Joint Turbo Equalization and BICM-ID-based IDMA over Frequency Selective Fading Channels

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***Abstract*—This paper proposes a joint turbo equalization and bit interleaved coded modulation with iterative detection-based interleave division multiple access (IDMA) technique over frequency selective fading channels. The transmission chain's parameters are optimized by using extrinsic information transfer-constrained Binary Switching Algorithm at a very low signal-to-noise power ratio range. A frequency domain turbo equalization is used together with IDMA signal detection to deal with all the simultaneous users. Simulation results show that the proposed technique can eliminate both intersymbol interference and multiple access interference, and achieve the excellent frame error rate performance in the cases of single user and 8 users, as well as of 10 users although the user number is larger than its equivalent spreading factor. Furthermore, we also propose a detection ordering technique to improve the efficiency of the detection scheme.**

I. Introduction

Nowadays, non-orthogonal Multiple Access (NOMA) has attracted more and more attention in wireless spread spectrum communications, since the NOMA outperforms over orthogonal signaling techniques in term of spectral efficiency advantage [1]. Interleave division multiple access (IDMA) is a new NOMA-based multiple access techniques, of which the original idea is inspired by [2] and proposed in [3]. After that, the IDMA concept was reformulated and introduced in [4], [5] and [6]. In IDMA, since the bandwidth is fully utilized for channel coding, very low rate code achieving near-capacity performance is needed. Recently, an idea of using single parity check and irregular repetition (SPC-IrR) codes in bit interleaved coded modulation with iterative detection (BICM-ID) scheme proposed in [7], and is found to be very suitable for designing very low rate code achieving near-capacity performance. In [7], the transmission chain's parameters are optimized in a systematic way by the extrinsic information transfer (EXIT)-constrained binary switching algorithm (EBSA) at a very low signal-to-noise power ratio (SNR) range in additive white gaussian noise (AWGN) channel. The optimized BICM-ID in IDMA systems has already been investigated in [8], where the excellent performances are shown via the convergence and rate region analyses.

In this paper, we propose a joint frequency domain turbo equalization [9] and IDMA signal detection technique based on the BICM-ID-based IDMA proposed in [8]. A frequency domain soft-cancelation minimum mean square error (FD-SC-MMSE) turbo equalization is considered as a compelling technique which well-performs equalizers without requiring excessive computational complexity. The joint use of turbo equalization and IDMA signal detection makes the BICM-ID-based IDMA system possible in fading channels. We determine the transmission chain's parameters by the EBSA, and investigate the performance of the system over frequency selective fading channels. It is shown that the BICM-ID optimized by EBSA is also effective in achieving excellent performance when applied to IDMA in fading channels; the proposed system can accommodate more users than its equivalent spreading factor.

In the soft-interference cancelation based multiuser detection (MUD), detection ordering (DO) is one of the important factors which impacts detection efficiency. This paper also proposes a DO technique which determines the detection order of simultaneous users. Results of simulations conducted to make comparison of the frame error rate (FER) performance between the systems with and without DO are presented. It is shown that the

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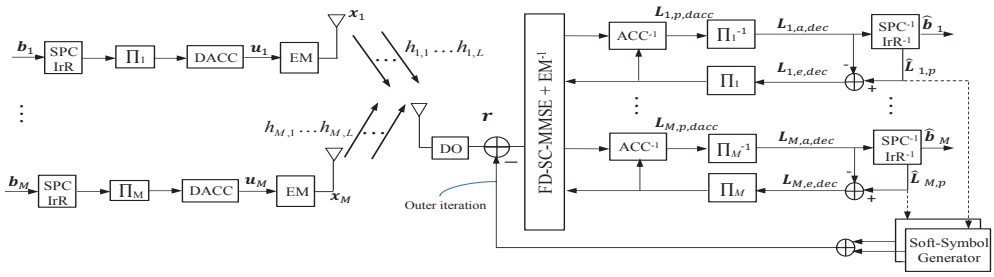

Fig. 1. A schematic diagram for the proposed joint turbo equalization and BICM-ID-based IDMA system.

detection process can be performed more efficiently with DO over without DO.

This paper is organized as follows: The schematic diagram is described in Section II. The joint turbo equalization and IDMA signal detection is elaborated in Section III. The algorithm of DO is presented in Section IV. The EXIT chart and numerical results are shown in Section V. Finally Section VI concludes this paper.

II. System Model

A schematic diagram of the BICM-ID-based IDMA system is depicted in Fig. 1. Each user uses the same BICM transmission chain, where the bit sequence $\mathbf{b}_m$ of m-th user is firstly encoded by the SPC-IrR encoder with parameters d_c, d_v, a, and then interleaved by interleaver Π_m, after that, in doped accumulator (DACC), doped-accumulated [7] with doping ratio p [10] and output a new bits sequence $\mathbf{u}_m$. The output $\mathbf{u}_m$ are mapped on to a 4-QAM[1] signal point, in part, according to the labeling pattern for extended mapping (EM) determined by EBSA, and in part, according to the non-Gray labeling pattern [11] to generate transmission symbols $\mathbf{x}_m$, with modulation mixing ratio D. The parameters of the transmission chain are shown in the EXIT chart in section V.

The modulated symbol sequence $\mathbf{x}_m$ is then transmitted over frequency selective fading channels. l denotes the channel multipath index, $l \in \{1, \cdots, L\}$ with L being the number of the multipath. The fading channel gains are assumed to be constant during one block interval, but vary block-by-block. Let $\mathbf{H}_m$ denote the equivalent block-wise representation of the channel matrix for m-th user. Cyclic prefix (CP) transmission is also assumed in this paper. When CP is appended at the transmitter side and eliminated at the receiver side, the equivalent channel matrix $\mathbf{H}_m$ becomes circulant matrix $\mathbf{H}_m^c$ in multipath channels.

[1]The proposed technique is also applicable with high-order modulation to meet the current trend requirement of mobile terminals.

Then, the equivalent frequency domain channel matrix Ξ can be obtained by utilizing the property of the circularity of matrix $\mathbf{H}_m^c$, expressed as

$$\Xi = \mathbf{F}^H \mathbf{H}_m^c \mathbf{F}, \tag{1}$$

where the Fourier matrix $\mathbf{F} \in \mathbb{C}^{N \times N}$ has each element defined as $\mathbf{F}_{i,j} = K^{-\frac{1}{2}} e^{-\jmath \frac{2\pi}{K}(i-1)(j-1)}, \jmath = \sqrt{-1}, i, j = 1, \cdots, K$, with K being the block length.

When the block of symbol sequences $\mathbf{x}_m$ pass through the channels, the received signal $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \sum_{m=1}^{M} \sqrt{P_m} \cdot \mathbf{H}_m^c \cdot \mathbf{x}_m + \mathbf{n}, \tag{2}$$

where P_m and $\mathbf{n}$ denote the power of the m-th user and the AWGN component with variance σ_n^2, respectively.

III. Joint Turbo Equalization and IDMA Signal Detection

The structures and principles for DACC decoder and SPC-IrR channel decoder have been presented in detail [7] [10]. The frequency domain algorithm of FD-SC-MMSE equalizer is provided in [9]. In this paper, we derive the joint utilization of turbo equalization and IDMA signal detection.

The joint turbo equalization and IDMA signal detection consists of a FD-SC-MMSE equalizer and SPC-IrR soft-in-soft-out channel decoders. There are two types of iterations, one is inner iteration which happens between equalizer and channel decoder for each user, the other one is outer iteration which happens between different users. The outer iteration is activated once all users finish one round specific times inner iterations. In this paper, perfect channel knowledge is assumed.

A. Soft-interference Cancelation

Since the interference from the other users can be eliminated by performing soft interference cancelation, the soft symbol and the variance of the soft symbol, $\hat{x}_{m,k}$ and $\sigma_{m,k}^2$, are updated every outer iteration.

The $\hat{x}_{m,k}$ and $\sigma^2_{m,k}$ at timing index k are updated by

$$\hat{x}_{m,k} = \sum_{s\in S} s \prod_{\varpi=1}^{lmap} P(b_{m,\varpi} = \mp 1), \quad (3)$$

$$\sigma^2_{m,k} = 1 - |\hat{x}_{m,k}|^2, \quad (4)$$

with

$$P(b_{m,\varpi} = W) = \frac{e^{-b_{m,\varpi}\hat{L}_{p,m}}}{1 + e^{-\hat{L}_{p,m}}}, \quad (5)$$

where $W \in \{0, 1\}$ and ϖ is bit index of EM label. $lmap$ is the parameter of EM, $lmap = 4$ when one constellation point represents 4 labeling patterns. S is a set of constellation points. $\hat{L}_{m,p}$ denotes the *a posteriori* log likelihood ratio (LLR) fed back via the outer iteration to generate the soft symbol replica, defined as $\hat{L}_{m,p} = L_{m,a,dec} + L_{m,e,dec} + L_{m,p,dacc}$.

Before the first outer iteration is activated, the value of $\hat{x}_m$ and σ^2_m are initialized, as $\hat{x}_{m,k} = 0$, $\sigma^2_{m,k} = 1$. The residual of the intersymbol interference (ISI) $\tilde{\mathbf{r}}$ are updated every outer iteration by

$$\tilde{\mathbf{r}} = \mathbf{r} - \textstyle\sum_{m=1}^{M} \sqrt{P_m} \cdot \mathbf{H}^c_m \cdot \quad \hat{\mathbf{x}}_m, \quad (6)$$

where $\hat{\mathbf{x}}_m = [\hat{x}_{m,1}, \hat{x}_{m,2}, ..., \hat{x}_{m,k}, ..., \hat{x}_{m,K}]$. K is the length of soft symbol.

The corresponding variance of ISI, $\hat{\sigma}^2_m$, after the soft cancelation via the outer iteration is given by

$$\hat{\sigma}^2_m = \textstyle\sum_{g\neq m, g=1}^{M} E_g \cdot P_g \cdot \sigma^2_g + \sigma^2_n. \quad (7)$$

with total channel power of g-th user

$$E_g = \textstyle\sum_{l=1}^{L} |h_{g,l}|^2. \quad (8)$$

B. MMSE Filter

After soft cancelation, the ISI residual $\tilde{\mathbf{r}}$ goes to MMSE filter. In this frequency domain equalizer, the output vector of the MMSE estimates of the transmitted symbols for m-th user, can be expressed as

$$\mathbf{Z}_m = (1 + \bar{\gamma}_m \cdot \bar{\delta}_m)^{-1} \cdot [\bar{\gamma}_m \cdot \hat{\mathbf{x}}_m + \mathbf{F}^H \Psi_m \tilde{r}^f_m], \quad (9)$$

where the following definitions have been used

$$\bar{\gamma}_m = \frac{1}{K} tr[\Xi^{fH}(\Xi^f \Delta \Xi^{fH} + \hat{\sigma}^2_m I_{NK})^{-1} \Xi^f], \quad (10)$$

$$\bar{\delta}_m = \frac{1}{K} \sum_{k=1}^{K} |\hat{\mathbf{x}}_m(k)|^2, \quad (11)$$

$$\Psi_m = \Xi^{fH}(\Xi^f \Delta \Xi^{fH} + \hat{\sigma}^2_m I_{NK})^{-1}, \quad (12)$$

and where the values of Ξ and Δ are given by (1) and

$$\Delta = \mathbf{F}\Lambda\mathbf{F}^H \approx \frac{1}{K} tr\Lambda, \quad (13)$$

with $\Lambda = I_K - diag[|\hat{\mathbf{x}}_m|^2]$. Hence, the first and second moments of the MMSE filter output are expressed as

$$\mu_{z,m} = \bar{\gamma}_m (1 + \bar{\gamma}_m \bar{\delta}_m)^{-1}, \quad (14)$$

$$\sigma^2_{z,m} = \mu_{z,m}(1 - \mu_{z,m}). \quad (15)$$

C. EM Demapper

Now, we can convert the MMSE filter outputs into the extrinsic LLR for m-th user in EM demapper by using

$$L_{m,e,dem}[b_{m,d}] = \ln \frac{\sum_{s\in S_0} e^{-\frac{|\tilde{\mathbf{r}}_m - \mu_{z,m}\cdot s|^2}{\hat{\sigma}^2_{z,m}}} \prod_{q=1, q\neq d}^{lmap} e^{-b_q(s) L_{m,a,dem}(b_q(s))}}{\sum_{s\in S_1} e^{-\frac{|\tilde{\mathbf{r}}_m - \mu_{z,m}\cdot s|^2}{\hat{\sigma}^2_{z,m}}} \prod_{q=1, q\neq d}^{lmap} e^{-b_q(s) L_{m,a,dem}(b_q(s))}}, \quad (16)$$

where $S_0(S_1)$ and $L_{m,a,dem}(b_{m,q}(s))$ denote the labelling set, of which the d-th bit is $0(1)$, and the *a priori* LLR fed back from the decoder corresponding to the qth position in the label allocated to the signal point s, respectively. $L_{m,a,dem}$ is equivalent to extrinsic LLR $L_{m,e,dec}$ of the decoder forwarded via the deinterleaver. q indicates the position of the bits allocated in the symbol s in the constellation. $\mathbf{r}_m$, $\mu_{z,m}$ and $\sigma^2_{z,m}$ are updated every time when the outer iteration is activated, before they are provided to the demapper. After that, the output of the EM demapper $\mathbf{L}_{m,e,dem}$ are fed back to DACC decoder and the inner iteration for m-th user is activated.

IV. Detection Ordering

The box indicated by DO in Fig. 1 after the receiver antanna determines the detection order of the users by comparing the channel gains E of each user. For the g-th user, E_g is shown in (8). The algorithm of DO is summarized in Algorithm 1.

Input: $\mathbf{E}$ is a *set* containing total channel power of each user
$\mathbf{E} = \{E_1, E_2, ..., E_m, ..., E_M\}$ and the corresponding indexes are in *set* $\mathbf{M} = \{1, 2, ..., m, ..., M\}$
Output: Detection order *set* $\mathbf{D}$

```
D = ∅;
for j = 1; j ≤ M; j++ do
    T=set M\set D;
    while T ≠ ∅ do
        D(j) = argMAX_{k∈T}(E_k);
    end
end
return D;
```

Algorithm 1: Pseudocode of DO Algorithm.

The detection order $\mathbf{D}$ is determined by the DO box before the detector starts the detection process for the received composite signal $\mathbf{r}$. In MUD, detection order is

one of the important factors which makes significant impact on the efficiency of the detector. The improvement due to DO is to be investigated in section V.

V. Numerical Results

A series of computer simulations was conducted to verify the effectiveness of the proposed joint turbo equalization and BICM-ID-based IDMA system as well as to evaluate the impact of the DO over frequency selective fading channels. As described above, first of all, all the parameters of the transmission chain are optimized by EBSA[2] in AWGN channel; then, the optimal parameters are applied into the proposed system. It is shown that the proposed system with the optimal parameters can achieve excellent performance over frequency selective fading channels.

A. EXIT Chart

The EXIT chart presented in Fig. 2 shows the excellent matching between the demapper and decoder for 8 users with SNR of each user $SNR_m = 0$ dB. The parameters of the transmission chain obtained by EBSA are also provided in the figure. Such close matching between the demapper and decoder EXIT curves indicates that near-capacity performance can be expected in AWGN channel, as investigated in [8]. The effectiveness of the transmission chain parameters over frequency selective fading channels is to be investigated in subseciton V-B.

B. FER Performance

This subsection presents the results of computer simulations conducted to evaluate the performance of the proposed system over frequency selective fading channels with the parameters of the transmission chain obtained by EBSA. All simulations assumed that the channel frequency selectivity is due to an L-path propagation scenario with each path experiencing the block fading; L-path components have identical average power and independent complex gaussian distribution.

In Fig. 3, it is shown that the FER performances of the proposed technique in cases of single user, 8 users and 10 users with DO as well as the outage probability of the single user, where for all the cases with $L = 6$. The outage P_{out} is defined as $P_{out} = Pr(R > C)$, which was evaluated via Monte Carlo simulations: C is the capacity of each channel realization, and the

[2]EBSA has to be run once before the whole transmission (not per-transmission), its computational complexity increases exponentially as the labeling length increases. However, we can reduce the complexity with some approximations, e.g. [12].

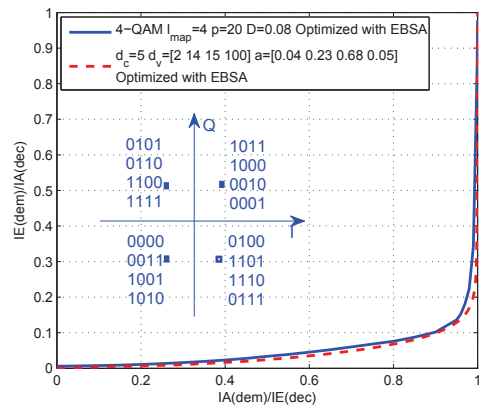

Fig. 2. EXIT Chart for 8 users with $SNR_m = 0$ dB.

data was generated from 10,000,000 channel realizations. $R = 0.1394$ bits/s/Hz is the total per-Hz transmission rate. Every outer iteration is followed by one round of inner iterations, expressed as (uO, vI), which represents that in total u outer iterations are performed, each outer iteration followed by v inner iterations (totaling $u \times v$ inner iterations).

It is found from Fig. 3 that the FER of single user with iterations (10O, 20I) shown by "□" has a very close performance (roughly 1 dB gap) to the outage probability bound shown by the dashed curve. Meanwhile, the FER performances of 8 users and 10 users with iterations (10O, 20I), shown by "○" and "◇", are not degraded too much compared with the single user's FER performance.

With a rate of 0.1394, the system is equivalent to CDMA with spreading factor of $7.1736 \approx 8$ users. However, our results show that the proposed system still work well with 10 users's case. The degradation is negligible compared with FER performance of 8 users, even though in this case, the equivalent spreading factor is larger than 8. The non-linear characteristic to the number of users can be observed, due to the benefit of FD-SC-MMSE turbo equalization.

Fig. 4 provides the average FER performance of the proposed system with and without DO, with several (u, v) pairs as a parameter. The solid curves in Fig. 4 present the performance with DO, while the dashed curves without DO. It is clearly found from the figure that the system with DO outperform that without DO, when numbers of the inner and outer iterations are not sufficient, such as (2O, 5I) and (4O, 5I), the interference from the other users can not be fully eliminated, as shown in "□" and "◇", respectively. The reason is

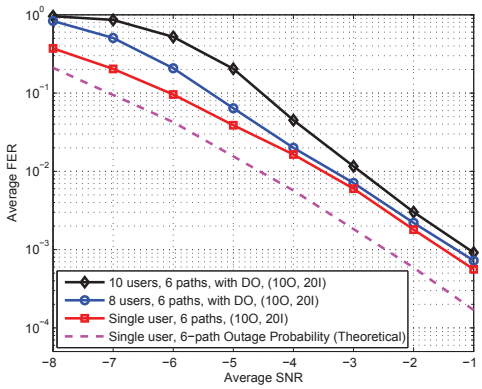

Fig. 3. FER performance of the proposed system with single user, 8 users and 10 users.

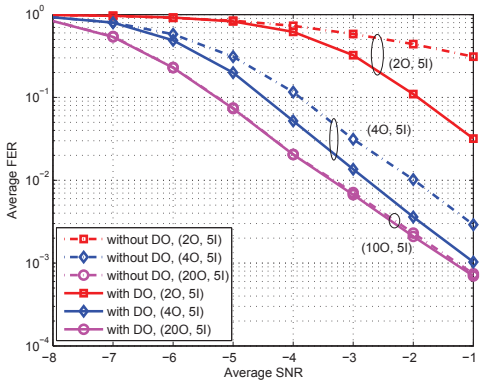

Fig. 4. FER performance of the proposed system with and without detection ordering, 8 users.

that the detection order of users in soft interference cancelation makes significant impact on the efficiency of the detector. With DO, the interference from the other users can be further eliminated by the same iteration times. However, from the average FER curves shown in "○", it can be found that the performances with the both cases become almost the same, as represented by (20O, 5I). This is because that, in the proposed system without DO, the interference from the other users can be also fully eliminated when sufficient iterations are performed. It can therefore be concluded that the proposed system performance can further be enhanced with DO over without DO.

VI. Conclusion

In this paper, we have proposed a joint turbo equalization and IDMA signal detection as well as DO technique for BICM-ID based IDMA at very low SNR range in frequency selective fading channels. The EBSA technique are also applied to optimize the transmission chain's parameters. The achieved performances of the proposed system demonstrated by the computer simulations are threefold: (1) close FER performance of single user with 6 paths to the outage probability; (2) less degradation on FER performance for 10 users even with the equivalent spreading factor of 8 (code rate $R = 0.1394$ bits/s/Hz); (3) significant performance improvement with DO technique compared with that of without DO technique, especially when the number of iteration is limited, due to, e.g., power constraint at the base stations. As a whole, the proposed joint turbo equalization and BICM-ID-based IDMA technique is suitable for future multiple access wireless communication systems, especially for reliable transmission at very low SNR range.

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