or 1-5-9. However, the inherent subjective vagueness or impreciseness in QFD presents a special challenge to effective prioritization of DRs [3]. Therefore, numerical studies have been conducted on how to prioritize DRs with fuzzy linguistic variables [16] semantically represented by fuzzy sets [15], e.g. [3,9]. Another type of uncertainty in QFD is the involvement of many customers and design team members in the evaluation of input information of QFD. Input information may have an uncertainty associated with user (customer or design team member) heterogeneity because each user may have a different opinion. In this context, several studies have considered fuzzy group decision-making approaches, e.g. [1,6,12].

In summary, existing studies perform calculations with the associated fuzzy membership functions of linguistic labels based on fuzzy extension principle [15]. However, quantification in terms of fuzzy sets is in fact the process of transforming an ordinal information into a cardinal scale that represents an "arbitrary passage", which may sometimes be dangerous [4], since it is easy to generate different results by choosing different scales from which to draw the ordinals. Moreover, the fuzzy-set-based semantics of linguistic labels is often defined subjectively and context-dependently, which may sensitively influence the final prioritization results. Even if the quantification process used is rational, existing approaches to prioritize DRs in QFD simultaneously has, as any fuzzy-computation-based approach, an unavoidable limitation of information loss caused by the process of linguistic approximation, which consequently implies a lack of precision in the final result [5]. Regarding the second type of uncertainty, it is necessary to consider the group behaviors of users (both customers and designer team members). On one hand, the information provided from several users can be combined to improve data reliability and accuracy and to include some features that are impossible to perceive with individual users [10]. The users can be treated unequally considering their possible importance differences reflecting the reliability of each information source. For example, product users can provide more valuable judgmental information than non-product users and potential product users [12]. On the other hand, as a basic element underlying group decision-making, the concept of *fuzzy majority* is accepted by most of its members in practice, since it is quite difficult for the solution to be accepted by all users.

Due to the above observations, the main focus of this paper is to propose an OWA based group decision-making approach to prioritize a number of DRs in uncertain QFD with an application to a flexible manufacturing system design. The proposed model, on one hand, performs computations solely based on the order-based semantics of linguistic labels so as to eliminate the burden of quantifying qualitative concepts. On the other hand, it performs group aggregations of fuzzy preference relations based on the weighted ordered weighted average method so as to incorporate the importance weights of users and the concept of fuzzy majority. Moreover, based on a quantifier-guided net flow score procedure, the proposed model derives a priority ranking with a classification of DRs into important and unimportant ones so as to provide a better decision-support to the decision-maker.

The rest of this paper is organized as follows. After presenting the basic notations in Section 2, Section 3 presents a novel group decision making-approach to prioritize DRs based on the order-based semantics of linguistic labels and aggregation techniques of fuzzy preference relations. Section 4 examines a comparative numerical example to show the effectiveness of the proposed model. The paper is concluded in Section 5.

2 Basic notations

Prioritizing DRs includes both "CR management" and "product development" systems, which begins by sampling the desires and preferences of customers of a product through marketing surveys or interviews, and organizes them as a set of CRs [2]. Formally, let $\mathcal{CR} = \{CR_1, CR_2, \ldots, CR_M\}$ be a set of CRs. The importance of CRs has then to be determined to continue the QFD process. In order to obtain the importance of the CRs, a set of customers to be surveyed in a target market is collected such that $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ with a weighting vector $(\gamma_1, \gamma_2, \ldots, \gamma_K), \sum_k \gamma_k = 1$. Each customer has to provide his/her judgment of importance toward each CR with linguistic variable $\mathcal{L}^1 = \left\{L_1^1, L_2^1, \ldots, L_{G_1}^1\right\}$. Formally, the importance judgement for customer requirement CR_m provided by customer C_k is denoted by $CRI_{mk} \in \mathcal{L}^1$.

Moreover, a set of DRs is collected from the design team for a QFD problem such that $\mathcal{DR} = \{DR_1, DR_2, \ldots, DR_N\}$. The QFD is based on a process involving teams of multidisciplinary representatives from all stages of product development and production. Translations from CRs to engineering DRs are carried out by a QFD design team. A design team is collected as $\mathcal{E} = \{E_1, E_2, \ldots, E_J\}$, which is also assigned a weighting vector $(\beta_1, \beta_2, \ldots, \beta_J), \sum_{j=1}^{J} \beta_j = 1$. The design team members are then asked to provide their judgments of the relationships between CRs and DRs using linguistic variable $\mathcal{L}^2 = \left\{L_1^2, L_2^2, \ldots, L_{G_2}^2\right\}$. Formally, the linguistic judgement for the relationship between customer need CR_m and design requirement DR_n given by design team member E_j is denoted by $R_{mnj} \in \mathcal{L}^2$

3 Proposed method

3.1 Fuzzy preference relations from linguistic information

We have a set of M customer needs $\mathcal{CR} = \{CR_1, CR_2, \ldots, CR_M\}$ and a set of K customers $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$. The importance judgment for customer need CR_m provided by customer C_k is denoted by $CRI_{mk} \in \mathcal{L}^1$. Then, an customer C_k's individual fuzzy preference relation in $\mathcal{CR} \times \mathcal{CR}$ assigns a value in the unit interval $[0, 1]$ for the preference of one customer need over another such that

$$\mu_{\mathbf{D}(C_k)} : (CR_m, CR_l) \in \mathcal{CR} \times \mathcal{CR} \longrightarrow D_{ml}(C_k) \in [0, 1], m, l = 1, \ldots, M. \quad (1)$$

The value $D_{ml}(C_k)$ reflects the degree of fuzzy preference relation of CR_m over CR_l under a customer C_k's subjective judgment, calculated by

$$D_{ml}(C_k) = \begin{cases} 1, & \text{if } CRI_{mk} > CRI_{lk} \\ 0.5, & \text{if } CRI_{mk} = CRI_{lk} \\ 0, & \text{if } CRI_{mk} < CRI_{lk} \end{cases}. \tag{2}$$

The matrix $\mathbf{D}(C_k) = [D_{ml}(C_k)]_{M\times M}$ has the following properties.

- When $D_{ml}(C_k) = 1$, it indicates that CR_m is absolutely preferred to CR_l, i.e. indicates the maximum degree of preference of CR_m over CR_l.
- When $0.5 < D_{ml}(C_k) < 1$, it indicates that CR_m is slightly preferred to CR_l.
- When $D_{ml}(C_k) = 0.5$, there is no preference (i.e. indifference) between CR_m and CR_l.
- When $0 < D_{ml}(C_k) < 0.5$, it indicates that CR_l is slightly preferred to CR_m.
- When $D_{ml}(C_k) = 0$, it indicates that CR_l is absolutely preferred to CR_m.

Such a value function is in fact based on the order-based semantics of linguistic labels, and consequently we will obtain K matrices of fuzzy preference relations of different customer needs $\mathcal{CR}$ under the K customers' judgments.

Then, for each customer C_k, a weighting vector of the M customer requirements can be induced from the matrix of fuzzy preference relations $\mathbf{D}(C_k)$ as $\mathbf{W^{CR}}(C_k) = \left(W_1^{CR}(C_k), W_2^{CR}(C_k), \ldots, W_M^{CR}(C_k)\right)$, where $W_m^{CR}(C_k)$ is calculated as follows

$$W_m^{CR}(C_k) = \frac{1}{M}\sum\nolimits_{l=1}^{M} \frac{D_{ml}(C_k)}{\sum_{n=1}^{M} D_{nl}(C_k)}, k = 1, \ldots, K, m = 1, \ldots, M. \tag{3}$$

Moreover, the linguistic judgment for the relationship between customer need CR_m and design requirement DR_n given by design team member E_j is denoted by $R_{mnj} \in \mathcal{L}^2$. Similarly, we can derive a matrix $\mathbf{D}(CR_m, E_j) = [D_{nl}(CR_m, E_j)]_{N\times N}$ of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ for each customer need under each designer team member, where $D_{nl}(CR_m, E_j)$ denotes the fuzzy preference relation of DR_n over DR_l under a design term member's E_j subjective judgment with respect to customer need CR_m, calculated by the following equation

$$D_{nl}(CR_m, E_j) = \begin{cases} 1, & \text{if } R_{mnj} > R_{mlj} \\ 0.5, & \text{if } R_{mnj} = R_{mlj} \\ 0, & \text{if } R_{mnj} < R_{mlj} \end{cases}. \tag{4}$$

Using the weighting vector $\mathbf{W^{CR}}(C_k)$ of the M customer requirements under each customer $C_k, k = 1, \ldots, K$, we are able to derive a matrix $\mathbf{D}(C_k, E_j) = [D_{nl}(C_k, E_j)]_{N\times N}$ of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ for a combination of each customer and each design team member, where

$$D_{nl}(C_k, E_j) = \sum\nolimits_{m=1}^{M} W_m^{CR}(C_k) \cdot D_{nl}(CR_m, E_j). \tag{5}$$

Obviously, there will be $K \times J$ combinations of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$. The matrix $\mathbf{D}(C_k, E_j)$ represents the matrix of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ under the combination of customer C_k and design team member E_j, where $k = 1, 2, \ldots, K$ and $j = 1, 2, \ldots, J$. Such a formulation is motivated by [12]. By this way, our approach takes into account the group behaviors of both customers and QFD design team members. In the next section, we will consider how to synthesize individual matrices of fuzzy preference relations into an overall one.

3.2 Group aggregations of individual fuzzy preference relations

The $K \times J$ combinations of different customers and different design team members can be viewed as combined information sources in our QFD context. Each combination produces an individual matrix of fuzzy preference relations for the design requirements. Consequently, the uncertain QFD transforms to a group decision-making problem, which needs to synthesize the individual matrices $\mathbf{D}(C_k, E_j), k = 1, \ldots, K, j = 1, \ldots, J$ into an overall representative matrix $\mathbf{D} = [D_{nl}]_{N \times N}$ in $\mathcal{DR} \times \mathcal{DR}$.

Note that the customers and the design team are assigned importance weighting vectors as $(\gamma_1, \gamma_2, \ldots, \gamma_K)$ and $(\beta_1, \beta_2, \ldots, \beta_J)$, respectively. Therefore, each preferential combination of a customer and a QFD design team member can be associated with an importance weight P. For the sake of convenience, let us re-denote the individual matrix $\mathbf{D}(C_k, E_j)$ of fuzzy preference relations of each preferential combination in $\mathcal{DR} \times \mathcal{DR}$ as $\mathbf{D}^i$ with its associated importance weight as

$$P_i = \gamma_k \cdot \beta_j, \tag{6}$$

where $k = 1, 2, \ldots, K, j = 1, 2, \ldots, J, i = 1, 2, \ldots, K \times J$. Since $\sum_k^K \gamma_k = 1$ and $\sum_j^J \beta_j = 1$, it is obvious that $\sum_i^{K \times J} P_i = 1$. In order to synthesize the individual matrix $\mathbf{D}^i$ of fuzzy preference relations into an overall representative matrix in $\mathcal{DR} \times \mathcal{DR}$, one commonly used way is to apply the weighted average (WA) method with the following value function

$$\begin{aligned} D_{nl} &= \mathcal{F}_{\mathrm{WA}}\left(D_{nl}^1, D_{nl}^2, \ldots, D_{nl}^{K \times J}\right) \\ &= \sum\nolimits_{i=1}^{K \times J} P_i \cdot D_{nl}^i \end{aligned} \tag{7}$$

where $n, l = 1, 2, \ldots, N$. The importance weight P_i associated with each combined information source in fact reflects its reliability, i.e., each combined information source has an attached weight that measures its reliability.

We also want to measure the importance of a value D_{nl}^i (in relation to other values) with independence of each information source that has captured it. A basic element underlying group decision-making is the concept of a *majority*, that is a solution is accepted by most of its members since in practice it is quite difficult for the solution to be accepted by all. A natural line of reasoning is to somehow make that strict concept of majority closer to its real human perception by making it more vague, called *fuzzy majority*. A natural manifestation of such a "soft" majority is the so-called linguistic quantifiers as, e.g., most, almost all, much more than half, etc. [17] suggested a formal representation of these linguistic quantifiers using fuzzy sets [15], i.e., any relative quantifier can be expressed as a fuzzy subset Q of the unit interval $[0,1]$. In this representation for any proportion $x \in [0,1]$, $Q(x)$ indicates the degree to which x satisfies the concept conveyed by the term Q. [13] further defined a Regular Increasing Monotone (RIM) quantifier to represent the linguistic quantifier, defined as follows.

Definition 1. *A fuzzy subset Q of the real line is called a Regular Increasing Monotone (RIM) quantifier if $Q(0)=0, Q(1)=1$, and $Q(x) \geq Q(y)$ for $x \geq y$.*

Examples of this kind of quantifier are all, most, many, at least α. A quantifier's membership function is often determined by intuition. For example, the following membership function for RIM quantifier has been widely used [14].

$$Q(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ 1, & \text{if } x > b \end{cases}$$

Using the RIM quantifiers, a linguistically quantified statement for our group aggregations can be written as "Q information sources are convinced", which may be exemplified by "most (Q) of information sources are convinced". Fortunately, Yager [14] proposed a special class of aggregation operators, called ordered weighted averaging (OWA for short) operators, which seem to provide an even better and general aggregation in the sense of being able to simply and uniformly model a large class of fuzzy linguistic quantifiers.

Definition 2. *Let $(D_1, D_2, \ldots, D_N)$ be a set of values, an OWA operator of dimension N is a mapping $\mathcal{F}_{\mathrm{OWA}} : \mathbb{R}^N \to \mathbb{R}$ if associated with $\mathcal{F}$ is a weighting vector $\mathbf{W} = (W_1, \ldots, W_N)$ such that: $W_n \in [0,1]$, $\sum_{n=1}^{N} W_n = 1$, and*

$$\mathcal{F}_{\mathrm{OWA}}(D_1, D_2, \ldots, D_N) = \sum\nolimits_{n=1}^{N} W_n \cdot D_{\sigma(n)}, \tag{8}$$

where $\{D_{\sigma(1)}, D_{\sigma(2)}, \ldots, D_{\sigma(N)}\}$ is a permutation of $D_1, D_2, \ldots, D_N$ such that $D_{\sigma(n-1)} \geq D_{\sigma(n)}$ for all $n = 2, \ldots, N$. (i.e., $D_{\sigma(n)}$ is the nth largest element in the collection $(D_1, D_2, \ldots, D_N)$. The weighting vector $\mathbf{W}$ is derived from the RIM quantifier Q.

Yager [13] proposed a method for obtaining the OWA weighting vector via linguistic quantifiers, especially the RIM quantifiers, which can provide information aggregation procedures guided by verbally expressed concepts and a dimension independent description of the desired aggregation. By using the OWA operator and a RIM quantifier Q, the overall matrix of fuzzy preference relations of different DRs is derived by

$$\begin{aligned} D_{nl} &= \mathcal{F}_{\mathrm{OWA}} \left(D_{nl}^1, D_{nl}^2, \ldots, D_{nl}^{K \times J} \right) \\ &= \sum\nolimits_{i=1}^{K \times J} W_i \cdot D_{nl}^{\sigma(i)} \\ &= \sum\nolimits_{i=1}^{K \times J} \left[Q\left(\frac{i}{K \times J} \right) - Q\left(\frac{i-1}{K \times J} \right) \right] \cdot D_{nl}^{\sigma(i)} \end{aligned} \tag{9}$$

where $\left\{D_{nl}^{\sigma(1)}, D_{nl}^{\sigma(2)}, \ldots, D_{nl}^{\sigma(K\times J)}\right\}$ is a permutation of $D_{nl}^{1}, D_{nl}^{2}, \ldots, D_{nl}^{K\times J}$ such that $D_{nl}^{\sigma(i-1)} \geq D_{nl}^{\sigma(i)}$ for all $i = 2, \ldots, K \times J$.

Essentially, we want to synthesize the individual matrices of fuzzy preference relations of different design requirements into an overall representative matrix by taking into account the importance weight of each information source in Eq. (7) and the weight of each value based on the concept of *fuzzy majority* in Eq. (9). In this case, a linguistically quantified statement may be generally written as "Q important information sources are convinced", which may be exemplified by "most (Q) of the important information sources are convinced". Such linguistic quantifiers can be, fortunately enough, dealt with by the weighted ordered weighted averaging (WOWA for short) operator [10], defined as follows.

Definition 3. *Let $(D_1, D_2, \ldots, D_N)$ be a set of values, $\mathbf{P}$ (importance weights) and $\mathbf{W}$ (value weights) be weighting vectors of dimension N ($\mathbf{P} = (P_1, P_2, \ldots, P_N)$, $\mathbf{W} = (W_1, W_2, \ldots, W_N)$) such that: (1) $P_n \in [0,1]$ and $\sum_n^N P_n = 1$; (2) $W_n \in [0,1]$ and $\sum_n^N W_n = 1$. In this case, a mapping $\mathcal{F}_{\text{WOWA}} : \mathbb{R}^N \to \mathbb{R}$ is a WOWA operator of dimension N if*

$$\mathcal{F}_{\text{WOWA}}(D_1, D_2, \ldots, D_N; P_1, P_2, \ldots, P_N) = \sum\nolimits_{n=1}^{N} \omega_n \cdot D_{\sigma(n)} \qquad (10)$$

where $\{D_{\sigma(1)}, D_{\sigma(2)}, \ldots, D_{\sigma(N)}\}$ is a permutation of $D_1, D_2, \ldots, D_N$ such that $D_{\sigma(n-1)} \geq D_{\sigma(n)}$ for all $n = 2, \ldots, N$, i.e., $D_{\sigma(n)}$ is the nth largest element in the collection $(D_1, D_2, \ldots, D_N)$, and the weight ω_n is defined as

$$\omega_n = W^*\left(\sum\nolimits_{l\leq n} P_{\sigma(l)}\right) - W^*\left(\sum\nolimits_{l<n} P_{\sigma(l)}\right) \qquad (11)$$

with W^* a monotone increasing function that interpolates the points $\left(i/n, \sum_{l\leq n} P_{\sigma(l)}\right)$ together with the point $(0,0)$. The value $P_{\sigma(l)}$ means the permutation according to $\{D_{\sigma(1)}, D_{\sigma(2)}, \ldots, D_{\sigma(N)}\}$.

When W^* is replaced with a RIM quantifier introduced in Definition 1, then $\omega_n = Q\left(\sum_{l\leq n} P_{\sigma(l)}\right) - Q\left(\sum_{l<n} P_{\sigma(l)}\right), n = 1, \ldots, N$, which indicates that the WOWA operator becomes the importance weighted quantifier guided aggregation method [13], i.e., the the importance weighted quantifier guided aggregation method is a special case of the WOWA operator. Using the WOWA operator and RIM quantifiers, the overall matrix of fuzzy preference relations of different DRs is derived by

$$\begin{aligned} D_{nl} &= \mathcal{F}_{\text{WOWA}}\left(D_{nl}^{1}, D_{nl}^{2}, \ldots, D_{nl}^{K\times J}; P_1, P_2, \ldots, P_{K\times J}\right) \\ &= \sum\nolimits_{i=1}^{K\times J} \omega_i \cdot D_{nl}^{\sigma(i)} \\ &= \sum\nolimits_{i=1}^{K\times J} \left[Q\left(\sum\nolimits_{l\leq i} P_{\sigma(l)}\right) - Q\left(\sum\nolimits_{l<i} P_{\sigma(l)}\right)\right] \cdot D_{nl}^{\sigma(i)} \end{aligned} \qquad (12)$$

where Q is a RIM quantifier introduced in Definition 1.

3.3 Choice function

We will use two quantifier-guided choice degrees of design requirements: a dominance degree and a non-dominance degree. In particular,

- let Φ_n^+ be the dominant degree which is a measure that design requirement DR_n is dominating the other design requirements, referred to as *leaving flow* in the terminology of decision-making;
- let Φ_n^- be the non-dominant degree which is a measure that design requirement DR_n is dominated by the remaining design requirements, referred to as *entering flow* in the terminology of decision-making.

Here, Φ_m^+ and Φ_m^- can be defined by the following formulas, respectively:

$$\begin{aligned} \Phi_n^+ &= \mathcal{F}_{\text{OWA}}\left(D_{nl}, l = 1, \ldots, N, n \neq l\right) \\ \Phi_n^- &= \mathcal{F}_{\text{OWA}}\left(D_{ln}, l = 1, \ldots, N, n \neq l\right) \end{aligned} \tag{13}$$

In addition, let Φ_n be the relative dominant degree which measures the difference between dominant degree and non-dominant degree of design requirement DR_n over the remaining design requirements such that $\Phi_n = \Phi_n^+ - \Phi_n^-, n = 1, 2, \ldots, N$. Φ_n is referred to as *net flow* in the terminology of decision-making. The greater Φ_n is, the higher priority design requirement DR_n will be. When Φ_n is a positive value, DR_n is important; when Φ_n is a negative value, DR_n is unimportant. Therefore, according to net flows $\{\Phi_1, \Phi_2, \ldots, \Phi_N\}$, we can not only determine a priority order of all the design requirements, but also divide the design requirements into important and unimportant classes, which is important for the decision-makers.

4 A comparative application case study

In this section, we shall apply our model to prioritize the basic design requirements of a flexible manufacturing system.

4.1 Problem descriptions

The example discussing the basic design requirements of a flexible manufacturing system (FMS) [3,12] is applied here to illustrate the idea proposed. In that example, eight major CRs are identified to represent the biggest concerns of the customers for the design of an FMS, as shown in Table 1 (indexed by CRs). Based on the design team's experience and expert knowledge, 10 DRs are identified responding to the eight major CRs, as shown in Table 1 (indexed by DRs).

In the QFD planning process, the first step is to determine the weighting vector of customer needs $\mathcal{CR}$. The linguistic variable for representing the importance of $\mathcal{CR}$ is provided in Eq. (14), indexed by $\mathcal{L}^1$. The 10 surveyed customers, $\mathcal{C} = \{C_1, C_2, \ldots, C_{10}\}$ were then asked to assess the 8 customer needs by making use of linguistic variable $\mathcal{L}^1$. Similar to the CRs, the relationship measure

Table 1. Customer requirements and design engineering requirements

Customer requirements (CRs)		Design requirements (DRs)	
CR_1	`High production volume`	DR_1	`Automatic gauging`
CR_2	`Short setup time`	DR_2	`Tool change system`
CR_3	`Load-carrying capacity`	DR_3	`Tool monitoring system`
CR_4	`User-friendliness`	DR_4	`Coordinate measuring machine`
CR_5	`Associated functions`	DR_5	`Automated guided vehicle`
CR_6	`Modularity`	DR_6	`Conveyor`
CR_7	`Wide tool variety`	DR_7	`Programmable logic controller (PLC)`
CR_8	`Wide product variety`	DR_8	`Storage and retrieval system (S&R system)`
		DR_9	`Modular fixture`
		DR_{10}	`Robots`

between CRs and DRs was assessed by a QFD design team with three team members, $\mathcal{E} = \{E_1, E_2, E_3\}$. The linguistic variable used for assessing the relationships between CRs and DRs by the design team is presented in Eq. (14), indexed by $\mathcal{L}^2$.

$$\begin{aligned}
\mathcal{L}^{\mathrm{I}} &= \{L_1^{\mathrm{I}}, L_2^{\mathrm{I}}, L_3^{\mathrm{I}}, L_4^{\mathrm{I}}, L_5^{\mathrm{I}}, L_6^{\mathrm{I}}, L_7^{\mathrm{I}}\} \\
&= \{\texttt{Very unimportant}, \texttt{Quite unimportant}, \texttt{Unimportant}, \texttt{Slightly important}, \\
&\quad \texttt{Moderately important}, \texttt{Important}, \texttt{Very important}\} \\
\mathcal{L}^{\mathrm{R}} &= \{L_1^{\mathrm{R}}, L_2^{\mathrm{R}}, L_3^{\mathrm{R}}, L_4^{\mathrm{R}}\} \\
&= \{\texttt{Very weak}, \texttt{Weak}, \texttt{Moderate}, \texttt{Strong}\} \qquad (14)
\end{aligned}$$

The fuzzy linguistic importance of the 8 CRs assessed by the 10 surveyed customers and the fuzzy linguistic relationship matrix between the 8 customer needs and the 10 design requirements assessed by each of the 3 design team members, are shown in [12], respectively.

4.2 Prioritizing engineering design requirements

First, we calculate a matrix $\mathbf{D}(C_k)$ of fuzzy preference relations in $\mathcal{CR}\times\mathcal{CR}$ under each customer C_k's linguistic judgment. A weighting vector $\mathbf{W}(C_k)$ can then be derived under each customer C_k's linguistic judgment according to Eq. (3). The weighting vectors of the 8 customer needs under different customers' judgments are shown in Table 2.

Second, we can obtain a matrix $\mathbf{D}(CR_m, E_j)$ of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ under each design team member's $E_j, j = 1, 2, 3$ judgment with respect to each customer need $CR_m, m = 1, \ldots, 8$. Thirdly, by incorporating the importance weights of the 8 customer needs under different customers, we can derive a matrix $\mathbf{D}(C_k, E_j)$ of the fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ under each combination of a customer C_k and a design team member E_j, where

Table 2. Importance weights of the 8 CRs under each customer

Customers	Importance weights							
	CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7	CR_8
C_1	0.203125	0.0625	0.203125	0.0625	0.203125	0.109375	0.140625	0.015625
C_2	0.09375	0.09375	0.15625	0.21875	0.015625	0.046875	0.15625	0.21875
C_3	0.046875	0.21875	0.109375	0.15625	0.046875	0.046875	0.15625	0.21875
C_4	0.203125	0.109375	0.203125	0.015625	0.078125	0.140625	0.203125	0.046875
C_5	0.125	0.0625	0.0625	0.1875	0.125	0.015625	0.234375	0.1875
C_6	0.21875	0.140625	0.015625	0.171875	0.109375	0.0625	0.21875	0.0625
C_7	0.234375	0.078125	0.1875	0.03125	0.109375	0.140625	0.03125	0.1875
C_8	0.125	0.203125	0.078125	0.203125	0.203125	0.125	0.03125	0.03125
C_9	0.1875	0.015625	0.109375	0.046875	0.1875	0.078125	0.1875	0.1875
C_{10}	0.078125	0.140625	0.21875	0.078125	0.21875	0.015625	0.171875	0.078125

$k = 1, \dots, K, j = 1, \dots, J$. It is obvious that the combinations generate $8 \times 3 = 24$ matrices of fuzzy preference relations of DRs. Taking customer C_1 and design team member E_3 as an example, the derived matrix of fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ is derived as

$$\mathbf{D}(C_1, E_3) = \begin{array}{c|cccccccccc} & DR_1 & DR_2 & DR_3 & DR_4 & DR_5 & DR_6 & DR_7 & DR_8 & DR_9 & DR_{10} \\ \hline DR_1 & 0.5 & 0.41 & 0.43 & 0.35 & 0.42 & 0.31 & 0.29 & 0.36 & 0.52 & 0.23 \\ DR_2 & 0.59 & 0.5 & 0.45 & 0.59 & 0.66 & 0.52 & 0.63 & 0.48 & 0.55 & 0.39 \\ DR_3 & 0.57 & 0.55 & 0.5 & 0.55 & 0.55 & 0.55 & 0.45 & 0.43 & 0.66 & 0.32 \\ DR_4 & 0.65 & 0.41 & 0.45 & 0.5 & 0.48 & 0.32 & 0.44 & 0.48 & 0.48 & 0.22 \\ DR_5 & 0.58 & 0.34 & 0.45 & 0.52 & 0.5 & 0.52 & 0.49 & 0.45 & 0.6 & 0.45 \\ DR_6 & 0.69 & 0.48 & 0.45 & 0.68 & 0.48 & 0.5 & 0.64 & 0.45 & 0.59 & 0.42 \\ DR_7 & 0.71 & 0.38 & 0.55 & 0.56 & 0.51 & 0.36 & 0.5 & 0.58 & 0.41 & 0.31 \\ DR_8 & 0.64 & 0.52 & 0.57 & 0.52 & 0.55 & 0.55 & 0.42 & 0.5 & 0.66 & 0.42 \\ DR_9 & 0.48 & 0.45 & 0.34 & 0.52 & 0.4 & 0.41 & 0.59 & 0.34 & 0.5 & 0.34 \\ DR_{10} & 0.77 & 0.61 & 0.68 & 0.78 & 0.55 & 0.58 & 0.69 & 0.58 & 0.66 & 0.5 \end{array} .$$

Fourthly, we have to aggregate the individual matrices of the fuzzy preference relations in $\mathcal{DR} \times \mathcal{DR}$ generated by each combination of each customer and each design team member into an overall one. As mentioned previously, each customer and each design team member may have a different importance weight in light of his/her design experience and domain knowledge, respectively. Similar to Wang [12], the importance weights of customers and design team members are assumed as $\gamma_1 = \gamma_2 = \frac{3}{14}, \gamma_3 = \cdots = \gamma_{10} = \frac{1}{14}$ and $\beta_1 = \frac{1}{2}, \beta_2 = \frac{1}{3}, \beta_3 = \frac{1}{6}$. In order to incorporate the concept of fuzzy majority to model the group behaviors, the WOWA method is used to perform the group aggregations of individual matrices of fuzzy preference relations into an overall one. The linguistic quantifiers [17] will be used in our aggregation. According to Section 3.3, the choice information with its ranking order is calculated, as shown in Table 3.

Table 3. Choice values of different design requirements under linguistic quantifiers

Q	Index	Design requirements									
		DR_1	DR_2	DR_3	DR_4	DR_5	DR_6	DR_7	DR_8	DR_9	DR_{10}
there exists	Φ	0.0	0.09	0.28	0.13	-0.16	-0.07	0.04	0.11	-0.04	-0.08
	$DR_3 \succ DR_4 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_9 \succ DR_6 \succ DR_{10} \succ DR_5$										
for all	Φ	0.0	0.09	0.28	0.13	-0.16	-0.07	0.04	0.11	-0.04	-0.08
	$DR_3 \succ DR_4 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_9 \succ DR_6 \succ DR_{10} \succ DR_5$										
identity	Φ	0.042	0.166	0.384	0.034	-0.365	-0.178	0.058	0.207	-0.185	-0.162
	$DR_3 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_4 \succ DR_{10} \succ DR_6 \succ DR_9 \succ DR_5$										
at least half	Φ	0.109	0.155	0.423	0.073	-0.36	-0.172	0.111	0.168	-0.129	-0.184
	$DR_3 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_4 \succ DR_9 \succ DR_6 \succ DR_{10} \succ DR_5$										
as many as possible	Φ	0.109	0.155	0.423	0.073	-0.359	-0.172	0.111	0.167	-0.129	-0.183
	$DR_3 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_4 \succ DR_9 \succ DR_6 \succ DR_{10} \succ DR_5$										
most	Φ	0.087	0.187	0.415	0.035	-0.375	-0.18	0.095	0.199	-0.187	-0.223
	$DR_3 \succ DR_8 \succ DR_2 \succ DR_7 \succ DR_1 \succ DR_4 \succ DR_6 \succ DR_9 \succ DR_{10} \succ DR_5$										

It is clear that different priority rankings are obtained with different linguistic quantifiers. There are some interesting observations.

- The priority ranking generated by "there exists" is consistent with the one generated by "for all".
- The design requirement DR_3 is always the most important one with a positive priority value and the design requirement DR_5 is always the most unimportant one with a negative priority value.
- The six DRs $\{DR_1, DR_2, DR_3, DR_4, DR_7, DR_8\}$ are always important ones with positive priority values regardless of the priority rankings of them; whereas, the four DRs $\{DR_5, DR_6, DR_9, DR_{10}\}$ are always unimportant ones with negative priority values, regardless of the rankings of them.

4.3 Comparative studies with two known approaches

Our uncertain QFD context has been widely investigated in the literature by quantifying qualitative concepts with fuzzy sets. For example, the linguistic variables in Eq. (14) can be semantically represented by the following function [6,12].

$$\begin{aligned}
\mathcal{L}^1 &= \left\{L_1^1, L_2^1, L_3^1, L_4^1, L_5^1, L_6^1, L_7^1\right\} \\
&= \{(0,0,2),(0,2,4),(2,3.5,5),(3,5,7),(5,6.5,8),(6,8,10),(8,10,10)\} \\
\mathcal{L}^2 &= \left\{L_1^2, L_2^2, L_3^2, L_4^2\right\} \\
&= \{(0.1,0.2,0.3),(0.3,0.4,0.5),(0.5,0.6,0.7),(0.7,0.8,0.9)\}
\end{aligned} \tag{15}$$

where $(\cdot,\cdot,\cdot)$ is used to represent a triangular fuzzy number. Two well-known fuzzy approaches to uncertain QFD are proposed by Chen *et al.* [3] and Wang [12], both of which are based on the nonlinear programming based fuzzy weighted average (NLP-FWA for short) method, introduced as follows.

In their seminal work of fuzzy linguistic QFD, Chen *et al.* [3] proposed a method by means of the NLP-FWA method and fuzzy expected value operator [7] to prioritize DRs. In particular, their approach can be summarized as follows.

1. First, fuzzy importance weights of CRs and fuzzy relationships between CRs and DRs provided by multiple customers and QFD team members are averaged.
2. Second, the NLP-FWA method is used to determine the fuzzy weights of DRs.
3. Finally, the fuzzy weights of DRs are defuzzified using the fuzzy expected value operator for prioritizing DRs.

Wang [12] proposed a different fuzzy group decision-making procedure for prioritizing DRs under uncertainty. The proposed approach does not aggregate the individual judgments of customers and QFD design team members, but rather aggregates the technical importance ratings of DRs, which can be summarized as follows.

1. The NLP-FWA method is used to determine the fuzzy weights of DRs with respect to each customer and design team member.
2. The NLP-FWA method is used to determine the overall fuzzy weights of DRS involving different combinations of any customer and any design team member.
3. The fuzzy weights of DRs are defuzzified using the centroid method [11] for prioritizing DRs.

With the importance weights of customers and design team members, the prioritization results by Chen *et al.*'s, and Wang's approaches are shown as

– Chen *et al.*: $DR_3 \succ DR_8 \succ DR_2 \succ \underbrace{DR_1 \succ DR_7} \succ DR_4 \succ DR_{10} \succ DR_6 \succ DR_9 \succ DR_5$.
– Wang: $DR_3 \succ DR_8 \succ DR_2 \succ \underbrace{DR_7 \succ DR_1} \succ DR_4 \succ DR_{10} \succ DR_6 \succ DR_9 \succ DR_5$.

It is obvious that the priority ranking generated by Chen *et al.* is slightly different from the one generated by Wang in terms of the ranking between DR_1 and DR_7, the main reason comes from the group behaviors of QFD. Both our approach and Wang have incorporated the group behavior of QFD; whereas, Chen *et al.* have ignored such a phenomenon. With different linguistic quantifers, different priority rankings may be obtained, which is missed in Wang's work.

5 Conclusion

It is a crucial step to derive the priority order of DRs from CRs in QFD. However, it is not straightforward to prioritize DRs due to two types of uncertainties: human subjective perception and user variability. To address the two uncertainties simultaneously in prioritizing DRs, a novel group decision-making method was proposed in this paper. First, the order-based semantics of linguistic information was used to derive the individual matrix of fuzzy preference relations with respect to each customer and each design team member. Second, the weighted OWA method was used to synthesize the individual matrix of fuzzy preference relations into an overall one. Thirdly, a quantifier-guided choice approach was developed to prioritize the DRs with a classification. A sample FMS design was used to illustrate the proposed approach.

In summary, our model can eliminate the burden of quantifying the qualitative concepts and capture the group behaviors of uncertain QFD. Moreover, since the quantifier-guided net flow score procedure is used to prioritize DRs with a classification of DRs into positive and negative ones, our model will provide a better decision-support the decision-maker.

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