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Description	

LDPC Code Optimization with Joint Source-Channel Decoding of Quantized Gauss-Markov Signals

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***Abstract*—This paper proposes an extrinsic information transfer (EXIT)-chart based optimization technique of LDPC codes for the transmission of quantized Gauss-Markov (GM) source samples over additive white Gaussian (AWGN) noise channels. A joint source and channel (JSC) decoding technique of the proposed code is also devised. In the proposed scheme, no interleaving is performed between the source and the JSC encoder so that the decoder can well exploit the relatively low entropy of the source with memory compared to memory-less sources. At the transmitter, the quantized samples are converted to bit sequences with an injective mapping and the bit sequences are encoded using a systematic binary LDPC code. The proposed JSC decoder is a concatenation of a multi-state BCJR Markov decoder and a sum-product (SP) LDPC decoder. Decoding thresholds of the optimized codes at certain code rates are investigated for both uniform and Lloyd-Max quantizations in different numbers of bits. The decoding thresholds are close to the Gaussian code book Shannon limits for code rate $R_c \leq 0.5$, although the gap to the Shannon limit notably increases at the higher rates. Finally, the simulation results confirm the significant improvement of coding gain on the bit error rate (BER) performances of the optimized LDPC codes with both the quantization schemes.**

I. INTRODUCTION

Transmission of *auto-correlated* source samples over noisy channels has motivated a huge flurry of research activity to source and channel coding during recent years [1], [2]. Recent investigations confirm that the joint source-channel (JSC) coding/decoding achieves significant coding gain improvement over the separate coding in finite block-length transmission [1], [2]. Application dependant variable length codes (VLC) that can achieve near entropy compression have been intensively researched towards their practical implementations, there still remains residual source redundancy in the source coding which can be utilized to improve the channel coding gain [3], [4]. Furthermore, the VLC based JSC coding suffers from the error propagation because of its sensitivity to the symbol boundary error [2]. In addition, certain applications, e.g., sensor networks, may have serious power consumption constraint and hence communication is only possible in very low signal-to-noise ratio (SNR) regime. As a remedy to these drawbacks, the framework of JSC coding which directly exploits the memory structure of the source has been recognized as being of significant importance, when the proper operability of the system in the low SNR regime is considered.

Majority of the JSC coding methods have taken binary Markov model into account for the sources with memory [5], [6], and references therein. However, binary Markov sources can simply be extended to the auto-correlated signals such as quantized *Gauss-Markov (GM)* sources. A serial concatenation of a multi-state BCJR decoder to a sum-product (SP) decoder has been developed recently in [7] for JSC decoding of multi-state Markov sources encoded by a short-length binary LDPC code. Since the BCJR decoder of [7] operates in the probability domain, it is numerically unstable for long-length codewords. Furthermore, the LDPC codes used in [7] were not optimized.

The main contributions of this paper are summarized as follows. First, we describe an analytic formulation of a JSC decoding method to directly exploit the *temporal correlation* of quantized GM samples with both uniform and Lloyd-Max quantizers [8]. Binary *irregular* LDPC codes are used to encode the bit-wise representation of the quantized samples of the scheme. The proposed decoder is composed of a log-domain multi-state BCJR decoder and an SP decoder which even performs in a numerically stable way for long-length codewords. Second, the *extrinsic* mutual information transfer (EXIT) chart [9], [10] analysis for the proposed JSC decoder is performed to optimize the asymptotic performance of the LDPC codes in the sense that the *decoding threshold* [11] of the code is minimized under the *stability* and *convergence* constraints [10], [11] for a given code rate. Our main result is the precise calculation of *decoding thresholds* of the optimized codes under the JSC decoder for both the uniform and Lloyd-Max quantizers at various channel code rates.

II. SYSTEM MODEL

A. Transmission Model

A simple signal model for specifying the temporal correlation between samples is the *first-order autoregressive* process which is denoted, with n being the discrete time index, as follows [12]

$$x_n = ax_{n-1} + u_n \quad n \geq 0, \tag{1}$$

where x_n and u_n denote samples of the process output and a white noise, respectively, and a is a correlation constant with $|a| < 1$. If u_n and the initial value of x_n, i.e., x_{-1}, are assumed to be independent zero mean white Gaussian with variances σ_u^2 and σ_x^2, respectively, the process is called *Gauss-Markov (GM)* process. If the x_{-1} is drawn from a Gaussian distribution with zero mean and variance $\sigma_x^2 = \sigma_u^2/(1-a^2)$, the process $\{x_n\}$ will be *wide sense stationary (WSS)* [1] [12].

The source samples are clipped symmetrically at a threshold c_q and then quantized by a scalar quantization, either uniformly or nonuniformly, with a q bit resolution. We primarily focus on the uniform quantization and then the derived results for the uniform quantization are straightforwardly generalized to the nonuniform cases such as Lloyd-Max quantization. Consider 2^q *quantization levels* between the range $[-c_q, c_q]$ and each of the $(2^q - 1)$ *quantization intervals* between the levels is represented by a corresponding integer at the center of the interval, called *quantization index*. *Quantization step size* of each interval, denoted by δ, is the width of the related interval calculated as $\delta = \frac{2c_q}{2^q-1}$ for the uniform quantization. Thus, a quantization index i_n associated to a sample x_n at the time instant n in the uniform quantization is calculated by

$$i_n = \lfloor \frac{x_n}{\delta} + 0.5 \rfloor, \quad (2)$$

where $-(2^{q-1}-1) \leq i_n \leq 2^{q-1}-1$ and $\lfloor . \rfloor$ denotes the largest integer not larger than its argument. Therefore, $\hat{x}_n = \delta i_n$ denotes the uniformly quantized value of the x_n. Due to the Gaussian distribution of the GM samples, the probability mass function of quantization index i_n for the uniform quantization is given by

$$p(i_n) = \begin{cases} 1 - Q(\frac{\hat{x}_n+\delta/2}{\sigma_x}) & \text{if } i_n = -\kappa_q \\ Q(\frac{\hat{x}_n-\delta/2}{\sigma_x}) - Q(\frac{\hat{x}_n+\delta/2}{\sigma_x}) & \text{if } -\kappa_q < i_n < \kappa_q \\ Q(\frac{\hat{x}_n-\delta/2}{\sigma_x}) & \text{if } i_n = \kappa_q \end{cases}, \quad (3)$$

where $\kappa_q = (2^{q-1} - 1)$ and $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp\left(-\frac{t^2}{2}\right) dt$ is the Q-function. On contrary, in case of Lloyd-Max quantization, the levels are optimized to minimize the quantization error and they are not uniformly apportioned [8]. Hence, the associated quantization levels of the nonuniform quantizer are used to numerically calculate the probability of quantization indices instead of $\hat{x}_n \pm \delta/2$ in (3).

Let $\mathbf{b}(i) = (b_1(i), \ldots, b_q(i))^{\mathrm{T}}$ denote a column vector whose entries $b_k(i) \in \{0,1\}$, $k \in \{1, \ldots, q\}$, are representative bits corresponding to its argument integer i where $(.)^{\mathrm{T}}$ denotes the transpose of a vector. To represent a quantization index i_n with q bits, we use decimal-to-bit conversion of $i'_n = (i_n + 2^{q-1})$ instead of i_n to avoid negative integers. This conversion denoted as $i'_n = \sum_{k=1}^{q} b_k(i_n) 2^{k-1}$ specifies an injective mapping[2] from the quantization samples to a q bit binary vector and the binary vectors of consecutive samples are then converted to the serial bit string. Hence, a k-th bit in the binary representation of a quantization index i_n at time n corresponds to the v-th bit of the serial bit sequence where $v = q(n-1) + k$.

After the parallel to serial converter, the serial bit string is encoded by a systematic binary LDPC code with rate R_c. The encoded bits are modulated using binary phase-shift keying (BPSK) and the modulated bits are sent over AWGN channel with the channel noise variance σ_n^2 and the noise power spectral density $N_0 = 2\sigma_n^2$. The proposed transmission model of the LDPC coded quantized samples is concisely presented in Fig. 1. The average energy per source bit, E_{s}, is related to the energy per information bit, E_{b}, and the energy per sent symbol, E_{sym}, over the channel as follows

$$E_{\mathrm{s}} = \frac{H(\mathcal{I}_q)}{q} E_{\mathrm{b}} = \frac{E_{\mathrm{sym}}}{R_c}, \quad (4)$$

where $H(\mathcal{I}_q)$ is the entropy rate of the quantization indices and $E_{\mathrm{sym}} = 1$ with BPSK. The total information rate, R bits per channel use, is defined as the ratio between the information generated by the source and the total number of bits to be sent over the channel. Hence, the source code rate, R_{s}, and the total information rate, R, are calculated, respectively, as $R_{\mathrm{s}} = H(\mathcal{I}_q)/q$ and $R = R_{\mathrm{s}} R_c$. The Shannon capacity of AWGN channel at the specific information rate R, theoretically limits the minimum energy E_{s}-to-noise ratio denoted by $\frac{E_{\mathrm{s}}}{N_0}|_{\mathrm{lim}}$.

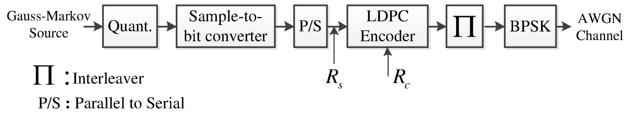

Fig. 1: Block diagram of bit-interleaved LDPC coded Gauss-Markov source.

B. LDPC Codes for JSC Coding

Consider a binary systematic LDPC code $\mathcal{C}$ and let $n_{\mathcal{I}}$ denote the number of quantization indices to be encoded by the $\mathcal{C}$. Hence, the number of source bits K and the codeword length N of $\mathcal{C}$ are $qn_{\mathcal{I}}$ and K/R_c, respectively. The code is represented by a Tanner graph $G = (V_b \cup V_c, E)$, where $V_b = \{v_1, \ldots, v_N\}$ and $V_c = \{c_1, \ldots, c_M\}$ are the sets of variable nodes and check nodes, respectively, and E is the set of edges. The variable node set V_b is divided into two sets, source nodes $V_{b_s} = \{v_1, \ldots, v_K\}$ and parity nodes $V_{b_p} = \{v_{K+1}, \ldots, v_N\}$. Corresponding to G, we have an $M \times N$ parity-check matrix $\mathbf{H} = [h_{ij}]$ of $\mathcal{C}$, where $h_{ij} = 1$ if and only if (iff) the node $c_i \in V_c$ is connected to the node $v_j \in V_b$ in G through an interleaver, or equivalently, iff $\{v_j, c_i\} \in E$. Each variable node $v_j \in V_{b_s}$ is bijectively connected to a source decoder which is a multi-state BCJR decoder in this paper. The joint graph of source and systematic LDPC code is depicted in Fig. 2.

The *degree* of a variable or a check node is the number of nodes connected to it. If the nodes in the set V_b and/or the nodes in the set V_c have different degrees, the corresponding LDPC code is called *irregular*. For an *irregular* systematic LDPC code, the *degree distributions* of source and parity variable nodes are described, respectively, by the two polynomials $\lambda_s(x) = \sum_{i=2}^{D_v} \lambda_{s_i} x^{i-1}$ and $\lambda_p(x) = \sum_{i=2}^{D_v} \lambda_{p_i} x^{i-1}$ where D_v is the maximum variable node degree, and λ_{s_i} and λ_{p_i} are, respectively, the fractions of edges connected to the source and parity nodes of degree i. Hence, $\lambda_s(1) + \lambda_p(1) = 1$. Similarly,

[1] We will assume that the GM process is WSS and the correlation parameter, a, is known to both the sender and the receiver.

[2] The proposed mapping is not the optimum mapping for transmission of the quantized GM samples. Although the optimum mapping increases the coding gain [1], devising it is out of scope of this work.

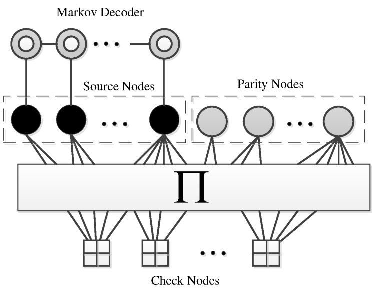

Fig. 2: Joint graph representation of Markov decoder and systematic LDPC code.

the degree distribution of the check nodes is specified by a polynomial $\rho(x) = \sum_{i=2}^{D_c} \rho_i x^{i-1}$, where ρ_i is the fraction of the edges connected to the check nodes of degree i and D_c is the maximum check node degree. The *decoding threshold* of a given degree distribution is the supremum of the channel noise variance for which the decoding error probability can be reduced to arbitrarily small for which, however, the number of decoding iterations tends to infinity [11]. The *decoding threshold* in terms of decibel, denoted by $\frac{E_s}{N_0}^*$, is used to specify the asymptotic gap between the Shannon's theoretical limit and the *decoding threshold* with the optimized degree distributions.

III. ENTROPY RATE OF THE SOURCE

To calculate the entropy rate of the source with different numbers of quantization bits, the corresponding Markov chain of the quantization indices are required. Consider two consecutive quantization indices i_n and i_{n-1} associated, respectively, to the samples x_n and x_{n-1}. Hence, the conditional probability $p(i_n|i_{n-1})$ is given by

$$p(i_n|i_{n-1}) = p(U = \hat{x}_n - a\hat{x}_{n-1}), \tag{5}$$

where U is a random variable corresponding to the input white Gaussian *driving noise* of the GM process. Thus, we have

$$p(i_n|i_{n-1}) = \begin{cases} 1 - Q(\frac{\hat{u}_n+\delta/2}{\sigma_u}) & \text{if } i_n = -\kappa_q \\ Q(\frac{\hat{u}_n-\delta/2}{\sigma_u}) - Q(\frac{\hat{u}_n+\delta/2}{\sigma_u}) & \text{if } -\kappa_q < i_n < \kappa_q \\ Q(\frac{\hat{u}_n-\delta/2}{\sigma_u}) & \text{if } i_n = \kappa_q \end{cases} \tag{6}$$

where $\hat{u}_n = \hat{x}_n - a\hat{x}_{n-1}$. The associated quantization levels of the nonuniform quantization are used instead of $\hat{x}_n \pm \delta/2$ for the probability calculation, in the case a nonuniform quantizer is used.

Let matrix $\mathbf{T} = [t_{rs}]$ denote the probability transition matrix associated to the Markov model of the quantization indices where $t_{rs} = p\left(i_n = (s - 2^{q-1})|i_{n-1} = (r - 2^{q-1})\right)$, $1 \le r, s \le 2^q - 1$.

Lemma 1: Consider quantized samples of the GM process with q-bit quantization. The entropy rate of the quantization indices is

$$H(\mathcal{I}_q) = \sum_{r=1}^{2^q-1} p(i_{n-1} = r - 2^{q-1}) \left(- \sum_{s=1}^{2^q-1} t_{rs} \log_2 t_{rs} \right). \tag{7}$$

IV. PROPOSED DECODER AND CODE OPTIMIZATION

A. Joint BCJR-SP Decoder

The transition matrix $\mathbf{T}$ describes the equivalent multi-state Markov chain corresponding to the quantization indices of the GM samples. Hence, an associated trellis diagram of the source Markov model perfectly presents the source statistics.

One straightforward implementation of the JSC decoder for the proposed transmission scheme is to exchange the output *extrinsic log-likelihood ratio* (LLR) of the concatenated decoders in each iteration of the JSC decoder. To devise a numerically robust decoder, a two-level JSC decoder consisting of two nested iteration loops [7] is applied herein. The inner loop is called as the *local* iteration of the SP decoder and the outer loop is called the *global* iteration exchanging the *extrinsic* LLR between the source nodes of the SP decoder and the BCJR decoder.

Since the quantization indices are converted to the bit string before transmission, the probability of quantization indices is calculated using the probability of the representative bits. Hence, the probability of a quantization index i_n is calculated as

$$p(i_n) = \prod_{k=1}^{q} 0.5 \left(1 + \left(1 - 2b_k(i_n)\right) \tanh(\frac{L_{v,B}}{2})\right), \tag{8}$$

where $L_{v,B}$ denotes the *extrinsic* LLR, sent from the v-th bit of the source node corresponding to the k-th bit of the i_n, $v = q(n-1) + k$, to the BCJR decoder. Then, probability of the trellis states and correspondingly the *extrinsic* LLR sending to the source nodes are calculated as (9)–(12). There is a one-to-one correspondence between each state s, $1 \le s \le 2^q - 1$, of the trellis diagram at an instant n, $1 \le n \le n_{\mathcal{I}}$, and a quantization index denoted by $i_n(s) = s - 2^{q-1}$. The forward recursion at the instant n is given by [7], [13]

$$\tilde{\alpha}_n(s) = \max_r{}^* \Big(\tilde{\alpha}_{n-1}(r) + \log t_{rs} + \log p\big(i_n(s)\big)\Big), \tag{9}$$

where $\max^*(x, y) = \max(x, y) + \log\left(1 + e^{-|x-y|}\right)$ is performed over the states of the trellis diagram r and s. Furthermore, $\tilde{\alpha}_0(s) = -\infty$ are the initial conditions of the above equation. Then, the backward recursion is calculated as

$$\tilde{\beta}_{n-1}(r) = \max_s{}^* \left(\tilde{\beta}_n(s) + \log t_{rs} + \log p\big(i_n(s)\big)\right), \tag{10}$$

with the initial conditions $\tilde{\beta}_{n_{\mathcal{I}}}(s) = -\infty$. Next, the log-probability of the states is calculated as

$$\tilde{\gamma}_n(s) = \max_r{}^* \left(\tilde{\alpha}_{n-1}(r) + \log t_{rs} + \log p\big(i_n(s)\big) + \tilde{\beta}_n(s)\right), \tag{11}$$

and the probability of $i_n(s)$ should be finally changed to the *extrinsic* LLR of the associated source nodes. The *extrinsic* LLR sent from the BCJR decoder to a node $v \in V_{b_s}$ is given by

$$L_{B,v} = \Big(\sum_{s:\, b_k(i_n(s))=0} \tilde{\gamma}_n(s) - \sum_{s:\, b_k(i_n(s))=1} \tilde{\gamma}_n(s)\Big) - L_{v,B}, \tag{12}$$

where the v corresponds to the k-th bit of the i_n. The above first and second summations are performed over the states s in which the k-th bit of the representative bits of $i_n(s)$ are zero and one, respectively.

Consider the ℓ-th local and the ℓ_g-th global iteration of the proposed JSC decoder. Let $L_{v,c}^{(\ell)}$ and $L_{c,v}^{(\ell)}$ denote the LLR messages sent from a variable node v to a check node c and from the check node c to the variable node v at the ℓ-th iteration, respectively. Furthermore, let L_v^{cc} denote the LLR obtained from the channel, being an input to a variable node $v \in V_b$. Thus, $L_v^{cc} = 2r_v/\sigma_n^2$ where r_v is the received noisy signal corresponding to the v-th variable node. The LLR message $L_{v,c}^{(\ell)}$ is updated as

$$L_{v,c}^{(\ell)} = L_v^{cc} + \sum_{c' \neq c} L_{c',v}^{(\ell-1)} + L_{B,v}^{(\ell_g-1)}, \tag{13}$$

where $v \in V_{b_s}$ and $L_{B,v}^{(\ell_g)}$ denotes the *extrinsic* LLR sent from the BCJR decoder at the iteration ℓ_g. Similarly, for the $v \in V_{b_p}$ we have

$$L_{v,c}^{(\ell)} = L_v^{cc} + \sum_{c' \neq c} L_{c',v}^{(\ell-1)}. \tag{14}$$

The messages $L_{c,v}^{(\ell)}$ are also updated in the same way as the standard SP decoding. *A posteriori* LLRs $\tilde{L}_v^{(\ell)}$ of the variable nodes at the ℓ-th iteration are calculated as

$$\tilde{L}_v^{(\ell)} = L_v^{cc} + \sum_{c'} L_{c',v}^{(\ell-1)} + L_{B,v}^{(\ell_g-1)}, \tag{15}$$

where $v \in V_{b_s}$ and

$$\tilde{L}_v^{(\ell)} = L_v^{cc} + \sum_{c'} L_{c',v}^{(\ell-1)}, \tag{16}$$

where $v \in V_{b_p}$. Thus, the estimated bit $\hat{b}_v^{(\ell)}$ is given by

$$\hat{b}_v^{(\ell)} = \begin{cases} 0 & \text{if } \tilde{L}_v^{(\ell)} \geq 0 \\ 1 & \text{if } \tilde{L}_v^{(\ell)} < 0. \end{cases} \tag{17}$$

Then, a stopping criterion $s_c^{(\ell)}$ based on the parity-check syndromes at the ℓ-th iteration is computed as

$$s_c^{(\ell)} = \sum_{c \in V_c} \sum_{v \in V_b} h_{cv} \hat{b}_v^{(\ell)}, \tag{18}$$

where h_{cv} is the (c,v)-th entry of the parity-check matrix. If $s_c^{(\ell)} = 0$ happens during the local iterations, the decoder finds a valid codeword and the decoding procedure successfully terminates. Otherwise, the *extrinsic* LLR of the source nodes are passed on to the BCJR decoder. Hence, the *extrinsic* LLR sent from a source node $v \in V_{b_s}$ to the BCJR decoder is given by

$$L_{v,B}^{(\ell_g)} = L_v^{cc} + \sum_{c'} L_{c',v}^{(\ell_{\max})}, \tag{19}$$

where $\ell_{\max}$ is the maximum local iteration number. Finally, the message $L_{v,B}^{(\ell_g)}$ is used as an input LLR of the BCJR decoder to calculate the $L_{B,v}^{(\ell_g)}$ messages through the (8)–(12). The *extrinsic* LLRs are exchanged between the SP and the BCJR decoders until either estimated bits form a valid codeword of the LDPC code or a maximum number of global iterations is reached.

B. EXIT Chart Analysis and Code Optimization

The EXIT chart technique is one of the most efficient tools developed to analyze and optimize LDPC codes based on the evolution of the *extrinsic* mutual information (MI) during successive iterations of an iterative decoder [9]. An optimization method based on the EXIT chart analysis was recently proposed in [10] for JSC decoders comprised of a general *soft-in soft-out (SISO)* source decoder and a SP decoder of a systematic LDPC code. This method is a semi-analytic technique which includes two major parts, the MI calculation of the SP decoder based on *Gaussian approximation* [9] and the EXIT curve approximation of the SISO source decoder. Moreover, it is assumed that the MI of the source variable nodes of the systematic LDPC code is updated by the source decoder in each iteration. In contrast, the LLR message sent from the BCJR decoder does not change during the local iterations due to practical reasoning in the proposed JSC decoder. However, the optimized degree distributions are achieved when the MI of the BCJR decoder is considered in each local iteration of the SP decoder. In this paper, the same technique described in [10] are applied to optimize the degree distributions of LDPC codes under the JSC decoder. However, the performance of the optimized codes is degraded when the source decoder is just updated through the global iterations in the two-level decoding procedure presented in the previous section.

Consider the BCJR decoder described in (8)–(12) as a source decoder and let I_A^{s} and I_E^{s}, respectively, denote the MI of input *a priori* LLR and the MI of output *extrinsic* LLR of the source decoder. A relationship between the source code rate and the area under a plot of the I_E^{s} and I_A^{s} is given as [14]

$$\int_0^1 I_E^{\mathrm{s}}\, dI_A^{\mathrm{s}} = 1 - R_{\mathrm{s}}, \tag{20}$$

when the representative bits of the source are equally likely to be zero and one.

An optimization procedure of the degree distribution of the LDPC codes with the JSC decoding includes the following steps. First, the EXIT function of the source decoder, in terms of I_A^{s} versus I_E^{s}, is estimated using Monte-Carlo simulations. Then, piece-wise curve-fitting methods are applied to derive approximate functions very close to the results of the Monte-Carlo simulations. Next, the code rate optimization problem is stated as a linear function of the $\lambda_s(x)$ and $\lambda_p(x)$ degree distributions with subject to the *convergence* and *stability* constraints [10], [11] using the same equation as in [10, Eq. (6)]. Finally, a linear programming (LP) technique is used to find the degree distributions of the variable nodes which maximizes the code rate. In addition, $\rho(x)$ is assumed to be in a form of *concentrated* degree distribution, i.e., $\rho(x) = \rho x^{D_c-2} + (1-\rho)x^{D_c-1}$, $0 \leq \rho \leq 1$. Thus, we perform the optimization algorithm for each value of ρ and

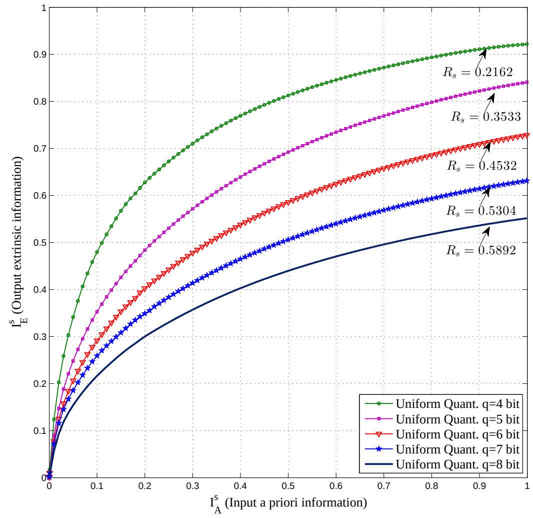

Fig. 3: EXIT function of the source decoder for uniformly quantized GM samples with $a = 0.98$, $\sigma_u^2 = 0.1$, and $q \in \{5, 6, 7\}$ numbers of quantization bits (Example 1).

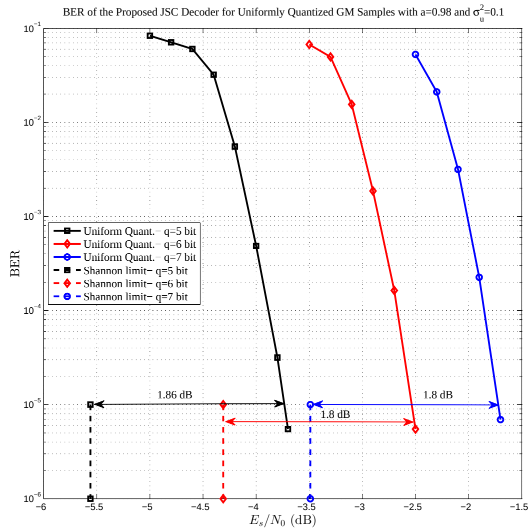

Fig. 4: BER performances of the proposed JSC decoder using optimized degree distributions with $R_c = 0.5$ for encoding of the uniformly quantized GM samples with $a = 0.98$, $\sigma_u^2 = 0.1$, and $q \in \{5, 6, 7\}$ numbers of quantization bits (Example 1).

D_c and select a code whose the rate is the maximum. The code rate optimization algorithm can be simply modified to minimize the *decoding threshold* of the code in a specified code rate [11].

V. SIMULATION RESULTS AND DISCUSSION

In the following examples, the codeword length of the LDPC codes was 21000 and the maximum number of local and global iterations were, respectively, 30 and 10 for the proposed JSC decoding. Furthermore, a highly correlated GM signal with $a = 0.98$ and $\sigma_u^2 = 0.1$ was applied to emphasize the impact of the proposed method. However, the JSC decoder also properly performs at the smaller values of a.

Example 1: The optimized degree distributions of LDPC codes at the rate $R_c = 0.5$ are presented for the uniformly quantized GM samples in different numbers of quantization bits. The EXIT function of the source decoder is obtained using the method as described in the previous section with $c_q = 4\sigma_x$ clipping threshold. The EXIT decoding trajectories are shown in Fig. 3 for $q \in \{4, \ldots, 8\}$. Numerical calculation of the entropy rate of the quantized samples shows that the rate R_s of the source increases with the higher values of q bits. Hence, the output *extrinsic* LLR of the source decoder achieves the higher MI at the lower values of q due to the area property of the EXIT chart [14], as shown in Fig. 3. The optimized degree distributions are given for $q \in \{5, 6, 7\}$ bits in Table I.

The BER performances of the optimized degree distributions under the proposed JSC decoder are demonstrated for $q \in \{5, 6, 7\}$ bits in Fig. 4. We can see that the JSC decoder operates on the BER 10^{-5} at roughly $E_s/N_0 = \{-3.7, -2.5, -1.7\}$ dB values for different q bits, respectively. Since the Shannon limit at the rate $R_c = 0.5$ is 0.19 dB for the sources without redundancy, the JSC decoder approximately improves $\{3.9, 2.7, 1.9\}$ dB, in terms of $\frac{E_s}{N_0}$ values, for $q \in \{5, 6, 7\}$ bits, respectively, compared to the Shannon limit of a decoder at the same rate in which the temporal correlation is not considered. However, the gap between the simulation results and the associated Shannon limits of $q \in \{5, 6, 7\}$ bits are, respectively, $\{1.86, 1.8, 1.8\}$ dB.

Example 2: The Shannon limits and the *decoding thresholds* of optimized codes are derived at different code rates R_c, in the range of $0.2 \le R_c \le 0.9$, with $q \in \{5, 6, 7\}$ bits in both the uniform and Lloyd-Max quantizations of the GM samples. The gap to the Shannon limit is depicted versus channel code rate for both the quantization schemes with $q \in \{5, 6, 7\}$ bits in Fig. 5. The asymptotic performance of the LDPC codes under the JSC decoding is theoretically close to the Shannon limits at the rates in the range of $R_c \le 0.5$. Moreover, the gap to the Shannon limit with the Lloyd-Max quantization is either less than or almost equal to the gap of uniform quantization with the same R_c and q values.

VI. CONCLUSION

We proposed an efficient joint source-channel decoding method in which the temporal correlation of the quantized Gauss-Markov samples is directly utilized to improve the channel coding gain over AWGN channel. The bit-wise representation of the quantized samples are encoded by a binary *irregular* LDPC code. The proposed decoding consists of a sum-product decoder and a multi-state BCJR decoder matched to the trellis diagram corresponding to the temporal correlation of the quantized samples. We optimized the degree distribution of the LDPC codes based on the EXIT function of the proposed decoder. The *decoding thresholds* of the optimized

TABLE I: OPTIMIZED LDPC CODES WITH $R_c = 0.5$ AND $D_v = 100$ FOR THE UNIFORMLY QUANTIZED GM SAMPLES WITH $a = 0.98$, $\sigma_u^2 = 0.1$, AND $q \in \{5, 6, 7\}$ NUMBERS OF QUANTIZATION BITS (Example 1).

	$q = 5$ bit		$q = 6$ bit		$q = 7$ bit	
i	λ_{s_i}	λ_{p_i}	λ_{s_i}	λ_{p_i}	λ_{s_i}	λ_{p_i}
2	0.0594	0.0811	0.0455	0.0871	0.0349	0.0903
3	0.0075	0.0739	0.0465	0.0718	0.0030	0.0914
6		0.0580		0.0454		
7					0.2983	
8			0.2776		0.0512	
9	0.3453					
25	0.0167				0.0072	
26					0.1284	
27				0.0402		
28				0.0059		
100	0.3581		0.3799		0.2955	
ρ_{13}	0.62		0.97		0.76	
ρ_{14}	0.38		0.03		0.24	
$\frac{E_s}{N_0}^*$ (dB)	-5.00		-3.61		-2.77	
$\frac{E_s}{N_0}\|_{\text{lim}}$ (dB)	-5.56		-4.31		-3.49	
gap (dB)	0.56		0.70		0.72	

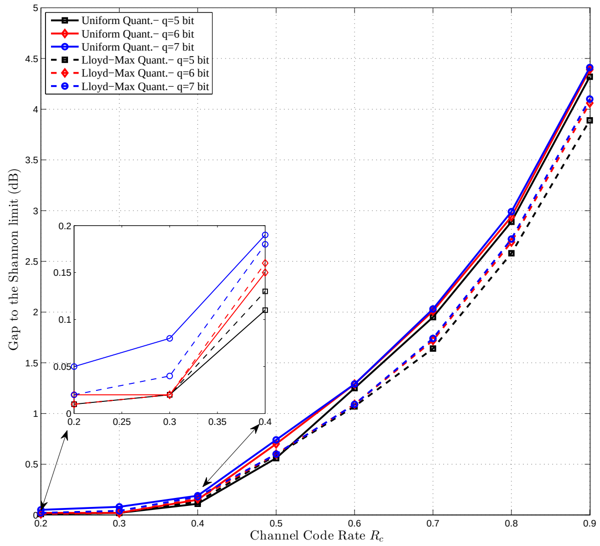

Fig. 5: Gap to the Shannon limit versus channel code rate of the optimized LDPC codes for encoding of the quantized GM samples with $a = 0.98$, $\sigma_u^2 = 0.1$, and $q \in \{5, 6, 7\}$ numbers of quantization bits (Example 2).

codes and the gap to the Shannon theoretical limits were calculated at certain code rates with both the uniform and Lloyd-Max quantization schemes. The gap is close to the Shannon limit at the code rate $R_c \leq 0.5$, but it significantly increases at the higher code rates in both the quantization methods. The gap to the Shannon limit with the Lloyd-Max quantization is either less than or almost equal to that of the uniform quantization at the same code rate and the numbers of quantization bits. Our simulation results have confirmed that the proposed decoder notably increases the coding gain over AWGN channel using the optimized LDPC codes and, hence, the proposed scheme is efficiently suited for applications with strict power limits.

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