Quantifying Engagement of Various Games

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***Abstract*—Game refinement idea is a unique theory that has been proposed based on the uncertainty of game outcome. A game refinement measure was derived from the game information progress model and has been applied in the traditional board games. The present challenge is to apply the game refinement theory in the domain of various games such as RTS(StarCraft II), MOBA(DotA), crane game and score limited sports. In conclusion, this paper makes contribution to apply game refinement theory in these new areas and supports the effectiveness of game refinement theory. This theory can be widely applied to various type of games to assess the entertainment impact of target games. Keywords: Game refinement theory, StarCraft II, DotA, crane game, score limited sports**

I. Introduction

Many efforts have been devoted to the study of strategic decision making in the framework of game theory with focus on mathematical models of conflict and cooperation between intelligent rational decision-makers or game-players. Game theory originated in the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games [2], which has been widely recognized as a useful tool in many fields such as economics, political science, psychology, logic and biology.

However, little is known about mathematical theory from the game creator's point of view. An early work in this direction has been done by Iida *et al.* [1], in which a measure of game refinement was proposed based on the concept of game outcome uncertainty. A logistic model was constructed in the framework of game refinement theory and applied to many board games including chess variants.

In this paper, first we sketch the basis of game refinement theory and then show some applications to various games. We thus support the effectiveness of game refinement thory and these various games are compared using game refinement values.

II. Game Refinement Theory

A. Mathematical model of game refinement

Recently a general model of game refinement was proposed based on the concept of game progress and game information progress [3]. It bridges a gap between board games such as chess and sports games such as soccer and basketball.

Game information progress presents how certain is the result of the game in a certain time or steps. Let G and T be the average number of successful shoots and the average number of shoots per game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation (5).

$$x(t) = \frac{G}{T}\ t \tag{1}$$

However, the game information progress given by Equation (5) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is given by Equation (2).

$$x(t) = G(\frac{t}{T})^n \tag{2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Then acceleration of game information progress is obtained by deriving Equation (2) twice. Solving it at $t = T$, the equation becomes

$$x''(T) = \frac{Gn(n-1)}{T^n}t^{n-2} = \frac{G}{T^2}n(n-1)$$

It is assumed in the current model that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in our minds, but it is likely that the acceleration of information progress is related to the force in mind. Hence, it is reasonably expected that the larger the value $\frac{G}{T^2}$ is, the more the game becomes exciting due to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{G}}{T}$, as a game refinement measure for the game considered.

B. Game progress model and board games

Here we consider the gap between board games and sports games by deriving a formula to calculate the game information progress of board games. Let B and D be average branching factor (number of possible options) and game length (depth of whole game tree), respectively. One round in board games can be illustrated as decision tree. At each depth of the game tree, one will choose a move and the game will progress. Figure 1 illustrates one level of game tree. The distance d, which has been shown in Figure 1, can be found by using simple Pythagoras theorem, thus resulting in $d = \sqrt{\Delta l^2 + 1}$.

Assuming that the approximate value of horizontal difference between nodes is $\frac{B}{2}$, then we can make a substitution and get $d = \sqrt{(\frac{B}{2})^2 + 1}$. The game progress for one game is the total level of game tree times d. For the meantime, we do not consider Δt^2 because the value ($\Delta t^2 = 1$) is assumed

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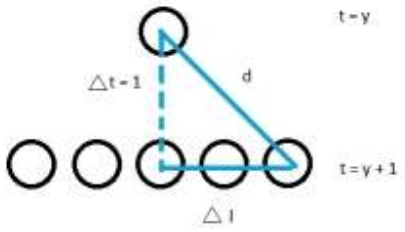

Fig. 1. Illustration of one level of game tree

to be much smaller compared to B. The game length will be normalized by the average game length D, then the game progress $x(t)$ is given by $x(t) = \frac{t}{D} \cdot d = \frac{t}{D}\sqrt{(\frac{B}{2})^2} = \frac{Bt}{2D}$. Then, in general we have, $x(t) = c\frac{B}{D}t$, where c is a different constant which depends on the game considered. However, we manage to explain how to obtain the game information progress value itself. The game progress in the domain of board games forms a linear graph with the maximum value $x(t)$ of B. Assuming $c = 1$, then we have a realistic game progress model for board games, which is given by

$$x(t) = B(\frac{t}{D})^n. \quad (3)$$

Equation (3) shows that the game progress in board games corresponds to that of sports games as shown in Equation (2).

To support the effectiveness of proposed game refinement measures, some data of games such as Chess and Go [4] from board games and two sports games [3] are compared. We show, in Table VI, a comparison of game refinement measures for various type of games. From Table VI, we see that sophisticated games have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games. Note that average branching factor B and game length D instead of G and T can be used in the board game case [3].

III. Further Investigation with Various Games

In this study, we show further investigation in the domains of sore-limited sports games such as Badminton and Table Tennis, Arcade games such as UFO catcher, MOBA games such as DotA, RTS games such as StarCraft II.

To support the effectiveness of proposed game refinement measures, we show, in Table VI, game refinement measures for various games. We see that sophisticated games have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games. In the every subsection, we will show how to apply game refinement theory in different game areas and types. For the body game, we will use the function of $\frac{\sqrt{G}}{T}$ (G means average number of successful shoots and T means the average number of shoots per game); while a game belong to brain game, then we should use the model of $\frac{\sqrt{B}}{D}$ (B means average branch fact and D means the depth of the game)for reference. As we know, the physical formula momentum theorem $I = mv = Ft$, the format of I is the same, but the meaning of mv and Ft is quite different.

TABLE I
Measures of game refinement for board games and sports games

Game	G	T	R
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073
Badminton	46.336	79.344	0.086
Table tennis	54.863	96.465	0.077
DotA ver 6.80	68.6	106.2	0.078
UFO catcher	0.967	13.30	0.074
StarCraft II Terran	1.64	16	0.0805

A. Score limited sport

The sports game can be divided into two types, Score limit game such as tennis and badminton, Time limit game such as basketball and soccer.In a score-based game, the measure of refinement was proposed based on the information gained from the game and the average game length. So we choose the formula for body game and redefine the G and T. Because the score limit game full length depend on the winner player achieve the goal points and plus the points which the loser got, so the T stands for the total score of the entire game. In time limit game such as soccer, representation of successful shoots is POINT or SCORE, as same as time limit game, G stands for the total score successfully made by the winning side. According to the meaning of Equation 1, we can take the example of badminton. In recent years, the rules of badminton had been changed by serval times, depend on the rules, competition data can be corrected and calculate the game refinement value of badminton as Table II shows[7].

TABLE II
Measure of game refinement for Badminton

Scoring system	Winning score (G)	Total score (T)	R
Old	30.070	45.145	0.121
Current	46.336	79.344	0.086

B. MOBA game: DotA

Multi-player Online Battle Arena(MOBA) game , in which a player controls a single character at one of two teams. The objective is to destroy the opposing team's main structure with the assistance of periodically spawned computer-controlled units that march forward along set paths[8]. Player characters typically have various abilities and advantages that improve over the course of a game and that contribute to a team's overall strategy. Usually, the behind side will input GG (good game) when they find that there is no hope to win, which means that they give up and quit the game. The map of DotA as the Figure 2 shows

DotA pits two teams of players against each other: the Sentinel and the Scurge. Players on the Sentinel team are based at the southwest corner of the map, and those on the Scourge team are based at the northeast corner. Each base is defended by towers and waves of units which guard the main paths leading to their base. In the center of each base is the "Ancient", a building that must be destroyed to win the game

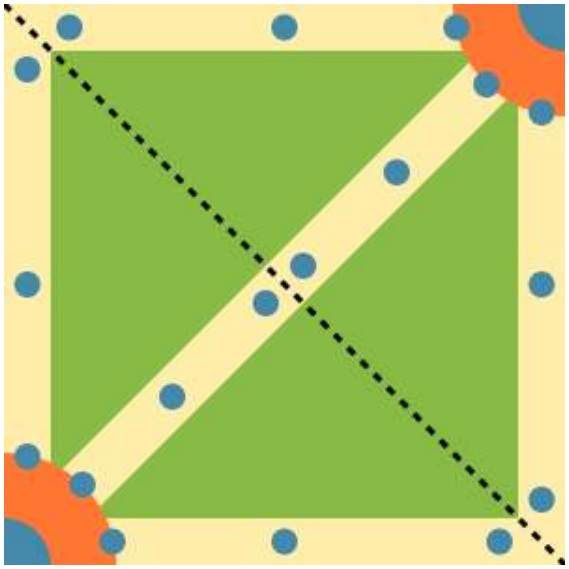

Fig. 2. Map of MOBA

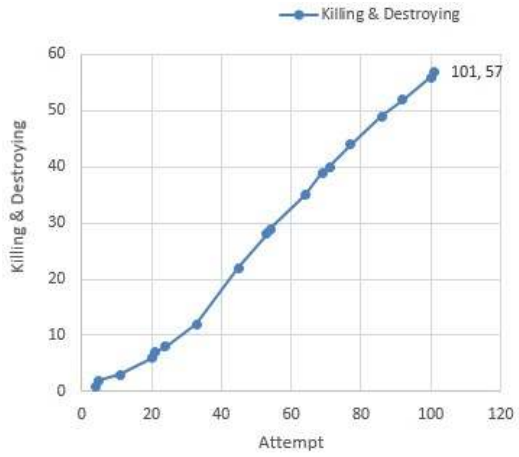

Fig. 3. Game progress of a replay on DotA ver. 6.74

TABLE III
MEASURES OF GAME REFINEMENT FOR HISTORICAL VERSIONS OF DOTA

version	released	K & D	A	R-value
6.48	Aug 2007	69.2	110.8	0.075
6.51	Mar 2008	68.4	110.2	0.074
6.59	Jan 2009	69.8	110.0	0.076
6.61	Aug 2009	70.0	111.6	0.075
6.64	Oct 2009	68.4	110.4	0.075
6.69	Oct 2010	67.8	108.4	0.076
6.74	Mar 2012	62.4	102.6	0.077
6.77	Dec 2012	62.8	102.8	0.077
6.80	Mar 2014	68.6	106.2	0.078

Each human player controls one Hero, a powerful unit with unique abilities. In Allstars, players on each side choose one of over one hundred different heroes, each with different abilities and tactical advantages over other heroes. The scenario is highly team-oriented; it is difficult for one player to carry the team to victory alone. Nevertheless, some heroes, given enough time, can change the outcome single-handedly, while countering the opposing team's heroes. Defense of the Ancients allows up to ten players in a five versus five format and an additional two slots for referees or observers, often with an equal number of players on each side.

Because the gameplay revolves around strengthening individual heroes, it does not require one to focus on resource management and base-building, as in most traditional real-time strategy games. Killing computer-controlled or neutral units earns the player experience points; when enough experience is accumulated, the player gains a level. Leveling up improves the hero's toughness and the damage it can inflict, and allows players to upgrade their spells or skills. In addition to accumulating experience, players also manage a single resource: gold. The typical resource gathering of Warcraft III is replaced by a combat-oriented money system; in addition to a small periodic income, heroes earn gold by killing hostile units, base structures, and enemy heroes. Using gold, players buy items to strengthen their hero and gain abilities. Certain items can be combined with recipes to create more powerful items. Buying items that suit one's hero is an important tactical element of the scenario.

Allstars offers a variety of game modes, selected by the game host at the beginning of the match. The game modes dictate the difficulty of the scenario, as well as whether people can choose their hero or are assigned one randomly. Many game modes can be combined (for example, an easy difficulty level and a random hero pick), allowing more flexible options[9].

We consider DotA's game progress. Although DotA or LOL belong to e-sports game, essentially access the body game, so we also can simulate the $\frac{\sqrt{G}}{T}$. It can be measured by two factors: to kill heroes and to destroy towers. Let K and A be the average number of successful killing heroes and destroying towers, and the average number of attempts per game, respectively. If one knows the game information progress, for example after the game, the game progress x(t) will be given by Equation 4.

$$x(t) = \frac{K}{A}\ t \tag{4}$$

Similarly, we have the game refinement formula

$$R = \frac{\sqrt{K}}{A} \tag{5}$$

According some auxiliary software, we can correct the data of DotA as Figure 3 shows

We download five replays of each version on website. The players in the game are all expert players in order to make the data more objective and reasonable. A software called replay manager is used for this study to collect the data of killing and the destroyed towers of each game. The attempt is counted by watching replays. We show, in Table III, the results of different DotA versions using game refinement measure by computer system. We played the related versions with other players on platform and collected five replays of each related version.

C. RTS game: StarCraft II

StarCraft II, which is one of the most popular RTS games.. In typical RTS games, players build armies and vie for control of the battlefield. The armies in play can be as small as a single squad of Marines or as large as a full-blown planetary invasion force. As commander, one observes the battlefield from a top-down perspective and issue orders to one's own units in real time. Strategic thinking is key to success. Players need to gather information about the opponents, anticipate their moves, outflank their attacks, and formulate a winning strategy. StarCraft II features three distinct races whose armies comprise entirely unique units and structures. Each race has its own strengths and weaknesses, and knowing their tactical profiles can mean the difference between glorious victory or crushing defeat[6].

Our present study focuses on StarCraft II which is a RTS game where the player's goal is to destroy their enemy's base by developing their own base and an army. In StarCraft II players cannot see their opponent's situation and they have the same power, StarCraft II does not rely on any chance. Therefore, in a sense this game is similar with board games such as chess. It means that we can use some similar tools $\frac{\sqrt{B}}{D}$ to analyze the game of StarCraft II.

According to the game features of StarCraft II, we should divide the game into four part: Opening, Mid-prophase, Mid-anaphase and Endgame. The game could finish in any time domain. For example, while players choose supervise attack or extremely rush strategy, the game must finish in 7 or 8 minutes or before; Normally, the average game time is 15 to 20 minutes (it means the most games will not enter into Mid-anaphase or Endgame time domain)[6]. As our experience, we find the game in different time domain, the **main elements** are completely disparate!

TABLE IV
FEATURE OF STARCRAFT II IN EVERY PROCESS

Domain	Timing	Character
Opening	0 to 10 minutes	Strategy
Mid-prophase	10 to 20 minutes	Economy and Management
Mid-anaphase	20 to 30 minutes	Economy and Operation
Endgame	Over 30 minutes	operation

In the opening, the StarCraft II is similar to real war or traditional board games. In other words, only in opening time domain, StarCraft II is an intellectual game. While a game enters into Mid-prophase or Mid-anaphase, the main elements are economy, management and operation, it means that in mid-game, the StarCraft II is similar to the Simulation Game! As we know, a good chess player not always can be a good manager, a strategy genius does not mean that he could be a nice executive.

For the endgame, the operation element will be more and more important, even occupy all the StarCraft II process. It means that on that time StarCraft II is similar to Super Mario. When we watch somebody playing Super Mario, we rarely focus on his intellectual strategy, we only focus on whether or not his operation skill is proficient. In this situation, StarCraft II is like sports games such as soccer and basketball.

Fig. 4. Feature of StarCraft II

According to the above, only in the opening stage, we have the strategy tree, and then find the B and D. Also in the opening stage, the game is highly similar to traditional board games or brain sports, we can take example by game tree model to establish new mathematical model. If we want to research mid-game or end game, we must find other model or method. At least, the meaning of B and D must be changed. Actually, the completion between profession players, the most exciting and wonderful part is mid-game.

Since StarCraft II is a RTS game, its minimax tree[5] cannot be built in a normal way. For example, the depth of tree is defined by each step or turn, while in Starcraft II, the depth might be given by time evolution. We show, in Figure 5, such an example. In Figure 5, we notice that the child node "Tokyo" and child node "Shanghai" have the different depth. This situation would never happen in traditional board games to build a minimax search tree. So we consider one method to solve it, while changing an unbalance depth tree into a balance tree. While add the temporary node, then we get another strategy tree of as shown in Figure 6.

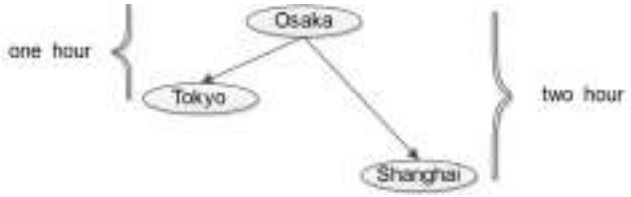

Fig. 5. An example of strategy tree with two unbalanced child nodes

Fig. 6. The strategy tree with temporary node

Then the game refinement value is calculated, as shown in Table V.

D. Crane game: UFO Catcher

Final one is the crane game, which is a type of arcade games, it is very popular among most people around the world, especially teenagers.

TABLE V
MEASURE OF GAME REFINEMENT FOR THREE RACES IN STARCRAFT II

Race	all nodes	all parent nodes	B	D	R-value
Terran	126	76	1.64	16	0.0805
Zerg	219	141	1.54	18	0.0692
Zerg*	564	210	1.61	20	0.0819
Protoss	116	74	1.55	18	0.0691

Over a long history of crane games, the gaes have been changed and modified and become one of the most favored arcade games. There are many variants of crane games regarding different machine models, different types of mechanical arm: 2-clawed and 3-clawed, and control panels. One of the variants, a popular Japanese crane games, so called SEGA UFO Catcher (see Figure 7) is used to measure game refinement value in the experiment. Most of the time, the machine is typically filled with prizes such as plush toys, dolls, snacks, etc.

In order to win prizes, players have to manipulate the mechanical arm inside the machine by using a joy stick, which controls the claw to a desired position. The crane starts at the corner of the machine. When a player inserts a coin, the joy stick is activated for a limited period of time. Once the time is up or a "drop" button is pressed, the crane drops to a certain depth and then raises, closing its claw on the way and returning to the drop hatch in the corner. Thus the player is required to judge position accurately in one attempt. Crane game is a game of skill and strategy. Skillful players have their own techniques to win prizes within a few attempts. However, the success rate also depends on other several factors, for instance, strength of the claw, size and weight of prizes, operator settings, and so forth.

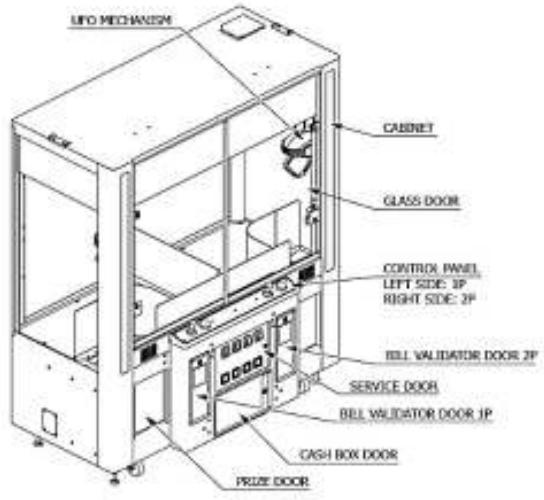

Fig. 7. Part description of UFO Catcher

From the game characteristic which is coin-operated, the playing cost is one of the factors which affects player's enjoyment in game. We propose c as a cost per each attempt normalized by the average cost per attempt of each country, since the playing costs are varied for different machines. Let P and T be average number of prizes captured, and average number of attempts, respectively. Similarly as the Section 1 wrote, we have the function:

$$x(t) = \frac{P}{cT}\ t \tag{6}$$

A model of game information progress for crane game is given by Equation (7).

$$x(t) = P(\frac{t}{cT})^n \tag{7}$$

Here n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Then acceleration of game information progress is obtained by deriving Equation (7) twice. Solving it at $t = cT$, the equation becomes

$$x''(cT) = \frac{Pn(n-1)}{c^n T^n} t^{n-2} = \frac{P}{c^2 T^2} n(n-1)$$

then expect $\frac{P}{c^2T^2}$ or its root square $\frac{\sqrt{P}}{cT}$ to be a game refinement measure for crane games. Consequently, we suppose that the larger the game refinement value is, the more attractive the game becomes. An experiment has been preliminarily carried out by observing crane game players in amusement centers in different countries. We collected data of 30 and 60 game samples from Japan and Thailand respectively. Then game refinement theory was applied. The results of the experiments are compared in Table VI.

TABLE VI
MEASURES OF GAME REFINEMENT FOR CRANE GAME

country	P	T	R-value
Japan	0.967	13.13	0.075
Thailand	0.367	10.65	0.057

IV. CONCLUSION

In this paper we have presented some applicatoins to various types of games. It shows that game refinement theory can succussfully be used in every type of games with its appropriate model of game information progress. It can be a usuful tool to enable game designers to make a target game more sophisticated. As a tentative conclusion, we observed that any kind of attractive games would have the similar zone value (say $0.07 - 0.08$) of game refinement.

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