# Exact and Approximated Outage Probability Analyses for Decode-and-Forward Relaying System Allowing Intra-link Errors

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***Abstract*—In this paper, we theoretically analyze the outage probability of decode-and-forward (DF) relaying system allowing intra-link errors (DF-IE), where the relay always forwards the decoder output to the destination regardless of whether errors are detected after decoding in the information part or not. The results apply to practical fading scenarios where all the links between the nodes suffer from independent block Rayleigh fading. The key idea of DF-IE system is that the data sequence forwarded by the relay is highly correlated with the original information sequence sent from the source, and hence with a proper joint decoding technique at the destination, the correlation knowledge can well be exploited to improve the system performance. We analyze this problem in the information theoretical framework of correlated source coding. Using the theorems for lossy source-channel separation and for source coding with side information, the exact outage probability is derived. It is then shown that the exact expression can be reduced to a simple, yet accurate approximation by replacing *the theorem for source coding with side information* by the Slepian-Wolf theorem. Compared with conventional DF relaying where relay keeps silent if errors are detected after decoding, DF-IE can achieve even lower outage probability. Moreover, by allowing intra-link errors, the optimal position of the relay is found to be exactly the midpoint between the source and destination. Results of the simulations are provided to verify the accuracy of the analytical results.**

***Index Terms*—Decode-and-forward, relay channel, intra-link errors, source coding with side information theorem, Slepian-Wolf theorem, Shannon's lossy source-channel separation theorem, outage probability**

## I. Introduction

COOPERATIVE communication has been recognized as a promising technology for future ubiquitous communication systems, where there is an increasing demand for efficient and reliable information transmission from multiple sources to multiple destinations over wireless channels suffering from deep fading. By exploiting the broadcasting nature of the wireless signals, multiple nodes cooperate with each other in the network to form a virtual multiple-antenna system by sharing the single antenna of each node [1], thereby spatial diversity gain with multiple-input multiple-output (MIMO) techniques can be achieved. Furthermore, cooperative communications can be adopted by future cellular networks or wireless sensor/mesh networks, without requiring significant changes in infrastructure. On the contrary, techniques requiring *fixed* multiple antennas in each single node may not be suitable, if the bandwidth, size and energy limitations impose practical difficulties in achieving the MIMO gains [2].

Decode-and-Forward (DF) relaying is the most widely studied protocol for cooperative communications. For the DF protocol, after receiving signal transmitted from the source, the relay first decodes the received signal, re-encodes and then forwards it to the destination. With the help of the relay, diversity gain can be achieved. So far several derivative techniques of the DF protocol have been proposed from the perspective of diversity-multiplexing tradeoff (DMT) [3], [4]. In this contribution, however, our focus is mainly on the DF protocol which satisfies the half-duplex and orthogonal system setup, referred to as conventional DF relaying in this paper. Due to the half-duplex setup, the relay does not transmit and receive simultaneously on the same frequency. Various practical implementations of conventional DF relaying have been proposed using different code families, such as convolutional codes [5], Turbo codes [6], [7] and low density parity check (LDPC) codes [8], [9]. Besides the practical coding schemes, results of the theoretical analysis on the outage probability of conventional DF relaying in Rayleigh fading channels are provided in [10]–[12].

In the conventional DF systems, the recovered data sequence is discarded at the relay if errors are detected after decoding. It has been believed that if the relay re-encodes the data sequence containing errors and forwards it to the destination, error propagation will occur, resulting in even worse performance [13]. However, even if errors are detected at the relay, the data sequences transmitted from the source and relay are still highly correlated, and therefore Slepian-Wolf's correlated source coding theorem [14] can be well utilized in this scenario. Ref. [15] formulates this issue from the viewpoint of the Slepian-Wolf theorem, where the authors assume that the relay does not aim to perfectly correct the errors occurring in the source-relay link (referred to as intra-link). Instead, it interleaves the information sequence, re-encodes and then forwards the encoder output to the destination, even though there may remain some errors after decoding. It is shown in

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[15] that with iterative processing utilizing the Log-likelihood Ratio (LLR) updating function [16], the error probability can be utilized at the destination. The key idea of the coding technique provided in [15] is that the relay system can be seen as a distributed Turbo code, and hence it can achieve Turbo-cliff-like bit-error-rate (BER) performance in Additive White Gaussian Noise (AWGN) channels. In this paper, the scheme presented in [15] is referred to as *DF relaying system allowing intra-link errors* (DF-IE) for notational convenience.

Recently, outage probability of DF-IE system is investigated in [17], where the source-destination and relay-destination links are assumed to suffer from block Rayleigh fading while the intra-link is modeled by a binary symmetric channel (BSC) with a *fixed* crossover probability as a parameter. The admissible rate region of the source-destination link and relay-destination link are determined by the Slepian-Wolf theorem. The outage probability is expressed as a set of double integrals over the admissible rate region, with respect to the probability density function (*pdf*) of the instantaneous signal-to-noise ratios (SNRs) of the source-destination and relay-destination links. However, there are two fundamental drawbacks inherent in this approach. First of all, the assumption that the intra-link is modeled by a BSC with fixed crossover probability (intra-link error probability) is not realistic in practical applications because the intra-link also suffers from block Rayleigh fading [18]. In this case, the error probability is no longer a fixed value but a random variable that changes according to the variation of the intra-link. Secondly, in the DF-IE system considered, correlated data sequences are transmitted from the source and the relay to the destination, however the destination aims only to recover the information sent from the source. This system setup does not perfectly match the Slepian-Wolf theorem which intends to recover both the two correlated sources at the destination.

The primary goal of this paper is to overcome these two drawbacks and to theoretically investigate outage probability of DF-IE system, where account is taken of the fading variations of all the links. As mentioned before, in DF-IE system, the intra-link error probability is considered as a random variable rather than a fixed value, hence deriving the *pdf* of the intra-link error probability is of crucial importance and a challenging topic. In our method, instead of deriving the *pdf* of the intra link error probability, we establish the relationship between the intra-link error probability and the instantaneous SNR of the intra-link according to Shannon's lossy source-channel separation theorem [19]. Furthermore, we found the data transmission over DF-IE system falls exactly into the category of source coding with side information problem [20], where only one of the two correlated sources is to be recovered and the other is served as a helper. Hence, the admissible rate region for the source and relay can be accurately determined by *the theorem for source coding with side information* [21]. We then show that the exact outage probability of DF-IE system can be expressed by a set of triple integrals over the admissible rate region, with respect to the *pdf* of the instantaneous SNRs of each link. Moreover, inspired by [17], we found that the exact outage probability expression can be reduced to a simple, yet accurate enough approximation by replacing *the theorem for source coding with side information* by the Slepian-Wolf theorem. It is shown that this approximation is quite accurate so far as the relay location scenarios assumed in this paper are concerned. As stated above, the outage analyses provided in this paper purely follow the correlated source coding theorems and assume infinite frame length. Therefore the analytical results upper bound the performance of the practical signaling schemes.

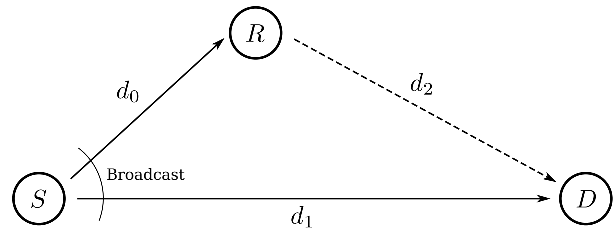


Fig. 1. The block diagram of the single relay system.

The rest of this paper is organized as follow. In Section II, we introduce the abstract model of DF-IE system assumed in this paper, and the corresponding channel model as well. The relationship between intra-link error probability and the instantaneous SNR of the source-relay link is established in Section III. In Section IV, we derive the exact outage probability of DF-IE system, based on *the theorem for source coding with side information*. In Section V, the approximated outage probability of DF-IE system is derived by using the Slepian-Wolf theorem. Numerical results are provided in Section VI to verify the theoretical analysis. Finally, conclusions are drawn in Section VII with some concluding remarks.

## II. System Model

### A. DF-IE system

We consider a simple one-way orthogonal half-duplex relay system, where a source $S$ and a relay $R$ cooperate to transmit a message to a destination $D$, as shown in Fig. 1. To guarantee orthogonal transmission, a time-division channel allocation is assumed; the transmission consists of two time slots. During the first time slot, $S$ encodes the original message and broadcasts it to both $R$ and $D$. $R$ always decodes to recover the information sequence, interleaves the information sequence, re-encodes it and forwards the encoder output to $D$ during the second time slot, even though the decoding result may contain errors in the original information sequence (such errors are referred to as intra-link errors). Note that the decoder output at $R$ is highly correlated with the original message sent from $S$. This correlation is referred to as *source-relay correlation* in this paper.

After receiving signals from $S$ and $R$, $D$ performs joint decoding by exploiting the source-relay correlation to retrieve the original message sent from $S$. The relay system assumed in this paper, as a whole, can be seen as a distributed Turbo code. Hence, an iterative decoding process is required at $D$ between the decoders for the codes used by $S$ and $R$ [15].

### B. Channel Model

The links between $S$ and $R$, $S$ and $D$, and $R$ and $D$ are assumed to suffer from independent block Rayleigh fading, where the channel gains keep constant within one transmission block but vary transmission-by-transmission. The received signals at $S$ and $D$ can be expressed as

$$y_0[k] = \sqrt{G_0} \cdot h_0 \cdot s[k] + n_0[k], \tag{1}$$
$$y_1[k] = \sqrt{G_1} \cdot h_1 \cdot s[k] + n_1[k], \tag{2}$$
$$y_2[k] = \sqrt{G_2} \cdot h_2 \cdot s'[k] + n_2[k], \tag{3}$$

where $s[k]$ and $s'[k]$ denote the signals transmitted from $S$ and $R$, respectively, with $k$ being the timing index of the symbols. $h_i$ and $n_i[k]$ denote the complex channel gain and the zero-mean AWGN with the variance $\sigma_i^2$ per dimension, where $i \in \{0, 1, 2\}$ denotes the $SR$, $SD$ and $RD$ links, respectively. It is assumed that $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = N_0/2$ without loss of generality. The geometric-gain of each link is also considered in this paper, which is represented by $G_i,\ i \in \{0, 1, 2\}$. With $d_i$ denoting the distance of its corresponding link and $G_1$ being normalized to the unity, $G_0$ and $G_2$ can be defined as $G_0 = (d_1/d_0)^l$ and $G_2 = (d_1/d_2)^l$, respectively,[1] where $l$ is the pathloss exponent, which is empirically set at 3.52 as in [7]. Note further that we assume the transmit power per symbol at $S$ and $R$ is the same, which is denoted as $E_s$.

With the definitions described above, the instantaneous and average SNRs of $SD$ link are expressed as $\gamma_1 = G_1 |h_1|^2 E_s/N_0$ and $\Gamma_1 = G_1 E_s/N_0$, respectively. Similar definitions apply to $\gamma_0$, $\Gamma_0$, $\gamma_2$ and $\Gamma_2$. With Rayleigh fading assumption, the *pdf* of $\gamma_i$ is given by [22]

$$p(\gamma_i) = \frac{1}{\Gamma_i} \exp\left(-\frac{\gamma_i}{\Gamma_i}\right),\ i = 0, 1, 2. \tag{4}$$

## III. Intra-link Error Probability Analysis

With the block Rayleigh fading assumption, the intra-link error probability $p$ stays constant over one block, but varies transmission-by-transmission. In this section, we consider the point-to-point intra-link transmission and identify the relationship between $p$ and $\gamma_0$.

According to Shannon's lossy source-channel separation theorem [19], [23], $U_1$ can be transmitted over the intra-link, with a distortion level $\mathscr{D}$, if

$$R_{c,1} \cdot R(\mathscr{D}) \leq C(\gamma_0), \tag{5}$$

where $R_{c,1}$ and $R(\mathscr{D})$ are the spectrum efficiency[2] of the transmission chain of the intra-link and the source rate-distortion function [21], respectively. $C(\gamma_0)$ denotes the *instantaneous* channel capacity of the intra-link given $\gamma_0$.[3] With the Hamming distortion measure and for a given $\gamma_0$ value, the minimum distortion $\mathscr{D}_{min}$ is equivalent to the intra-link error

[1]The geometric-gain between two nodes with distance $d$ is defined as $G = (\frac{1}{d})^l$ according to [7].

[2]The spectrum efficiency includes both the channel coding rate and the modulation multiplicity.

[3]Assuming Gaussian codebook is used for modulation, $C(x) = \frac{N_D}{2}\log_2(1 + \frac{2x}{N_D})$, with its inverse function $C^{-1}(x) = \frac{N_D}{2}(2^{\frac{2x}{N_D}} - 1)$, where $N_D$ denotes the dimensionality of the channel input.

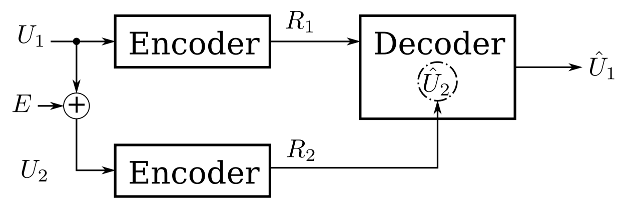


Fig. 2. Abstract model for the coding/decoding of $U_1$ and $U_2$ from the viewpoint of source coding with side information. $U_2$ is the bit-flipped version of $U_1$ and served as a helper for the decoding of $U_1$ at the decoder.

probability $p$ [24]. By substituting $p$ into (5) and taking the equality, we have $R(p) = \Phi_1(\gamma_0) = \frac{C(\gamma_0)}{R_{c,1}}$. Therefore, $p$ can be further expressed as[4]

$$p = \begin{cases} H_b^{-1}[1 - \Phi_1(\gamma_0)], & \text{for } \Phi_1^{-1}(0) \leq \gamma_0 \leq \Phi_1^{-1}(1), \\ 0, & \text{for } \gamma_0 \geq \Phi_1^{-1}(1), \end{cases} \tag{6}$$

where $H_b^{-1}(\cdot)$ denotes the inverse function of the binary entropy function $H_b(x) = -x\log_2 x - (1-x)\log_2(1-x)$, and $\Phi_1^{-1}(\cdot)$ is the inverse function of $\Phi_1(\cdot)$.

## IV. Exact Outage Probability Analysis

### A. Admissible Rate Region Based on Source Coding with Side Information

As mentioned before, the intra-link error probability $p$ stays fixed within one transmission. In this subsection, we consider only one transmission, and hence $p$ is regarded as a fixed parameter. Let $U_1$ denote the original binary information sequence transmitted from $S$, and $U_2$ the decoder output at $R$. As described above, $U_2$ may contain some errors. Hence, $U_2$ can be regarded as the bit-flipped version of $U_1$, as $U_2 = U_1 \oplus E$, where $\oplus$ indicates modulus-2 addition and $E$ is a binary random variable with probability $\Pr(E = 1) = 1 - \Pr(E = 0) = p$, as shown in Fig. 2. The correlation between $U_1$ and $U_2$ is characterized by $p$, where $p = 0$ indicates perfect decoding at $R$, and $0 < p \leq 0.5$ indicates errors occurring in the intra-link.

Assume $U_1$ and $U_2$ are described with rates $R_1$ and $R_2$, respectively, as shown in Fig. 2. As stated before, in DF-IE system, the objective of $D$ is only to retrieve $U_1$, which was sent from $S$. On the other hand, $U_2$ sent from $R$ does not need to be successfully recovered at $D$. The coding/decoding of $U_1$ and $U_2$ falls exactly into the category of source coding with side information problem [20], where two correlated sources are encoded separately and transmitted to the same decoder at the destination, but only one of the two sources is to be recovered at the destination and the other one serves as a helper (side information). Obviously, in DF-IE system, $U_2$ provides side information to help the decoding of $U_1$ at the destination. According to *the theorem for source coding with side information* [21], successful recovery of $U_1$ after joint decoding at $D$ can be achieved if $R_1$ and $R_2$ satisfies

$$\begin{cases} R_1 & \geq & H(U_1|\hat{U}_2), \\ R_2 & \geq & I(U_2; \hat{U}_2), \end{cases} \tag{7}$$

[4]For i.i.d. binary source, $R(\mathscr{D}) = 1 - H_b(\mathscr{D})$.

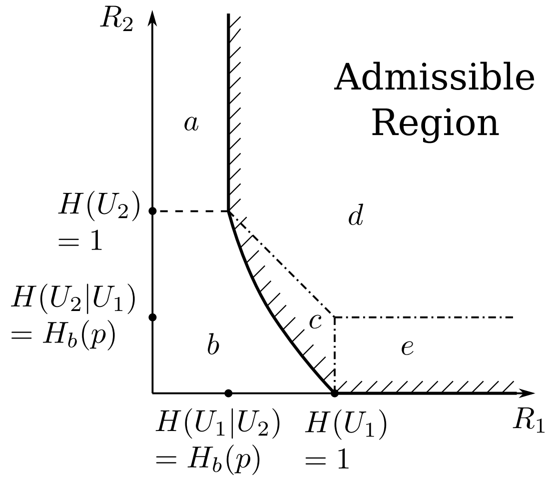


Fig. 3. The admissible rate region for $S$ and $R$ determined by *the theorem for source coding with side information*.

where $\hat{U}_2$ is the estimate of $U_2$ at the final output, as shown in Fig. 2. The relationship between $U_2$ and $\hat{U}_2$ can also be expressed as a bit-flipping model with a error probability $\alpha$, where $\alpha \in [0, 0.5]$. Let $H(U_1|\hat{U}_2)$ and $I(U_2;\hat{U}_2)$ denote the entropy of $U_1$ conditioned on $\hat{U}_2$ and the mutual information between $U_2$ and $\hat{U}_2$, respectively. In this paper, we only consider independent and identically distributed (i.i.d.) source, therefore it is easily found that $H(U_1|\hat{U}_2) = H_b(\alpha * p)$ and $I(U_2;\hat{U}_2) = H(\hat{U}_2) - H(\hat{U}_2|U_2) = 1 - H_b(\alpha)$, where $\alpha * p = (1-\alpha)p + \alpha(1-p)$.

Let $\mathscr{R}_{SI}$ denote the admissible rate region specified by (7). To facilitate the rate region comparison to be provided later in this paper, we divide the entire rate region into five sub-regions, $\mathscr{R}_a$, $\mathscr{R}_b$, $\mathscr{R}_c$, $\mathscr{R}_d$ and $\mathscr{R}_e$, as shown in Fig. 3. From this figure, we have $\mathscr{R}_{SI} = \mathscr{R}_c \cup \mathscr{R}_d \cup \mathscr{R}_e$. Consider two extreme cases: (1) In the case $U_2$ can be successfully decoded at the decoder, $\hat{U}_2 = U_2$ and $\alpha = 0$. Therefore, the conditions become as $R_1 \geq H_b(0*p) = H_b(p)$ and $R_2 \geq 1 - H_b(0) = 1$, which corresponds to the linear boundary between $\mathscr{R}_a$ and $\mathscr{R}_d$. (2) In the case the estimate $\hat{U}_2$ of $U_2$ after decoding is totally wrong, $\hat{U}_2$ does not contain any information about $U_2$ and $\alpha = 0.5$. Therefore, the conditions become as $R_1 \geq H_b(0.5*p) = 1$ and $R_2 = 1 - H_b(0.5) = 0$, which corresponds to the lower linear boundary of $\mathscr{R}_e$. In all other cases ($0 < \alpha < 0.5$), it is easily to know the conditions become as $R_1 \geq H_b(\alpha * p)$ and $R_2 \geq 1 - H_b(\alpha)$, which corresponds to the nonlinear boundary between $\mathscr{R}_b$ and $\mathscr{R}_c$. According to the discussions presented above, $\mathscr{R}_{SI}$ can be expressed even in an explicit way as

$$R_1 \geq \begin{cases} H_b(p), & \text{for} \quad R_2 \geq 1, \\ H_b(\alpha * p), & \text{for} \quad 0 \leq R_2 \leq 1. \end{cases} \tag{8}$$

The three boundaries between $\mathscr{R}_d$ and $\mathscr{R}_c$, $\mathscr{R}_d$ and $\mathscr{R}_e$, and $\mathscr{R}_c$ and $\mathscr{R}_e$ will be discussed in Section V-A.

### *B. Relationship Between $R_1$, $R_2$ and Their Corresponding Channel SNRs*

It should be emphasized here that, in the system considered in this paper, specific source coding for compression is performed neither at $S$ nor at $R$. Instead, the correlation knowledge between $U_1$ and $U_2$ is exploited at $D$ to enhance the error correction capability of the system. Now consider the transmission of the $SD$ and $RD$ links. According to Shannon's separation theorem, if the *total* information transmission rates over these two independent channels satisfy [16]

$$\begin{cases} R_1 R_{c,1} & \leq & C(\gamma_1), \\ R_2 R_{c,2} & \leq & C(\gamma_2), \end{cases} \tag{9}$$

the message error probability can be made arbitrarily small. Here, $R_{c,1}$ and $R_{c,2}$ indicate the spectrum efficiency of the transmission chain of the $SD$ and $RD$ links, respectively. $C(\gamma_1)$ and $C(\gamma_2)$ denote the channel capacity of the $SD$ and $RD$ links, respectively, given the instantaneous SNRs of the $SD$ and $RD$ links being $\gamma_1$ and $\gamma_2$.

In the theoretical analysis, we only consider the equality of (8) and (9). The relationship between rate $R_i$ and its corresponding instantaneous channel SNR $\gamma_i$ is given by

$$R_i = \Phi_i(\gamma_i) = \frac{C(\gamma_i)}{R_{c,i}}, \tag{10}$$

with its inverse function

$$\gamma_i = \Phi_i^{-1}(R_i) = C^{-1}(R_i R_{c,i}), \tag{11}$$

where $i = 1, 2$ and $C^{-1}(\cdot)$ denotes the inverse function of channel capacity.

### *C. Exact Outage Calculation Based on $\mathscr{R}_{SI}$*

Within one transmission and for a given $p$ value, the outage event happens when $(R_1, R_2)$ falls outside the admissible rate region $\mathscr{R}_{SI}$, i.e., the set $(R_1, R_2)$ is in $\mathscr{R}_a$ or $\mathscr{R}_b$. Note that the intra-link error probability $p$ changes, according to the variation of $\gamma_0$, as described in Section III. Taking into account the impact of the variation of the intra-link, the outage probability of the system is defined by taking average over all the transmissions, which results in

$$\begin{aligned} P_{out}^{SI} &= \Pr\{0 \leq p \leq 0.5,\ (R_1, R_2) \notin \mathscr{R}_{SI}\} \\ &= \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_a \cup \mathscr{R}_b\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_a \cup \mathscr{R}_b\} \\ &= \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_a\} \\ &\quad + \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_b\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_a\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_b\}. \end{aligned} \tag{12}$$

Note that the intra-link error probability $p$ and the rates $R_1$, $R_2$ can be converted into the instantaneous channel SNR of their corresponding links, as shown in Section IV-B and III. Moreover, since all the three links are suffering from statistically independent block Rayleigh fading, the joint *pdf* of the instantaneous SNRs can be expressed as $p(\gamma_0, \gamma_1, \gamma_2) = p(\gamma_0) \cdot p(\gamma_1) \cdot p(\gamma_2)$. Given the facts described above, the last four terms of (12), denoted as $P_{1,a}$, $P_{1,b}$, $P_{2,a}$ and $P_{2,b}$, respectively, can be further expressed as

$$
\begin{aligned}
P_{1,a} &= \Pr\{p=0, R_2 \geq 1, 0 \leq R_1 \leq H_b(p)\} \\
&= \Pr\{\gamma_0 \geq \Phi_1^{-1}(1), \gamma_2 \geq \Phi_2^{-1}(1), \\
&\qquad \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}(0)\} \\
&= \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_0 \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(1)} d\gamma_2 \\
&\quad \cdot \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(0)} p(\gamma_0)\cdot p(\gamma_1)\cdot p(\gamma_2) d\gamma_1 \\
&= 0,
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
P_{1,b} &= \Pr\{p=0, 0 \leq R_2 \leq 1, 0 \leq R_1 \leq H_b(\alpha * p)\} \\
&= \Pr\{\gamma_0 \geq \Phi_1^{-1}(1), \Phi_2^{-1}(0) \leq \gamma_2 \leq \Phi_2^{-1}(1), \\
&\qquad \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}[1-\Phi_2(\gamma_2)]\} \\
&= \int_{\Phi_1^{-1}(1)}^{\Phi_1^{-1}(\infty)} d\gamma_0 \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(1)} d\gamma_2 \\
&\quad \cdot \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}[1-\Phi_2(\gamma_2)]} p(\gamma_0)\cdot p(\gamma_1)\cdot p(\gamma_2) d\gamma_1 \\
&= \frac{1}{\Gamma_2} \exp\left[-\frac{\Phi_1^{-1}(1)}{\Gamma_0}\right] \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(1)} \exp(-\frac{\gamma_2}{\Gamma_2}) \\
&\quad \cdot \left[1-\exp(-\frac{\Phi_1^{-1}[1-\Phi_2(\gamma_2)]}{\Gamma_1})\right] d\gamma_2,
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
P_{2,a} &= \Pr\{0 < p \leq 0.5, R_2 \geq 1, 0 \leq R_1 \leq H_b(p)\} \\
&= \Pr\{\Phi_1^{-1}(0) \leq \gamma_0 \leq \Phi_1^{-1}(1), \gamma_2 \geq \Phi_2^{-1}(1), \\
&\qquad \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}[1-\Phi_1(\gamma_0)]\} \\
&= \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_0 \int_{\Phi_2^{-1}(1)}^{\Phi_2^{-1}(\infty)} d\gamma_2 \\
&\quad \cdot \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}[1-\Phi_1(\gamma_0)]} p(\gamma_0)\cdot p(\gamma_1)\cdot p(\gamma_2) d\gamma_1 \\
&= \frac{1}{\Gamma_0} \exp\left[-\frac{\Phi_2^{-1}(1)}{\Gamma_2}\right] \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} \exp(-\frac{\gamma_0}{\Gamma_0}) \\
&\quad \cdot \left[1-\exp(-\frac{\Phi_1^{-1}[1-\Phi_1(\gamma_0)]}{\Gamma_1})\right] d\gamma_0,
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
P_{2,b} &= \Pr\{0 < p \leq 0.5, 0 \leq R_2 \leq 1, 0 \leq R_1 \leq H_b(\alpha * p)\} \\
&= \Pr\{\Phi_1^{-1}(0) \leq \gamma_0 \leq \Phi_1^{-1}(1), \Phi_2^{-1}(0) \leq \gamma_2 \leq \Phi_2^{-1}(1), \\
&\qquad \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}[\Psi(\gamma_0,\gamma_2)]\} \\
&= \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_0 \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(1)} d\gamma_2 \\
&\quad \cdot \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}[\Psi(\gamma_0,\gamma_2)]} p(\gamma_0)\cdot p(\gamma_1)\cdot p(\gamma_2) d\gamma_1 \\
&= \frac{1}{\Gamma_0\Gamma_2} \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(1)} \exp(-\frac{\gamma_0}{\Gamma_0} - \frac{\gamma_2}{\Gamma_2}) \\
&\quad \cdot \left\{1-\exp\left[-\frac{\Phi_1^{-1}[\Psi(\gamma_0,\gamma_2)]}{\Gamma_1}\right]\right\} d\gamma_0 d\gamma_2
\end{aligned} \tag{16}
$$

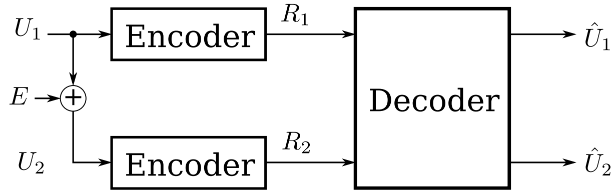


Fig. 4. Abstract model for the coding/decoding of $U_1$ and $U_2$ from the viewpoint of Slepian-Wolf theorem. $U_2$ is the bit-flipped version of $U_1$ and also needs to be recovered at the decoder.

with $\Psi(\gamma_0,\gamma_2) = H_b\{H_b^{-1}[1-\Phi_1(\gamma_0)] * H_b^{-1}[1-\Phi_2(\gamma_2)]\}$.

As indicated by (13), the value of $P_{1,a}$ is found to be always equal to 0. Since the derivation for the explicit expressions of the integrals in (14), (15) and (16) may not be possible, we use a numerical method [25] to calculated the values of $P_{1,b}$, $P_{2,a}$ and $P_{2,b}$. Note that the boundary between $\mathscr{R}_b$ and $\mathscr{R}_c$ is nonlinear, and the calculation of $P_{out}^{SI}$ requires an inverse binary entropy function $H_b^{-1}(\cdot)$. However, it is difficult to derive explicit expression of $H_b^{-1}(\cdot)$ and we use an approximation technique which is described in Appendix B.

## V. Approximated Outage Probability Analysis

The Slepian-Wolf theorem is well known for lossless transmission of correlated sources. Unlike *the theorem for source coding with side information*, the Slepian-Wolf theorem provides the admissible rate region required to recover all the correlated sources. In this section, we show that the rate region of the DF-IE system can also be approximated by the Slepian-Wolf theorem. With this assumption, the boundary can be expressed by a connection of linear lines. Based on the approximated admissible rate region with linear boundary, we derive the outage probability, which eliminates the difficulty in numerical calculation due to the nonlinear boundary of $\mathscr{R}_{SI}$.

### A. Approximated Admissible Rate Region Based on Slepian-Wolf Coding

First of all, we consider the successful transmission of both $U_1$ and $U_2$ from the viewpoint of Slepian-Wolf theorem, as shown in Fig. 4. According to the Slepian-Wolf theorem [14], successful recovery of both $U_1$ and $U_2$ after joint decoding at $D$ is possible if $R_1$ and $R_2$ satisfies

$$
\begin{cases}
R_1 \geq H(U_1|U_2), \\
R_2 \geq H(U_2|U_1), \\
R_1 + R_2 \geq H(U_1,U_2),
\end{cases} \tag{17}
$$

where $H(U_1|U_2)$ ($H(U_2|U_1)$) denotes the conditional entropy of $U_1$ ($U_2$) given $U_2$ ($U_1$), and $H(U_1,U_2)$ denotes the joint entropy of $U_1$ and $U_2$. It is obvious that $H(U_1) = H(U_2) = 1$, $H(U_1|U_2) = H(U_2|U_1) = H_b(p)$ and $H(U_1,U_2) = 1 + H_b(p)$. The admissible rate region identified by (17) is represented by $\mathscr{R}_d$, which is an unbounded polygon, as shown in Fig. 3. However, $\mathscr{R}_e$ should also be included as the admissible rate region for DF-IE system [17], [18], since we only focus on the transmission of $U_1$ and an arbitrary value of $R_2$ is satisfactory as long as $R_1 \geq H(U_1)$.

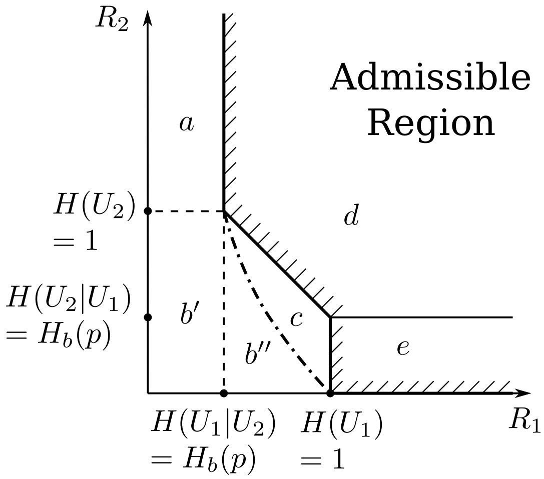


Fig. 5. The approximated admissible rate region for $S$ and $R$ determined by the Slepian-Wolf theorem.

Let $\mathscr{R}_{SW}$ denote the admissible rate region that includes both $\mathscr{R}_d$ and $\mathscr{R}_e$ ($\mathscr{R}_{SW} = \mathscr{R}_d \cup \mathscr{R}_e$), $\mathscr{R}_{SW}$ can be expressed as

$$R_1 \geq \begin{cases} H_b(p), & \text{for} \quad R_2 \geq 1, \\ 1 + H_b(p) - R_2, & \text{for} \quad H_b(p) \leq R_2 \leq 1, \\ 1, & \text{for} \quad 0 \leq R_2 \leq H_b(p). \end{cases} \tag{18}$$

According to (18), if $0 \leq R_2 \leq H_b(p)$, $R_1$ should always be larger than 1 to guarantee successful recovery of $U_1$ at the receiver. However, $U_2$ can be partially recovered at the receiver as long as $R_2 > 0$, and the partially recovered $U_2$ can serve as side information for recovering $U_1$. Intuitively, with the help of the side information at the receiver, $R_1$ can be further reduced to less than 1 while keeping the possibility that $U_1$ still can be successfully recovered at the receiver, which is obviously excluded in (18). This shows that $\mathscr{R}_{SW}$ is an approximation of the admissible rate region for $S$ and $R$. As shown in Fig. 3, $\mathscr{R}_{SI} \geq \mathscr{R}_{SW}$, and the difference is represented by $\mathscr{R}_c$. Although $\mathscr{R}_{SW}$ is an approximation of $\mathscr{R}_{SI}$, if we set $p = 0$ and according to (8) and (18), the boundaries of $\mathscr{R}_{SI}$ and $\mathscr{R}_{SW}$ become the same, and $\mathscr{R}_c$ vanishes. This indicates that as the instantaneous SNR of intra-link is large enough, $\mathscr{R}_c$ does not have any impact on the outage probability calculation.

### B. Approximated Outage Calculation Based on $\mathscr{R}_{SW}$

Similarly, given $\mathscr{R}_{SW}$, the outage event happens when $(R_1, R_2)$ falls outside $\mathscr{R}_{SW}$. In this case, the outage probability of DF-IE system is defined as

$$\begin{aligned} P_{out}^{SW} &= \Pr\{0 \leq p \leq 0.5, (R_1, R_2) \notin \mathscr{R}_{SW}\} \\ &= \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_a \cup \mathscr{R}_b \cup \mathscr{R}_c\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_a \cup \mathscr{R}_b \cup \mathscr{R}_c\} \\ &= \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_{ab'}\} \\ &\quad + \Pr\{p = 0, (R_1, R_2) \in \mathscr{R}_{b''c}\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_{ab'}\} \\ &\quad + \Pr\{0 < p \leq 0.5, (R_1, R_2) \in \mathscr{R}_{b''c}\}. \end{aligned} \tag{19}$$

To simplify the calculation of $P_{out}^{SW}$, we further divide $\mathscr{R}_b$ into two subregions, $\mathscr{R}_{b'}$ and $\mathscr{R}_{b''}$, i.e., $\mathscr{R}_b = \mathscr{R}_{b'} \cup \mathscr{R}_{b''}$, as shown in Fig. 5. In (19), $\mathscr{R}_{ab'}$ and $\mathscr{R}_{b''c}$ are defined as $\mathscr{R}_{ab'} = \mathscr{R}_a \cup \mathscr{R}_{b'}$ and $\mathscr{R}_{b''c} = \mathscr{R}_c \cup \mathscr{R}_{b''}$, respectively. It is obvious that $\mathscr{R}_{ab'} = \{(R_1, R_2) : 0 \leq R_1 \leq H_b(p), R_2 \geq 0\}$, and $\mathscr{R}_{b''c} = \{(R_1, R_2) : H_b(p) \leq R_1 \leq 1, R_1 + R_2 \leq 1 + H_b(p)\}$.

Note that $R_1$, $R_2$ and $p$ can be expressed as functions of their corresponding channel SNRs, as shown in Section IV-B and III. Let $P_{1,ab'}$, $P_{1,b''c}$, $P_{2,ab'}$ and $P_{2,b''c}$ denote the last four terms in (19), respectively. They can be calculated as

$$\begin{aligned} P_{1,ab'} &= \Pr\{p = 0,\ 0 \leq R_1 \leq H_b(p),\ R_2 \geq 0\} \\ &= \Pr\{\gamma_0 \geq \Phi_1^{-1}(1), \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}(0), \\ &\qquad \gamma_2 \geq \Phi_2^{-1}(0)\} \\ &= \int_{\Phi_1^{-1}(1)}^{\Phi_1^{-1}(\infty)} d\gamma_0 \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(0)} d\gamma_1 \\ &\quad \cdot \int_{\Phi_2^{-1}(1)}^{\Phi_2^{-1}(\infty)} p(\gamma_0) \cdot p(\gamma_1) \cdot p(\gamma_2) d\gamma_2 \\ &= 0, \end{aligned} \tag{20}$$

$$\begin{aligned} P_{1,b''c} &= \Pr\{p = 0, H_b(p) \leq R_1 \leq 1, R_1 + R_2 \leq 1 + H_b(p)\} \\ &= \Pr\{\gamma_0 \geq \Phi_1^{-1}(1), \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}(1), \\ &\qquad \Phi_2^{-1}(0) \leq \gamma_2 \leq \Phi_2^{-1}[1 - \Phi_1(\gamma_1)]\} \\ &= \int_{\Phi_1^{-1}(1)}^{\Phi_1^{-1}(\infty)} d\gamma_0 \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_1 \\ &\quad \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}[1-\Phi_1(\gamma_1)]} p(\gamma_0) \cdot p(\gamma_1) \cdot p(\gamma_2) d\gamma_2 \\ &= \frac{1}{\Gamma_1} \exp\left[-\frac{\Phi_1^{-1}(1)}{\Gamma_0}\right] \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} \exp(-\frac{\gamma_1}{\Gamma_1}) \\ &\quad \cdot \left[1 - \exp(-\frac{\Phi_2^{-1}[1 - \Phi_1(\gamma_1)]}{\Gamma_2})\right] d\gamma_1, \end{aligned} \tag{21}$$

$$\begin{aligned} P_{2,ab'} &= \Pr\{0 < p \leq 0.5,\ 0 \leq R_1 \leq H_b(p),\ R_2 \geq 0\} \\ &= \Pr\{\Phi_1^{-1}(0) \leq \gamma_0 \leq \Phi_1^{-1}(1), \\ &\qquad \Phi_1^{-1}(0) \leq \gamma_1 \leq \Phi_1^{-1}[1 - \Phi_1(\gamma_0)], \gamma_2 \geq \Phi_2^{-1}(0)\} \\ &= \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_0 \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}[1-\Phi_1(\gamma_0)]} d\gamma_1 \\ &\quad \cdot \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}(\infty)} p(\gamma_0) \cdot p(\gamma_1) \cdot p(\gamma_2) d\gamma_2 \\ &= \frac{1}{\Gamma_0} \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} \exp(-\frac{\gamma_0}{\Gamma_0}) \\ &\quad \cdot \left[1 - \exp(-\frac{\Phi_1^{-1}[1 - \Phi_1(\gamma_0)]}{\Gamma_1})\right] d\gamma_1, \end{aligned} \tag{22}$$

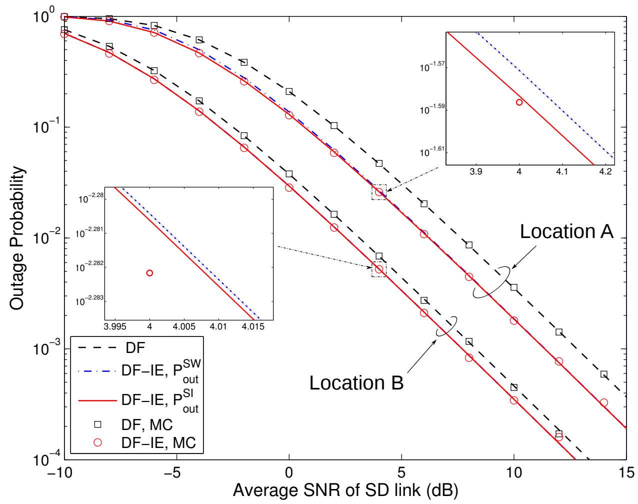


Fig. 6. Comparison of the outage probability of the DF-IE and conventional DF system.

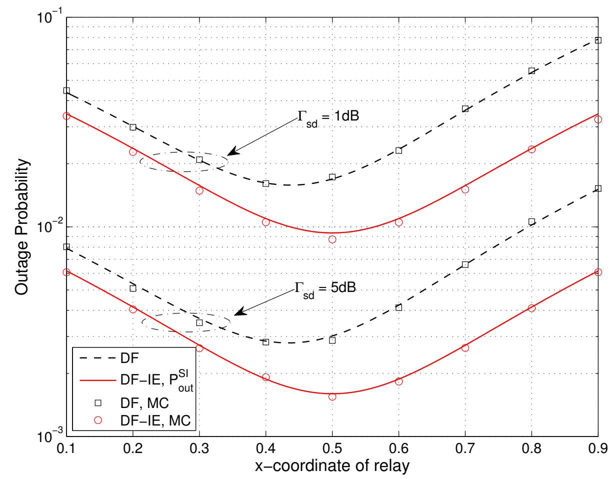


Fig. 7. Outage probability versus the relay location, where $\Gamma_{sd} = 1$ dB and $\Gamma_{sd} = 5$ dB.

and

$$\begin{aligned}
P_{2,b''c} &= \Pr\{0 < p \le 0.5,\ H_b(p) \le R_1 \le 1, \\
&\qquad R_1 + R_2 \le 1 + H_b(p)\} \\
&= \Pr\{\Phi_1^{-1}(0) \le \gamma_0 \le \Phi_1^{-1}(1), \\
&\qquad \Phi_1^{-1}[1-\Phi_1(\gamma_0)] \le \gamma_1 \le \Phi_1^{-1}(1), \\
&\qquad \Phi_2^{-1}(0) \le \gamma_2 \le \Phi_2^{-1}[2-\Phi_1(\gamma_0)-\Phi_1(\gamma_1)]\} \\
&= \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} d\gamma_0 \int_{\Phi_1^{-1}[1-\Phi_1(\gamma_0)]}^{\Phi_1^{-1}(1)} d\gamma_1 \\
&\quad \cdot \int_{\Phi_2^{-1}(0)}^{\Phi_2^{-1}[2-\Phi_1(\gamma_0)-\Phi_1(\gamma_1)]} p(\gamma_0)\cdot p(\gamma_1)\cdot p(\gamma_2) d\gamma_2 \\
&= \frac{1}{\Gamma_0\Gamma_1} \int_{\Phi_1^{-1}(0)}^{\Phi_1^{-1}(1)} \int_{\Phi_1^{-1}[1-\Phi_1(\gamma_0)]}^{\Phi_1^{-1}(1)} \exp(-\frac{\gamma_0}{\Gamma_0} - \frac{\gamma_1}{\Gamma_1}) \\
&\quad \cdot \left[1 - \exp(-\frac{\Phi_2^{-1}[2-\Phi_1(\gamma_0)-\Phi_1(\gamma_1)]}{\Gamma_2})\right] d\gamma_1 d\gamma_0.
\end{aligned} \tag{23}$$

Again, the value of $P_{1,ab'}$ is always equal to 0. To calculate the values of $P_{1,b''c}$, $P_{2,ab'}$ and $P_{2,b''c}$, the numerical method [25], used to calculate $P_{out}^{SI}$, was also used here. Note that the outage probability $P_{out}^{SW}$ and $P_{out}^{SI}$ both include the average SNR and the spectrum efficiency of all the three links, only as parameters, i.e., $P_{out} = g(\Gamma_0, \Gamma_1, \Gamma_2, R_{c,1}, R_{c,2})$. Therefore, they both can be applied to arbitrary relay location case. However, since $\mathscr{R}_{SI}$ is larger than $\mathscr{R}_{SW}$, it is expected that $P_{out}^{SI}$ is smaller than $P_{out}^{SW}$.

## VI. Numerical Results

In this section, the numerical results of the theoretical outage probability calculation and the simulation results using Monte Carlo (MC) method are presented. In the simulations, $R_{c,1}$ and $R_{c,2}$ both are set at $1/2$.

The outage curves of DF-IE, obtained using the analytical expressions of $P_{out}^{SW}$ and $P_{out}^{SI}$, respectively, are shown in Fig. 6. Here, two different relay location scenarios are considered: in Location A, $d_0 = d_1 = d_2$; in Location B, $d_0 = \frac{1}{4}d_1$ and $d_2 = \frac{3}{4}d_1$.[5] From the figure we can see the difference between $P_{out}^{SW}$ and $P_{out}^{SI}$ is roughly 0.01 dB in Location A, and 0.001 dB in Location B. This observation indicates $P_{out}^{SW}$ is an accurate approximation to the exact outage probability $P_{out}^{SI}$. Moreover, it is found that $P_{out}^{SI}$ is always smaller than $P_{out}^{SW}$, which is consistent with the theoretical analysis provided in Section V-B. As a reference, the outage probability of a conventional DF system [12], where the relay keeps silent in case of intra-link error is detected, is also provided in the same figure. As can be seen from the figure, DF-IE system can achieve better performance than the conventional DF system,[6] which also agrees with the theoretical analysis. However, the performance gain obtained in Location B ($\approx 0.6$ dB) is smaller than that obtained in Location A ($\approx 1.5$ dB). This is because, in Location B, the quality of the intra-link is better than that in Location A, resulting in lower probability of the intra-link transmission failure. Note that the theoretical outage probabilities of the both DF-IE and conventional DF system are in excellent agreement with their corresponding simulation results, respectively.

The impact of the relay location on the outage probability of the both DF-IE and conventional DF systems is depicted in Fig. 7, where the average SNR of $SD$ link is kept at $\Gamma_{sd} = 1$ dB and $\Gamma_{sd} = 5$ dB. Here, the position of $R$ is assumed to vary along the line between $S$ ($x = 0$) and $D$ ($x = 1$). With the conventional DF relaying system, the lowest outage probability is achieved at a certain point ($x \approx 0.43$) between $S$ and the midpoint. With DF-IE system, interestingly, we found that when the relay is located at the midpoint ($x = 0.5$), the lowest outage probability can be achieved. In general, the lowest outage probability can be achieved at a point where the contributions of both $SR$ and $RD$ links to outage are

[5] According to (5), the average SNR of $L_{sr}$ and $L_{rd}$ are $\Gamma_{sr} = \Gamma_{sd} + 10\log_{10}[(d_{sd}/d_{sr})^{3.52}]$ (dB) and $\Gamma_{rd} = \Gamma_{sd} + 10\log_{10}[(d_{sd}/d_{rd})^{3.52}]$ (dB), respectively.

[6] Note that DF-IE system causes more complexity or energy consumption than conventional DF system when the intra-link errors are detected. However, the increase in complexity or energy consumption over the conventional DF is negligible, especially in high SNR regime.

balanced. Since in conventional DF, the relay stops forwarding the information sequence when errors are found, the quality of the $SR$ link has to be good enough, which results in the optimal position shifted more closer to the side of the source. In DF-IE system, the contributions of the two links to the outage probability are balanced because there is a chance that the errors occurring in the $SR$ link can be corrected at the destination, and as a consequence, the optimal location is the midpoint.

## VII. Conclusions

In this paper, the outage probability of DF-IE system was theoretically analyzed, where all the links between source, relay and destination are subject to independent block Rayleigh fading. The exact and approximated outage probabilities are derived from the viewpoint of *the theorem for source coding with side information* and the Slepian-Wolf theorem, respectively. The admissible rate region is determined by *the theorem for source coding with side information*, and found to be larger than that determined by the Slepian-Wolf theorem. The exact outage probability is lower than its approximation by the Slepian-Wolf theorem, however, the difference is negligible.

Compared with the conventional DF system, DF-IE can always achieve better outage performance. The most significant finding of this paper is, with the DF-IE system, the optimal relay location is shifted to exactly the midpoint between the source and destination, where the contributions of the $SR$ and $RD$ links are balanced. The accuracy of the theoretical analysis has been verified through computer simulations. The analytical results presented in this paper provide a theoretical basis for designing cooperative networks where intra-link error is allowed. The extension of the analytical results to relay systems with multiple sources and/or multiple relays is straightforward. This is left as a future study.

## Appendix A
## Proof of $\mathscr{R}_{SI} \geq \mathscr{R}_{SW}$

According to (8) and (18), for a given $p$ value, $\mathscr{R}_{SI} \geq \mathscr{R}_{SW}$ is equivalent to $f_1(x) \leq f_2(x)$, where

$$f_1(x) = \begin{cases} H_b(p), & \text{for} \quad x \geq 1, \\ H_b(\alpha * p), & \text{for} \quad 0 \leq x \leq 1, \end{cases} \tag{24}$$

and

$$f_2(x) = \begin{cases} H_b(p), & \text{for} \quad x \geq 1, \\ 1 + H_b(p) - x, & \text{for} \quad H_b(p) \leq x \leq 1, \\ 1, & \text{for} \quad 0 \leq x \leq H_b(p), \end{cases} \tag{25}$$

are the boundaries for the sets of the inequalities for $\mathscr{R}_{SI}$ and $\mathscr{R}_{SW}$, respectively. It is obviously that $f_1(x) = f_2(x)$ for $x \geq 1$. For $0 \leq x \leq H_b(p)$, $f_1(x) \leq f_2(x)$ since $H_b(\alpha * p) \leq 1$ always holds. For $H_b(p) \leq x \leq 1$, we can obtain $x = R_2 = 1 - H_b(\alpha)$ according to (7). In this case,

$$\begin{aligned} f_2(x) - f_1(x) &= 1 + H_b(p) - x - H_b(\alpha * p) \\ &= H_b(\alpha) + H_b(p) - H_b(\alpha * p). \end{aligned} \tag{26}$$

To prove $f_2(x) - f_1(x) \geq 0$ for $H_b(p) \leq x \leq 1$, we consider the joint entropy of two binary random variable $X$ and $Y$,

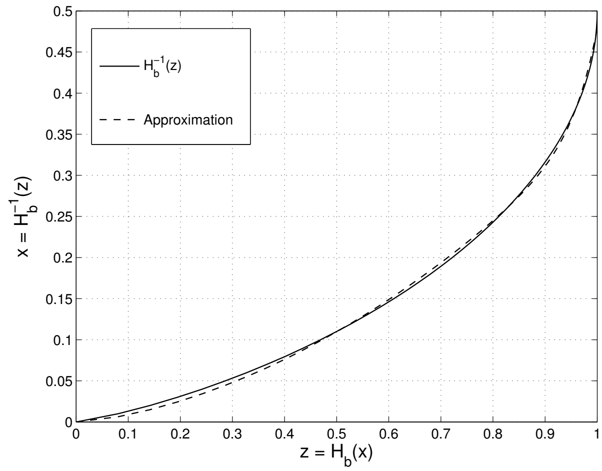


Fig. 8. The inverse entropy function and its approximation.

where $X$ follows a Bernoulli distribution with parameter $\alpha$, and $Y$ is the observation of $X$ over a BSC with a crossover probability $p$. The joint entropy of $X$ and $Y$ can be expressed as

$$\begin{aligned} H(X,Y) &= H(X) + H(Y|X) \\ &= H(Y) + H(X|Y), \end{aligned} \tag{27}$$

where $H(X) = H_b(\alpha)$, $H(Y) = H_b(\alpha * p)$ and $H(Y|X) = H_b(p)$. From (27), we get

$$\begin{aligned} H(X|Y) &= H(X) + H(Y|X) - H(Y) \\ &= H_b(\alpha) + H_b(p) - H_b(\alpha * p) \\ &\geq 0. \end{aligned} \tag{28}$$

Hence, it can be concluded that $f_2(x) - f_1(x) \geq 0$ for $H_b(p) \leq x \leq 1$.

In summary, $f_1(x) \leq f_2(x)$ for $x \geq 0$, and $\mathscr{R}_{SI} \geq \mathscr{R}_{SW}$ has been proven.

## Appendix B
## Approximation of $H_b^{-1}(\cdot)$

The binary entropy function is defined as

$$H_b(x) = z = -x \log_2 x - (1 - x) \log_2 (1 - x). \tag{29}$$

For $0 \leq x \leq 0.5$, $H_b(x)$ is monotonically increasing and therefore has a unique inverse function

$$x = H_b^{-1}(z). \tag{30}$$

However, it may not be possible to derive the explicit expressions of $H_b^{-1}(z)$, according to (29). By using a curve fitting technique [26], it can be well approximated by

$$H_b^{-1}(z) \approx (2^{c_1 z^{c_2}} - 2^{-c_3 z^{c_4}})^{c_5}, \tag{31}$$

with $c_1 = 0.6794$, $c_2 = 0.7244$, $c_3 = 0.1357$, $c_4 = 21.8026$ and $c_5 = 1.9920$. The numerically calculated $H_b^{-1}(z)$ and its approximated curves are shown in Fig. 8.

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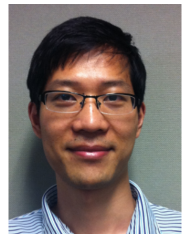

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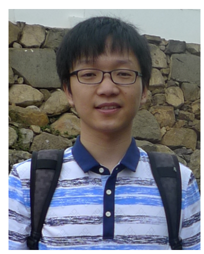

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