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Citation	2013 IEEE Globecom Workshops (GC Wkshps): 807-811
Issue Date	2013-12
Type	Conference Paper
Text version	author
URL	http://hdl.handle.net/10119/12340
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Description	

Outage Based Power Allocation: Slepian-Wolf Relaying Viewpoint

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***Abstract*—Cloud-processing techniques in wireless network are emerging in recent years. Due to the resource energy restriction, power optimization schemes are demanded for designing future transmission strategies. However, it is not realistic to consider the techniques of the whole cloud network having massive amount of nodes. Instead, it is quite reasonable that we first decompose the whole network topology into several typical and simple structures, and identify the most suitable strategies, in general, for them. This work presents an optimal power allocation scheme for a simple Slepian-Wolf relay system over block Rayleigh fading channels. In the assumed decode-and-forward relay model, the information bit sequence obtained as the result of decoding at the relay may contains some errors, however they are highly correlated with the original information bit sequence sent from the source. It is well known that the exploitation of the correlation knowledge between the sequences from the source and relay nodes at the destination achieves significant performance improvement, according to the Slepian-Wolf theorem. In this paper, we obtain an approximated closed-form outage probability expression, based on our previous work for the outage analysis. It is shown that the power allocation for the proposed Slepian-Wolf relay system can be formulated as a convex optimization problem. Specifically, we aim to minimize the outage probability while keeping the total power fixed, and to minimize the total power under an given outage threshold. It is shown that the data obtained from the optimization is very close to the results of numerical calculation.**

I. INTRODUCTION

Improving energy- and spectrum-efficiencies, as a whole network, as well as achieving high reliability and robustness against the network topology change are of crucial importance, when designing future wireless communication networks. As in the distributed source coding (DSC) scenarios, the spatially distributed nodes encode the information data sequence and cooperatively transmit them to a destination. It should be noted that the information sent from different nodes are somehow correlated with each other, and the correlation is depending on the network topology and signaling schemes. According to the Slepian-Wolf theorem [1], the DSC is able to achieve the same compression rate as the case when information data are jointly encoded, by best exploiting the correlation knowledge at the destination.

This research is supported in part by the Japan Society for the Promotion of Science (JSPS) Grant under the Scientific Research KIBAN (B) No. 2360170.

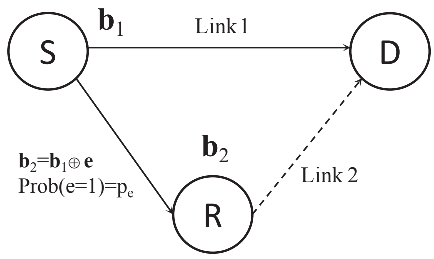

Fig. 1: A simple Slepian-Wolf relay model

In [2], a simple Slepian-Wolf relay system is proposed with decode-and-forward (DF) relay strategy, where the relay only extracts the information part by performing one round of Viterbi decoding of the channel code used, which eliminates heavy computational complexity at the relay. With the conventional DF scheme, the reconstructed information sequence at the relay node are discarded if it is found to contain error(s). With the system proposed in [2], the reconstructed information sequence is interleaved, re-encoded, and forwarded to the destination because the information sequence sent from the relay is highly correlated with the original information sequence sent from the source, even though the sequence reconstructed at the relay may contain some errors. The correlation knowledge is estimated and exploited at the destination node which significantly improve the system performance.

In [3], the outage probability is theoretically derived for the Slepian-Wolf relay system, where the source-destination (SD) link and the relay-destination (RD) link are assumed to suffer from block Rayleigh fading. Instead of performing practical transmission of the source-relay (SR) link, a bit-flipping model [4] is adopted where the correlated binary bit sequence sent from source and relay nodes can be regarded as bit-flipped versions with a flipping probability p_e. The outage probability can be expressed by a double integral with respect to the probability density functions (*pdfs*) of the instantaneous signal-to-noise power ratios (SNRs) of the SD and RD links, given the achievable Slepian-Wolf rate region. However, only numerical method is used in [3] for the calculation of the double integral. It is shown in [3] that the second order diversity can be achieved over entire value range of SNR only when the data bits sent from the source and relay are fully correlated.

This paper provides a closed-form expression of the outage probability of the Slepian-Wolf relay system, by setting the average SNRs of both the SD and RD links to sufficiently large while keeping the power ratio allocated to the source and relay constant. With a constraint that the total transmit power allocated to both the source and relay nodes are constant, it is shown in this paper that the optimal power allocation problem can be formulated as a convex optimization problem based on the asymptotic outage probability expression.

The organization of this paper is as follows. First of all, the Slepian-Wolf relay system is briefly described in Section II. The relationship between the relay system and the Slepian-Wolf theorem is also provided. In III, we derive the outage probability as well as its closed-form approximation using the technique described above. Furthermore, the optimal power allocation problems are formulated in IV, and solutions to the optimization problems are also provided in IV. Finally, this paper is concluded in V with some concluding remarks.

II. System Model

This paper assumes a very simple relay system as shown in Fig. 1. The source node broadcasts the coded original bit sequence $\mathbf{b}_1$ to both the relay and the destination nodes during the first time slot. After that, the relay node aims to recover the original information sequence by performing one round Viterbi decoding [5], then interleave the sequence $\mathbf{b}_2$ obtained as the results of decoding, re-encodes and forwards it to the destination node in the second time slot. The reconstructed information sequence $\mathbf{b}_2$ at the relay may contain some errors with a probability p_e, but still is highly correlated with the original information sequence $\mathbf{b}_1$ sent from the source. It should be noted that we simplify the source-relay link transmission by adopting the bit-flipping mode as $\mathbf{b}_2 = \mathbf{b}_1 \oplus \mathbf{e}$ with $Pr(e = 1) = p_e$, where $\mathbf{e}$ is the random error sequence and e represents its element. The SD and RD links, which are denoted as Link 1 and Link 2, are assumed to suffer from independent block Rayleigh fading, and the distances are assumed to be the same in this paper.

Suppose that two correlated information sequences $\mathbf{b}_1$ and $\mathbf{b}_2$ are transmitted. According to the Slepian-Wolf theorem, when $\mathbf{b}_1$ is transmitted at a rate R_1 which is equal to its entropy $H(\mathbf{b}_1)$, then $\mathbf{b}_2$ can be transmitted at a rate R_2 which is lower than $H(\mathbf{b}_2)$, but the rate R_2 has to be higher than the conditional entropy $H(\mathbf{b}_2 \mid \mathbf{b}_1)$, and vice verse. Specifically, if the three inequalities shown below are satisfied [1],

$$R_1 \geqslant H(\mathbf{b}_1 \mid \mathbf{b}_2), \tag{1}$$

$$R_2 \geqslant H(\mathbf{b}_2 \mid \mathbf{b}_1), \tag{2}$$

$$R_1 + R_2 \geqslant H(\mathbf{b}_1, \mathbf{b}_2), \tag{3}$$

$\mathbf{b}_1$ and $\mathbf{b}_2$ can be recovered with arbitrary small error rate, where $H(\mathbf{b}_1, \mathbf{b}_2)$ denotes the joint entropy of the correlated binary sequences $\mathbf{b}_1$ and $\mathbf{b}_2$. For the binary symmetric sources $(Pr(1) = Pr(0) = 0.5)$ adopted in this paper, we have $H(\mathbf{b}_1) = H(\mathbf{b}_2) = 1$, $H(\mathbf{b}_1 \mid \mathbf{b}_2) = H(\mathbf{b}_2 \mid \mathbf{b}_1) = H(p_e)$, $H(\mathbf{b}_1, \mathbf{b}_2) = 1 + H(p_e)$ with $H(p_e) = -p_e \log_2(p_e) - (1 - p_e) \log_2(1 - p_e)$.

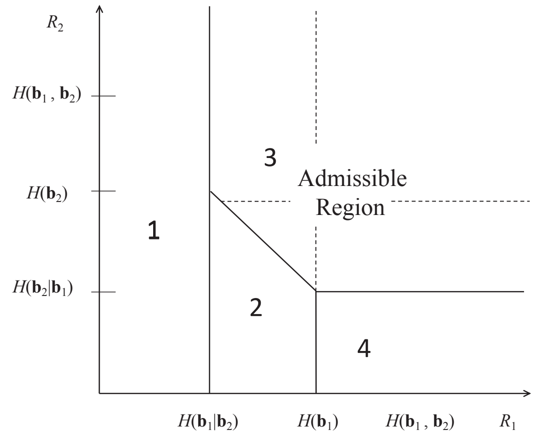

Fig. 2: Slepian-Wolf rate region

III. Outage probability

As shown in Fig. 2, the entire Slepian-Wolf rate region can be divided into 4 areas. Let P_i $(i = 1, 2, 3, 4)$ denote the probability that the rate pair R_1 and R_2 are within Part i of the whole rate region. According to Eq. (1)- (3), Part 3 indicates the admissible rate region while Part 1, Part 2 and Part 3 represent the inadmissible portion.

Unlike the case where two correlated sequences are sent from different users, in this paper, the Slepian-Wolf relay system only aims to recover the original information sequence $\mathbf{b}_1$, with the help of $\mathbf{b}_2$. It is known that $\mathbf{b}_1$ can be correctly detected as long as R_1 is larger than or equal to $H(\mathbf{b}_1)$, regardless of the value of R_2. Thus, Part 4 should also be included as the admissive region. With the mathematical formulations described above, the outage probability is defined as

$$P_{out} = P_1 + P_2. \tag{4}$$

The conditions on R_1 and R_2 to achieve arbitrary low bit error rate can be converted to the constraint of the instantaneous SNRs γ_1 and γ_2. Assuming that Gaussian codebook is used for channel coding, the relationship between the rate and the instantaneous SNRs is given by

$$R_i = \frac{1}{R_{ci}} \log(1 + \gamma_i), \quad i = 1, 2, \tag{5}$$

where R_{ci} represents the spectrum efficiency of the transmission chain of Link i, including the channel coding scheme and the modulation multiplicity. Specifically, P_1 and P_2 can be expressed as

$$\begin{aligned} P_1 =& \Pr\left[0 < R_1 < H(\mathbf{b}_1 \mid \mathbf{b}_2), R_2 > 0\right], \\ =& \Pr\left[0 < \gamma_1 < 2^{R_{c1} H(\mathbf{b}_1|\mathbf{b}_2)} - 1, \gamma_2 > 0\right], \end{aligned} \tag{6}$$

$$
\begin{aligned}
P_2 &= \Pr\left[H(\mathbf{b}_1 \mid \mathbf{b}_2) < R_1 < H(\mathbf{b}_1), R_1 + R_2 < H(\mathbf{b}_1, \mathbf{b}_2)\right], \\
&= \Pr\Big[2^{R_{c1}H(\mathbf{b}_1|\mathbf{b}_2)} - 1 < \gamma_1 < 2^{R_{c1}H(\mathbf{b}_1)} - 1, \\
&\qquad 0 < \gamma_2 < 2^{\left[R_{c2}H(\mathbf{b}_1,\mathbf{b}_2) - \frac{R_{c2}}{R_{c1}}\log(1+\gamma_1)\right]} - 1\Big]. \qquad (7)
\end{aligned}
$$

It is found that the outage probability can be calculated by using a double integral with respect to the joint *pdf* of the instantaneous SNRs $p(\gamma_1, \gamma_2)$ [3], given the range defined in Eq. (6) and (7). Suppose that the fading variance of Link 1 and Link 2 are statistically independent, the joint *pdf* of γ_1 and γ_2 can be expressed as $p(\gamma_1, \gamma_2) = p(\gamma_1)p(\gamma_2)$, where

$$
p(\gamma_i) = \frac{1}{\Gamma_i}\exp(-\frac{\gamma_i}{\Gamma_i}), \quad i = 1, 2, \qquad (8)
$$

with Γ_i denoting the average SNRs of Link i in [6]. In the following parts of this paper, we assume R_{c1} and R_{c2} are fixed to 1 (corresponding to the case where, for example, rate 1/2 channel codes are used with quadrature phase-shift keying (QPSK)). According to [3], P_{out} can be mathematically expressed as

$$
\begin{aligned}
P_{out} =& 1 - \exp\left(-\frac{1}{\Gamma_1}\right) - \exp\left(\frac{1}{\Gamma_2}\right)\frac{1}{\Gamma_1}\cdot \\
&\cdot \int_{2^{H(p_e)}-1}^{1} \exp\left(-\frac{\gamma_1}{\Gamma_1} - \frac{2^{1+H(p_e)}}{\Gamma_2(1+\gamma_1)}\right) d\gamma_1, \qquad (9)
\end{aligned}
$$

Now, let's assume that the average SNRs Γ_1 and Γ_2 are brought to infinity while keeping their ratio fixed. The closed-form expression of the outage can then be obtained as in Eq. (10), by invoking the approximation $\mathrm{e}^{-x} \approx 1 - x$ for a very small x.

$$
P_{out} \approx \frac{1 - C_1}{\Gamma_1} + \frac{C_2}{{\Gamma_1}^2} + \frac{C_3 - C_1}{\Gamma_1\Gamma_2} + \frac{C_2}{{\Gamma_1}^2\Gamma_2} + \frac{C_3}{\Gamma_1{\Gamma_2}^2}, \qquad (10)
$$

where the three constants are defined as $C_1 = 2 - 2^{H(p_e)}$, $C_2 = 2^{H(p_e)} - 2^{2H(p_e)-1}$ and $C_3 = 2^{1+H(p_e)}\left[\ln 2 - \ln 2(2^{H(p_e)})\right]$.

IV. Optimal power allocation

Given the total transmit power E_T fixed, the allocated transmit power of the source and the relay nodes are represented by E_1 and E_2, respectively. Let k $(0 < k < 1)$ be the transmit power ratio, as $E_1 = E_T k$ and $E_2 = E_T(1-k)$. The geometrical gain of both Link 1 and Link 2 are assumed to be 1 without the loss of generality with the system model described in II . By normalizing the noise variance σ_n^2 of both Link 1 and Link 2 to unity, E_1 and E_2 are equivalent to their corresponding average SNRs Γ_1 and Γ_2. Notice that the last two terms in Eq. (10) are negligible with high SNRs, then, the closed-form expression of $P_{out}(k, \Gamma_\mathrm{T})$ can be re-written as

$$
P_{out}(k, \Gamma_\mathrm{T}) \approx \frac{1 - C_1}{E_T k} + \frac{C_2}{E_T^2 k^2} + \frac{C_3 - C_1}{{E_T}^2 k(1-k)}. \qquad (11)
$$

Fig. 3 illustrates good matching of the outage probability curves obtained by using the approximation method Eq. (11) and the numerical calculations using Eq. (9). Moreover, Eq. (11) can be proven to be convex [7], of which proof is detailed in APPENDIX 1.

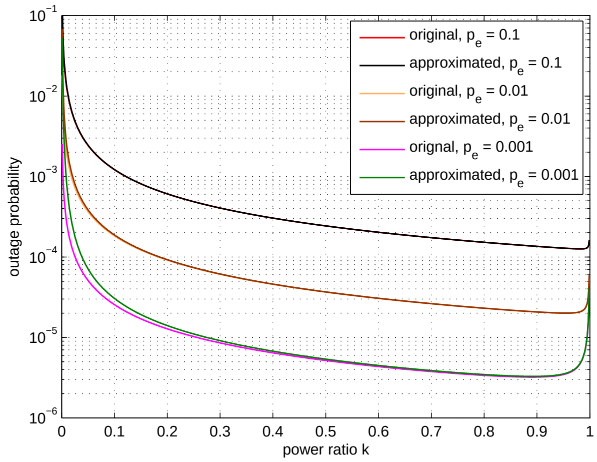

Fig. 3: Comparison of outage curves obtained by using approximation method Eq. (11) and the numerical calculation Eq. (9), when Γ_T = 35 dB.

A. Total power fixed

The goal of this sub-section is to minimize the outage probability while the total power E_T is fixed. The convex problem can be formulated as

$$
\begin{array}{ll}
\text{minimize} & P_{out}(k, \Gamma_\mathrm{T}) \\
\text{subject to} & k - 1 < 0 \\
& -k < 0
\end{array} \qquad (12)
$$

With the help of a convex optimization tool, the optimal values of k can be obtained as shown in TABLE I. Obviously, the larger the p_e value, the more transmit power should be allocated to the source node. Interestingly, it is found also from TABLE I that the optimal power ratio k becomes larger when increasing the total power Γ_T.

TABLE I: Optimal power ratio k

Γ_T(dB)	optimal k (p_e = 0.1)	optimal k (p_e = 0.01)
20	0.8904	0.7865
24	0.9316	0.8519
28	0.9575	0.9015
32	0.9735	0.9360

Fig. 4 presents the simulation results for the frame-error-rate (FER) performance with and without optimal power allocation. In the simulation, we employ the same transmission strategy of a simple one-way relay system with bit-interleaved coded modulation with iterative decoding (BICM-ID) technique presented in [8], where the correlation knowledge is utilized at the destination node. It is found that, by selecting the optimal k values, the Slepian-Wolf relay system can achieve roughly 2 dB gain compared with the cases with equal power allocation.

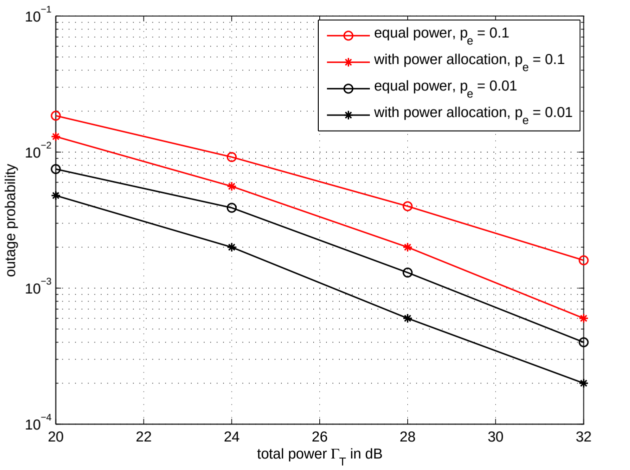

Fig. 4: Comparison of simulated FER with and without power allocation scheme

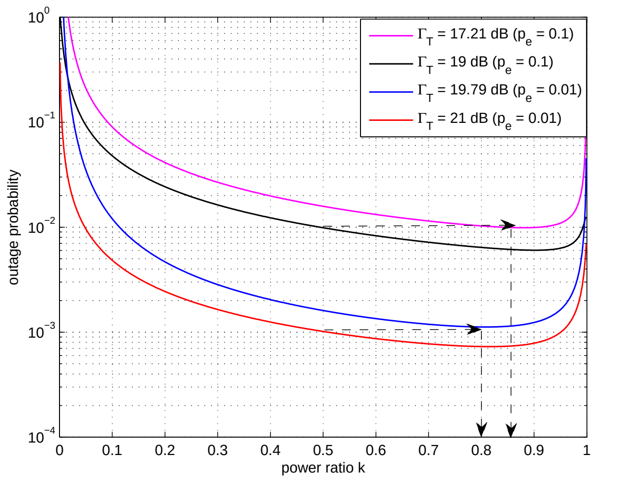

Fig. 5: Theoretical outage probabilities with different total power

B. Outage probability requirement fixed

The goal of this sub-section is to minimize the total transmit power Γ_{T} while keeping the outage probability fixed. We formulate the problem in the following way to find the minimum power as well as its corresponding k, given the outage requirement C_{out}:

$$\begin{array}{ll} \text{minimize} & \Gamma_{\mathrm{T}} + 0k \\ \text{subject to} & P_{out}(k, \Gamma_{\mathrm{T}}) - C_{out} \leqslant 0 \\ & k - 1 < 0 \\ & -k < 0 \\ & -\Gamma_{\mathrm{T}} < 0 \end{array} \tag{13}$$

This problem is proven to be convex, of which proof is detailed in APPENDIX 1. The Karush-Kuhn-Tucker (KKT) conditions [7] corresponding to this problem is shown in APPENDIX 2.

TABLE II: Optimized total power and k

P_{out} requirement	required Γ_{T} (equal power)	required Γ_{T} (optimized)	Gain
0.01 (p_e=0.1)	19 dB	17.21 dB (k=0.85)	1.79 dB
0.001 (p_e=0.01)	21 dB	19.79 dB (k=0.8)	1.21 dB

As shown in TABLE II, for the Slepian-Wolf relay system with equal power allocation scheme and $p_e = 0.1$, we need to allocate 19 dB total power in order to achieve the outage probability $P_{out} = 0.01$. However, by using the optimal $k = 0.85$ obtained as the solution to the optimization problem, it can be reduced to 17.21 dB. Fig. 5 shows the theoretical outage curves obtained by using numerical double integration technique [9], with total power as a parameter. It can be clearly seen that the optimal k values corresponding to p_e and outage requirements of 0.01 and 0.001 are exactly consistent to the the data obtained as the solution to the optimization problem.

V. Conclusion

This paper has provided an approximated closed-form expression of the outage probability for a simple Slepian-Wolf relay system, where the received SNRs Γ_1 and Γ_2 are set large enough while keeping their ratio Γ_1/Γ_2 constant. Based on the closed-form expression, it has been shown that power allocation for minimizing the outage probability while keeping the total transmit power constant, and for minimizing the transmit power while keeping the outage fixed are both formulated as convex optimization problems. It has been shown that the larger the error probability of the SR link, the more power should be allocated to the source node. Moreover, when the total transmit power becomes larger, the optimal power ratio k needs to be increased. The power allocation method presented in this paper can be extended to more complex network topology, such as in the wireless cloud scenarios for more energy savings. This is left as future study.

Acknowledgements

Thank Prof. Mineo Kaneko from Japan advanced institute of science and technology (JAIST) for the guidance and advices of the convex optimization problem.

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APPENDIX 1

The convexity of the approximated outage probability expression Eq. (11) is proven here. It is clear to see that Eq. (11) is comprised of three terms. If each of the three terms can be proven to be convex, Eq. (11) is also convex because it is a sum of the convex terms. The Hessian matrix of the first term $\frac{1-C_1}{k\Gamma_{\mathrm{T}}}$ can be calculated as

$$\mathbf{H}\left[\frac{1-C_1}{\Gamma_{\mathrm{T}}}\right]=\frac{1-C_1}{k^3\Gamma_{\mathrm{T}}^3}\begin{bmatrix}2\Gamma_{\mathrm{T}}^2 & k\Gamma_{\mathrm{T}}\\ k\Gamma_{\mathrm{T}} & 2k^2\end{bmatrix}. \tag{14}$$

Since $1-C_1 = 2^{H(p_e)-1 \geq 0}$, the eigenvalues $\lambda_{1,2} = 0.5\frac{1-C_1}{k^3\Gamma_{\mathrm{T}}^3}\left(\sqrt{k^2+\Gamma_{\mathrm{T}}^2}-\sqrt{k^2+\Gamma_{\mathrm{T}}^2-3k^2\Gamma_{\mathrm{T}}^2}\right)$ are clearly non-negative. Therefore, the Hessian matrix of $\left[\frac{1-C_1}{\Gamma_{\mathrm{T}}}\right]$ is positive semi-definite and hence its convexity has been proven.

The Hessian matrix of the second term $\frac{C_2}{k^2\Gamma_{\mathrm{T}}^2}$ can be calculated as

$$\mathbf{H}\left[\frac{C_2}{\Gamma_{\mathrm{T}}}\right]=\frac{2C_2}{k^4\Gamma_{\mathrm{T}}^4}\begin{bmatrix}3\Gamma_{\mathrm{T}}^2 & 2k\Gamma_{\mathrm{T}}\\ 2k\Gamma_{\mathrm{T}} & 3k^2\end{bmatrix}. \tag{15}$$

Since $C_2 = 2^{H(p)} - 2^{2H(p)=1} \geq 0$, the eigenvalues $\lambda_{1,2} = \frac{C_2}{k^4\Gamma_{\mathrm{T}}^4}\left(\sqrt{6k^2+3\Gamma_{\mathrm{T}}^2}-\sqrt{6k^2+3\Gamma_{\mathrm{T}}^2-56k^2\Gamma_{\mathrm{T}}^2}\right)$ are clearly non-negative. Therefore, the Hessian matrix of $\frac{C_2}{k^2\Gamma_{\mathrm{T}}^2}$ is positive semi-definite and hence its convexity has been proven.

The Hessian matrix of the third $\frac{C_3-C_1}{\Gamma_{\mathrm{T}}^2 k(k-1)}$ can be calculated as

$$\begin{aligned}&\mathbf{H}\left[\frac{C_3-C_1}{{\Gamma_{\mathrm{T}}}^2 k(1-k)}\right]\\ &=\frac{2(C_3-C_1)}{k^3(1-k)^3{\Gamma_{\mathrm{T}}}^4}\begin{bmatrix}k^2{\Gamma_{\mathrm{T}}}^2-k{\Gamma_{\mathrm{T}}}^2+{\Gamma_{\mathrm{T}}}^2 & k\Gamma_{\mathrm{T}}-k\Gamma_{\mathrm{T}}\\ k\Gamma_{\mathrm{T}}-k\Gamma_{\mathrm{T}} & 3k^4-6k^3+3k^2\end{bmatrix},\end{aligned} \tag{16}$$

Let $m(k) = 3k^2(k-1)^2 + {\Gamma_{\mathrm{T}}}^2(k^2+1-k)$ and $n(k) = 3k^4-9k^3+11k^2-7k+2$, the eigenvalues $\lambda_{1,2} = \frac{C_3-C_1}{k^3(1-k)^3\Gamma_{\mathrm{T}}^4}\left(m-\sqrt{m^2-4k^2\Gamma_{\mathrm{T}}^2 n}\right)$. For $0<k<1$, obviously, $m(k)>0$. Since that $n(k)^{''} = 36k^2-54k+22>0$, $n(k)^{'}$ is proven to be monotonically increasing until the boundary $n(k)^{'} < n(k=1) = 0$, and furthermore $n(k)^{'}<0$ indicates $n(k)$ is monotonically decreasing until the boundary $n(k) > n(k=1) = 0$. This proves the non-negativity of n(k).

Let $H(p_e) = x$, and $0 \leq x \leq 1$, $y = C_3 - C_1 = 2^x\left[2\ln 2 - 2\ln(2^x)+1\right]-2$. Due to the fact that

$$\begin{aligned}y^{''} &= 2^x \ln 2\left[\ln 2\left(2\ln 2 - 2\ln 2^x + 1\right) - 4\right]\\ &< 2^x \ln 2\left[\ln 2\left(2\ln 2 - 2\ln 2^0 + 1\right) - 4\right]\\ &< 0,\end{aligned} \tag{17}$$

y is concave and $C_3 - C_1 > min(y(0), y(1)) = 0$. Therefore, the Hessian matrix of $\frac{C_3-C_1}{\Gamma_{\mathrm{T}}^2 k(k-1)}$ is proven to be positive semi-definite and hence its convexity has been proven.

APPENDIX 2

The KKT condition for the optimization problem presented in sub-section IV-B is summarized below:

$$\begin{aligned}&P_{out}(k,\Gamma_{\mathrm{T}}) - C_{out} \leqslant 0\\ &k-1<0\\ &-k<0\\ &-\Gamma_{\mathrm{T}}<0\\ &\lambda_1 \geq 0\\ &1+\lambda_1\frac{\partial P_{out}(k,\Gamma_{\mathrm{T}})}{\partial \Gamma_{\mathrm{T}}} = 0\\ &\lambda_1\frac{\partial P_{out}(k,\Gamma_{\mathrm{T}})}{\partial k} = 0\end{aligned} \tag{18}$$

The formulation and notations are all consistent to [7], where $f_0 = \Gamma_{\mathrm{T}} + 0k$, $f_1 = P_{out}(k,\Gamma_{\mathrm{T}}) - C_{out}$, $f_2 = k-1$, $f_3 = -k$, $f_4 = -\Gamma_{\mathrm{T}}$ and they are all differentiable.