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Simplification of Statements for Automated Geometry Theorem Proving

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The great growth of automated geometry theorem proving has been brought by algebraic methods developed by Wen-Tsün Wu in 1977. These methods can be applied to many non-trivial geometry theorems with highly success rate. Generally, their proofs consist of operations for complicated polynomial expressions that are different from the traditional proof methods used by geometers. Therefore, it is extremely difficult to understand proofs intuitively because users who want to know proofs have to look at tedious computations of polynomials.

Readability is one of the most important factors in proof of automated geometry theorem proving. The traditional proof methods used by geometers usually give elegant, short and readable proofs. Researchers have studied automated generation of traditional proof since late fifties. In spite of the enormous amount of efforts and great improvements, the success has been limited in a number of non-trivial theorems.

In late eighties, by combining ideas from both the algebraic and the logic approach, Area Method has been presented which produces short and readable proofs as the traditional proof methods do. After that, many automated proving methods have been proposed such as Vector Method, Full-Angle Method and so on. Furthermore, automated geometry theorem proving in solid geometry has been studying in recent years.

In this paper, we consider relation between the statement description and its proof in a geometry theorem prover named GEX (Geometry Expert). GEX implements four proving methods as a basic engine based on Area Method, Vector Method, Full-Angle Method, and Fixpoint Method. Using these methods, we can get short and elegant proofs similar to geometers' proofs. GEX also implements two algebraic methods well-developed during the eighties: Wu's Method and Gröbner Basis Method. Users can learn how to prove a given statement and know the most effective method to prove the statement.

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In order to put a geometry statement to GEX, Users should transform it into a construction sequence that arranges points on a plane. An intuitive transformation of geometry statements usually leads to a construction sequences that include unnecessary geometric properties of points and lines, which causes a complicated proof.

Here we compare proofs of the construction sequences, which are produced by the same geometry statement, using Area Method.

[Example: The Orthocenter Theorem]

The three altitudes of a triangle are concurrent.

This geometry statement is described as construction sequences for GEX as follows.

Sequence1:

```
HYPOTHESES:
  POINT A B C;
  FOOT D A B C;
  FOOT E B C A;
  FOOT F C A B;
  INTERSECTION_LL O A D C F;
  INTERSECTION_LL P A D B E;
SHOW:
  (SQ_DIS O P)=0.
```

Sequence2:

```
HYPOTHESES:
  POINT A B C;
  FOOT D A B C;
  FOOT E B C A;
  INTERSECTION_LL O A D B E;
SHOW:
  PERPENDICULAR A B C O.
```

Sequence1 is more intuitive description of the above theorem than Sequence2. The proof of Sequence1 is longer than that of Sequence2 in the length of the largest algebraic expression. Such the length, denoted *maxt*, is one of important indices for readability of the proof. For this reason, it is very important to eliminate the unnecessary expressions in construction sequences in order to obtain more readable and shorter proofs.

This paper proposes an algorithm for simplification of a given construction sequence. The algorithm analyzes intuitive construction sequences, and simplifies them to get readable and shorter proofs. We introduce a structure called condition sequence that consists of three relations of lines: parallel, perpendicular and congruence. The simplification algorithm analyzes the condition sequence and eliminates unnecessary expressions in the construction sequence.

Our simplification method is carried out by these procedures as follows.

[Simplification Algorithm]

1. Analyze a given construction sequence and make a condition sequence.
2. Analyze the condition sequence and add new conditions, referring to the contexts.
3. Update the condition sequence with a transition rule applicable to a condition in the condition sequence.
4. Check the conclusion. If the conclusion is congruence of points then eliminate the latest construction and regard the last condition as the new conclusion.
5. Output the new construction sequence after searching and eliminating useless points.

We implement a system called GSS (Geometry Statement Simplifier) in Scheme based on the simplification algorithm. It tries to simplify some intuitive construction sequences that are made by users. Consequently, the construction sequences, which are simplified by GSS, are proved by GEX with lower $maxt$s than intuitive construction sequences.

There also exists another simplification method that not only the elimination of unnecessary points but also the arrangement of new points in a given construction sequence. Compared with such the method, the simplified construction sequence by GSS helps users to understand geometric properties easier.

Hence, the result shows that our algorithm is effective in improvement of readability of proofs in GEX.