

Title	Formal Topology における Bishop のコンパクト性
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Abstract

Since the publication of *Foundation of Constructive Analysis*, Bishop and coworkers have developed a large body of analysis constructively. However, the gap between the notion of compactness for topological spaces and for Bishop metric spaces has been a major obstacle to finding the right notion of general topology which naturally extends that of Bishop metric space.

Independently of Bishop, Sambin initiated a study of constructive general topology using a point-free approach. His notion, *formal topology*, has been quite successful in constructivising many results of classical general topology, and has established itself as the most promising approach to general topology in constructive mathematics.

However, the precise connection between Bishop metric space and formal topology has not been established, and this prevents us from applying the wealth of results obtained in formal topology to Bishop metric spaces. This thesis tries to improve this unsatisfactory situation by establishing a precise connection between the notions of compactness and local compactness for Bishop metric spaces and the corresponding notions for formal topologies.

As the first main result of this thesis, we obtained a point-free characterisation of compact metric spaces in terms of formal topology. We identified the full subcategory of formal topologies which is essentially equivalent to that of compact metric spaces. We show that the notion of compact overt enumerably completely regular formal topology characterises that of compact metric space up to isomorphism.

Our second main result generalises the above mentioned characterisation to the class of Bishop locally compact metric spaces. We show that the notion of inhabited enumerably locally compact regular formal topology characterises that of Bishop locally compact metric space up to isomorphism. As an application of these characterisations, we prove a point-free version of the well-known fact that any Bishop locally compact metric space has a one-point compactification. The point-free result immediately yields the corresponding result for Bishop locally compact metric spaces.

Keywords: Constructive mathematics; Formal topologies; Point-free characterisations; Compact metric spaces; Locally compact metric spaces