Game Refinement Theory and Its Application to Score Limit Games

Nathan Nossal and Hiroyuki Iida

***Abstract*—This paper presents an application of game refinement theory to score limited sport games badminton and table tennis, in order to observe the effect of recent rule changes on their sophistication. We apply the game refinement measure of information from world class tournaments. The results show game refinement values for badminton and table tennis. Although not considered in the present study, it can be seen that additional studies on a relationship between elapsed times and game refinement values for score limited games would be useful.**

***Index Terms*—game refinement theory, badminton, table tennis**

I. Introduction

Classical game theory [5] originated with the idea of the existence of mixed-strategy equilibria in two-person zero-sum games. This has been widely recognized as a powerful tool in many fields such as economics, political science, psychology, logic and biology. Game refinement theory is another game theory focusing on the attractiveness and the sophistication of games. The foundation of this direction was made by Iida et al. [2], in which a measure of game refinement was proposed based on the concept of information of game outcome uncertainty. A logistic model was constructed in the framework of game refinement theory and applied to many board games including chess variants and Mah Jong [3]. Recently a general model of game refinement was proposed based on the game information progress model and applied to time limit games such as soccer and basketball [7]. While game theory concerns a player's winning strategy, game refinement theory is concerned with the whole game, including quality of play and entertainment.

Our work now aims to develop a general model of game information progress. Sports are one of the more familiar kind of games and can be seen every day nearly ubiquitously. Many modern sports have long and varied histories, having evolved over decades or centuries of play – a process we call game sophistication. Changing the rules of the games in order to improve commercial and broadcasting appeal is common practice. Interestingly, it is observed that many sophisticated board games and popular time limit sports games have a similar value of game refinement, though exactly why this is remains to be discovered. The previous works support that game refinement is directly related to entertainment or engagement for spectators and players [2][3].

N. Nossal is with the School of Information Science, JAIST, Japan, e-mail: Nossal@jaist.ac.jp.

H. Iida is with the School of Information Science, JAIST, Japan, e-mail: iida@jaist.ac.jp

A similar study of video games applied game refinement theory to Pac-Man(R) and Defense of the Ancients(R), and that result is pending publication. It seems however that GRT can be relied upon to show the game refinement value of evolutionary changes to game information in video games as well. In this paper we consider a reasonable model of game information progress to derive the game refinement in score limit games such as badminton and table tennis.

II. Game Refinement Theory

A general model of game refinement was proposed based on the concept of game progress and game information progress [7]. It bridges a gap between board games and sports.

Game information progress presents the degree of certainty of a game's results in time or in steps. Let G and T be the average number of successful shots and the average number of shots per game, respectively. Having full information of the game progress, i.e. after its conclusion, game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation (1).

$$x(t) = \frac{G}{T}\ t \tag{1}$$

However, the game information progress given by Equation (1) is unknown during the in-game period. The presence of uncertainty during the game, often until the final moments of a game, renders exponential game progress. Hence, a realistic model of game information progress is given by Equation (2).

$$x(t) = G(\frac{t}{T})^n \tag{2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer of the game considered. Then acceleration of game information progress is obtained by deriving Equation (2) twice. Solving it at $t = T$, the equation becomes

$$x''(T) = \frac{Gn(n-1)}{T^n}t^{n-2} = \frac{G}{T^2}n(n-1)$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our brains. We do not yet know about the physics of information in the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Therefore we expect that the larger the value $\frac{G}{T^2}$ is, the more the game becomes exciting, due in part to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{G}}{T}$, as a game refinement measure for the game under consideration. We can call it GR value for short.

III. Application to Score Limit Sports

In a previous study using volleyball, the game refinement value of the different scoring systems were measured using the average score in each "set" with the result shown in [9]. In this sense "game" is defined as the information of winning or losing one sequence on a pre-determined game length continuum, e.g. three 20-minute periods, or 15 points, in the context of the rules governing the selected sport. "Set" is the sum of games in a specified series or sub-series, such as best-of-five or best-of-seven. "Match" is the final result determining the winner of a set, or best-of-a number of sets. In common parlance, set and match are sometimes confused, and often used interchangeably.

A higher or lower ordinal value of GR indicates greater or lesser game complexity and length. According to the results with board games [3], the game refinement value of sophisticated games is around 0.07-0.08. We believe this to be a generally appropriate range, based on previous findings of similar near-perfect play as is found in world class competition.

Volleyball and table tennis are decided not by a single game, but by the winner of sets and matches. With the game refinement measure based on sets, the progress graph would reach its end at every end of the set, while the game overall progress should only reach its end at the end of the match. Therefore, we use the average overall score of both sides from the entire match to calculate the game refinement value.

A. Badminton

The ancients played a game "battledore and shuttlecock" similar to badminton more than 2000 years ago. Badminton was played in India in the 1800s and brought back to England by British officers. The first use of the name badminton in England was around the mid-1800s [6]. There have been numerous changes in rules and equipment. We first explain the current scoring of badminton and compare the results for game refinement, applied to the scoring system used now, and the most recent one. Then, we discuss the relationship between the different scoring systems.

Badminton's official rules are described on the official site of BWF [1]. There are two sides of player(s), either singles or doubles. There are five types of events: Men's singles, Women's singles, Men's doubles, Women's doubles, and Mixed doubles. Under the new current scoring system, the side which wins the rally scores the point regardless of which side serves. The information length of game is 21 points, while maintaining a minimum two-point lead. The first side to win two games out of three wins the match. This is the 3 x 21 rally point scoring system.

In the past, a 3 x 15 side-out scoring system was used (3 x 11 for Women's singles). For this side-out scoring system, only the server can score the point. If the service side loses the rally, no point is awarded, and the service passes to the other side. It can be seen that this could, and often did, result in irregularly long match times. The scoring system was changed in December 2005.

Table I compares the average score for both sides and the game refinement value between the old scoring system and the current scoring system. Data was collected from BWF world championship [9][6].

TABLE I
Game refinement value of Badminton

scoring system	G	T	GR
Old	30.070	45.154	0.121
Current	46.336	79.344	0.086

Game refinement for the two variants is significantly different, indicating the change to game progress as affected by the change in scoring. The side-out system gives the outer value 0.121. Under the new system, the game refinement value recedes to a more balanced 0.086. Our result implies that the change in the scoring system makes the game more interesting and attractive for observation.

B. Table Tennis

Table tennis was first played in 1880s in England as an after dinner activity. It became popular with the introduction of the name "Ping-Pong" by J. Jacques & Son, and has since undergone a few change of rules and equipment [8]. In this part, we will explain the basic rules of table tennis and the change in equipment, and compare the results from applying game refinement theory on table tennis. The official rules are available on the official site of ITTF [4].

There are two sides of player(s), either singles or doubles. There are four types of events: Men's or Women's singles, and Men's or Women's teams. As with the new badminton and volleyball systems, in table tennis the side which wins the rally scores the point regardless of service. The information length of game is 11 points, while maintaining a minimum two-point lead. The match consists of any odd number of games, usually seven.

At first, table tennis rackets were pure wood. Around 1900, the use of layered foam-core rackets, topped with rubber sheeting, dramatically increased game speed and added some spin to the game. Prior to the 2000 Summer Olympics, table tennis used a 38 mm ball. The ball size was changed to 40 mm which effectively reduced the game's speed. The scoring system was also changed in September 2001. Until that year, game lengths were 21 points with a two-point lead, while matches were usually a best-of-three or best-of-five. Most recently, the doubles event was integrated as a part of team events for 2008 Olympics. Each team features three players consisting of one singles player and a doubles pair, playing a best-of-five series of two singles, followed by doubles, and followed by up to 2 more singles if necessary. The first side to win three wins the match.

We collected information of Olympics matches from 1988 to present from records in the ITTF official site [4]. Table II compares the results for both sides and the game refinement value between pre-2000 games, post-2000 games, and the new team events.

The result indicates that the change in the equipment and the scoring system does not affect the progress of the game

much. The stable GR value, around 0.076, also suggests the appropriate level of sophistication of table tennis since the 1988 Olympics. The result of team events is lower than for the other two. At the same time, the average score is much higher. It can be safely assumed that the change to integrate a doubles event in the team events, while allowing more players to join the tournament, is not designed to improve the game's sophistication.

TABLE II
GAME REFINEMENT VALUE OF TABLE TENNIS

	G	T	GR
pre-2000	57.869	101.530	0.075
post-2000	54.863	96.465	0.077
team	131.283	232.123	0.049

C. Discussion

In this paper, we proposed the use of match average scores to measure game refinement values of score limit sports. We are interested in the relationship between the rule changes and the game refinement value GR. According to earlier studies on board games and time limited games [3][9], the window of ideal game refinement in sophisticated games is between 0.07 - 0.08, shown in Table III.

TABLE III
COMPARISON OF REFINEMENT MEASURES OF DIFFERENT GAMES

Chess	0.074
Go	0.076
Basketball	0.073
Soccer	0.073
Badminton	0.086
Table tennis	0.077
Table tennis (team)	0.049

The change of scoring system in badminton caused a change in the GR value from 0.121 to 0.086, approaching the mentioned zone. We hypothesize that the appropriate level of sophistication of badminton is between 0.07 - 0.08 as well, and that the game was made more interesting and pleasant for observers by this rule change.

As for table tennis, excluding the team events, our result shows that the game refinement values remain almost unchanged within bounds of 0.075 and 0.077 since 1988. This result indicates that the changes of equipment and scoring did not affect table tennis' sophistication by any noticeable degree. If we can assume that the best GR value for table tennis is also between 0.07-0.08, then we could also assume that additional changes cannot improve the game refinement value under present conditions. The integration of a doubles game into team event of table tennis, on the other hand, reduces the game refinement value to 0.049. We view this as contradicting the ideal window. Of course it is also quite possible that the integration of doubles was not aiming to improve game sophistication.

The model excludes complicating factors, which are not considered in our equation. There have not yet been any studies to establish the relationship between elapsed time and information game length with consideration of game sophistication in score limit sports. The data we collected did not contain information of actual elapsed time. The current equations used for game refinement theory for score limit games do not provide comprehensive study for the difference between the side-out and the rally scoring system, of which one major difference is the actual time used per point. We have isolated and analyzed just one aspect of the game by measuring informational decision complexity and game length after changes to the rules governing the progress of games.

IV. CONCLUSION

We conclude that the resulting game refinement values of badminton and table tennis, as measured by game refinement theory, support the previous assumptions of a balanced window of game refinement around 0.07-0.08. We acknowledge there are numerous other factors, namely elapsed time, and others which could affect the measure of sophistication of score limit sports.

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