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Decidable Problem of Reachability Property in Term Rewriting System

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Term Rewriting Systems (TRS) can be regarded as a model for computation in which terms are reduced with a set of directed equations, called rewrite rules. They are used to represent abstract interpreters of programming languages and to model formal manipulating systems used in various applications, such as program optimization, program verification, and automatic theorem proving.

In a TRS, if a term can reach another term by the repetitive applications of rewrite rules, we say that two terms have reachability with respect to the TRS. If we can (not) always judge whether a property holds, we say the property is decidable (undecidable). A problem connecting with decidability and undecidability is said a decision problem. In general, reachability in TRSs is undecidable. Thus, these are various sufficient conditions for decidability of reachability proposed by researchers.

W.S.Brainerd (1969) proved that reachability was decidable for ground TRSs,i.e., TRSs without variables. Recently, the following results are shown: for monadic TRSs, which consist of rewrite rules whose right-hand (left-hand) sides have depth at most (least) one, by K.Salomaa(1988); semi-monadic TRSs, whose rewrite rules have the right-hand (left-hand) sides in depth at most (least) one, by J.Coquide(1994); and growing TRSs such that variables occurring in the both sides appear in the left-hand sides at the depth at most one by F.Jacquenmard(1996). In their TRSs we can judge reachability by analysis of tree automata. It is known that tree automata are very effective for solving decision problem.

In this paper we study decision problem of reachability by tree automata. We discuss the following two decision problems: general reachability for growing TRSs and reachability for Strong Membership Condition TRSs(SMC-TRSs). First, we propose general

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reachability. General reachability extends reachability. We discuss two kinds of general reachability as follows.

- A term can reach a set of terms by a TRS R.
- Every term in a set of terms can reach a term in another set of terms by a TRS R.

Using tree automata techniques, we show that general reachability for growing TRSs is decidable. A tree automaton is made of a growing TRS R and a set of terms. Whether a term is accepted by the tree automaton judges general reachability of the term and the set of terms. Since, as mentioned before, general reachability is an extension of reachability, our result property includes that of F.Jacquenmard(1996) for decision problem of reachability.

Second, we propose SMC-TRSs and show that reachability for SMC-TRSs is decidable. A SMC-TRS R satisfies the following conditions.

- Only variables occurring in both sides have membership conditions.
- Every set M in membership conditions is accompanied with a TRS R_M. The element of M with respect to $R \cup \biguplus_{M \in \mathcal{M}} R_M$ are confluence and terminating, and normal forms with respect to $R \cup \biguplus_{M \in \mathcal{M}} R_M$ is finite, where $\mathcal{M}$ is a collection of sets occurring in R.
- Left-thand sides of R does not have terms which are compose of signature of M.

The following TRS is an example of SMC-TRSs. This R satisfies the above conditions.

$$\mathcal{F} = \{f/1, g/1, \neg/1, \wedge/2, \vee/2, \mathsf{T}/0, \mathsf{F}/0\}, \quad f : Bool \to Bool, \quad g : Bool \to Bool$$

$$R = \left\{ \begin{array}{lcll} f(g(x)) & \to & g(x). & x \in Bool \\ g(g(x)) & \to & f(x). & x \in Bool \end{array} \right\}$$

$$\mathcal{F}_{Bool} = \{\mathsf{T}/0, \mathsf{F}/0, \neg/1, \wedge/2, \vee/2\}$$

$$R_{Bool} = \left\{ \begin{array}{ccccccc} \neg(\mathsf{T}) & \to & \mathsf{F}, & \neg(\mathsf{F}) & \to & \mathsf{T} \\ \wedge(\mathsf{T},\mathsf{T}) & \to & \mathsf{T}, & \wedge(\mathsf{T},\mathsf{F}) & \to & \mathsf{F} \\ \wedge(\mathsf{F},\mathsf{T}) & \to & \mathsf{F}, & \wedge(\mathsf{F},\mathsf{F}) & \to & \mathsf{F} \\ \vee(\mathsf{T},\mathsf{T}) & \to & \mathsf{T}, & \vee(\mathsf{T},\mathsf{F}) & \to & \mathsf{T} \\ \vee(\mathsf{F},\mathsf{T}) & \to & \mathsf{T}, & \vee(\mathsf{F},\mathsf{F}) & \to & \mathsf{F} \end{array} \right\}$$

SMC-TRSs enjoy decidability because their membership conditions of Membership Conditional TRS proposed by Toyama(1989) are restricted as decribed before. In Strong Membership Conditional TRSs variables occurring both sides of a rewrite rule should have membership conditions. Thus it removes the condition that such the variables in the left-hand sides should occur at the depth at most one, which is critical in growing TRSs. Using tree automata techniques again, we show that reachability for Strong Membership Conditional TRSs is decidable.

In the TRS with membership conditions, this result entends the recent results proposed by K.Salomaa (1988) and J.Coquide (1994),F.Jacquenmard (1996) proposed because Strong Membership Condition TRSs don't have the condition of depth while their TRSs have it. In addition Strong Membership Conditional TRS has a good effect in system analysis.