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**Formal Semantics of Core SQL Language  
based on the K Framework**

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# Formal Semantics of Core SQL Language based on the K Framework

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## Abstract

There are lots of SQL dialects, e.g., MySQL, various versions of Oracle, and Microsoft SQL Server, ProgreSQL, and more. They share common semantics on standard table operations (with slight syntax differences), which appear in a textbook of relational database management systems. However, formal semantics of non-standard operations, e.g., type violation like `SELECT 1 + "1a"`, varies in detail. Most of programmers in system development do not aware of such differences, which will be crucial when applying formal methods. They are typically *coercion*, `NULL`, the *name space*, and the *error handling*. Even a standard operation `JOIN` varies depending on detailed types (including the bit-width) of arguments.

This thesis investigates detailed semantics of the core of SQL, specifically on MySQL and Oracle11. First, we observe their formal semantics by testing queries on boundary cases. Next, the semantics of the core of MySQL is implemented on the K framework. We call it *KSQL*, which covers basic table operations, like *selection*, *creation*, *deletion*, *update*, and *insertion*. They are defined with the features of *coercion*, `NULL`, and the *name space convention*. Lastly, we discuss on current limitations and difficulties in KSQL implementation.

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# Chapter 1

## Introduction

Formal semantics of a programming language is required in many views. For instance, understanding detailed behaviour of languages reduces bugs of implementation. Automatic support by verification / analysis tools is constructed on formal semantics. Although formal semantics is often embedded into algorithms and/or implementation of such tools, there are several attempts to define formal semantics alone, e.g., Java [4], ANSI-C [3], PHP [11], Verilog [6], Scheme [5], x86 [2], and HTML5 [7]. Among them, for Java, ANSI-C, Verilog, and Scheme are implemented on K framework, thus executable.

Our aim is to give formal semantics of SQL, and clarify their differences among SQL dialects, e.g., MySQL, various versions of Oracle, Microsoft SQL Server, ProgreSQL, and more. They share common semantics on standard table operations (with slight syntax differences), like selection, insertion, deletion, creation, update, and join which are popular in a textbook of relational database management systems (RDBMSs). However, consider the following query. What do MySQL and Oracle11 return?

MySQL query : `SELECT 1 + "" ;`

Oracle query : `SELECT 1 + '' FROM DUAL;`

One possibility is simply an error, because the addition `+` accepts numbers as its arguments. However, the addition of integer and string is valid in MySQL and Oracle11, and they return `1` and `''` (empty string), respectively.

In this thesis, formal semantics of the core (a subset) of SQL is investigated. We first compare detailed semantics of MySQL and Oracle11, and next, the semantics of the core of MySQL is implemented on K-framework. We call it *KSQL*, which covers basic table operations, like *selection*, *creation*, *deletion*, *update*, and *insertion*.

In our study, we found two main issues to cause semantic differences between MySQL and Oracle11.

- Operations on boundary values, e.g., coercion, NULL, the name space convention, and error handling.



- Two different layers, logical and physical models of data types. Dialects have their own design of data types. For example, Oracle11 has NUMBER type, which allows users to specify the precision and the scale factor, while MySQL has predefined data types, such as TINYINT, SMALLINT, MEDIUMINT, INT, and BIGINT corresponding to 8-bit, 16-bit, 32-bit, 64-bit, and 128-bit integers, respectively.

In this thesis, we investigate the first issue, specifically, coercion, NULL, and the name space management, over basic operations. They are implemented on K framework [9], which is constructed on Maude, a programming language based on algebraic specification. Algebraic specification consists of rewriting rules (equations) over terms with sorts. We describe states of SQL as terms and SQL (small step) operational semantics of SQL as rewriting rules.

Lastly, we discuss on current limitations and difficulties in KSQL implementation.

## Contributions

Our contributions are:

- The formal semantics of the core SQL language including selection, insertion, creation, deletion, and update statements.
- Differences between MySQL and Oracle11: coercion, NULL, and the name space convention.
- Semantics definition on K framework of detailed behaviour: coercion, NULL, and the name space convention in MySQL.
- Explanation and analysis of difficulties for defining semantics of SQL.

This thesis is organized as follows:

- Chapter 2 provides technical background about term rewriting systems and algebraic specification.
- Chapter 3 provides a brief introduction of K framework.
- Chapter 4 explains differences and choices among SQL semantics, based on observation on boundary cases.
- Chapter 5 describes basic description of semantics of MySQL on K framework.
- Chapter 6 describes semantics of coercion, NULL, and the name space convention of MySQL on K framework.
- Chapter 7 discusses on related work.
- Chapter 8 concludes the thesis and mentions future direction.

# Chapter 2

## Preliminaries

In this chapter, we explain term rewriting in the first section and algebraic specification in the second section.

### 2.1 Term rewriting system

**Definition 2.1.** Let  $\mathcal{V}$  be a countable set of variables, and  $\mathcal{F}$  a set of function symbols associated with the arity mapping  $\text{ar} : \mathcal{F} \rightarrow \mathbb{N}$ . We call  $\mathcal{F}$  a signature and  $f \in \mathcal{F}$  has arity  $n$  if there exists  $n \in \mathbb{N}$  which satisfies  $\text{ar}(f) = n$ . We call a function  $f$  constant if  $\text{ar}(f) = 0$ .

**Definition 2.2.** The set  $\mathcal{T}(\mathcal{F}, \mathcal{V})$  of terms over the signature  $\mathcal{F}$  is the smallest set satisfying the following conditions:

- if  $x \in \mathcal{V}$  then  $x \in \mathcal{T}(\mathcal{F}, \mathcal{V})$ ,
- if  $t_1, \dots, t_n \in \mathcal{T}(\mathcal{F}, \mathcal{V})$  and  $f \in \mathcal{F}$  which has arity  $n$  then  $f(t_1, \dots, t_n) \in \mathcal{T}(\mathcal{F}, \mathcal{V})$ .

**Example 2.3.** Let  $\mathcal{V} = \{x, y\}$  and  $\mathcal{F} = \{0, \mathbf{s}, +\}$  with  $\text{ar}(0) = 0$ ,  $\text{ar}(\mathbf{s}) = 1$ , and  $\text{ar}(+) = 2$ . Then the following terms are members of  $\mathcal{T}(\mathcal{F}, \mathcal{V})$ :  $0$ ,  $\mathbf{s}(0)$ ,  $\mathbf{s}(x)$ ,  $\mathbf{s}(\mathbf{s}(x))$ ,  $0 + \mathbf{s}(0)$ , and  $x + \mathbf{s}(y)$ .

**Definition 2.4.** Let  $t \in \mathcal{T}(\mathcal{F}, \mathcal{V})$ . We inductively define the set  $V(t)$  of variables occurring in  $t$  as follows:

$$V(t) = \begin{cases} \{t\} & \text{if } t \in \mathcal{V} \\ \bigcup_{i=1}^n V(t_i) & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

**Definition 2.5.** A position is a sequence of positive integers. The position of empty sequence is denoted by  $\epsilon$  and the concatenation of positions  $p$  and  $q$  is  $p.q$ . The set  $Pos(t)$  of positions of a term  $t$  is

$$Pos(t) = \begin{cases} \{\epsilon\} & \text{if } t \in \mathcal{V} \\ \{\epsilon\} \cup \bigcup_{1 \leq i \leq n} \{i.p \mid p \in Pos(t_i)\} & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

**Definition 2.6.** A subterm  $t|_p$  of  $t$  at the position  $p$  is inductively defined as follows:

$$t|_p = \begin{cases} t & \text{if } p = \epsilon \\ t_i|_q & \text{if } t = f(t_1, \dots, t_n) \text{ and } p = i.q \end{cases}$$

**Definition 2.7.** If  $t'$  is a term, A term  $t[t']_p$  denotes a term that is obtained from  $t$  by replacing the subterm at the position  $p$  with  $t'$ :

$$t[t']_p = \begin{cases} t' & \text{if } p = \epsilon \\ f(t_1, \dots, t_i[t']_q, \dots, t_n) & \text{if } t = f(t_1, \dots, t_n) \text{ and } p = i.q \end{cases}$$

**Example 2.8** (Continued from Example 2.3). Let  $t = (0 + \mathbf{s}(x)) + (y + \mathbf{s}(x))$ . Then we have  $Pos(t) = \{\epsilon, 1, 11, 12, 121, 2, 21, 22, 221\}$ . We have  $t|_{11} = 0, t|_{221} = x$  and  $t|_{2[0]_1} = 0 + \mathbf{s}(x)$ .

**Definition 2.9.** A rewrite rule is a pair  $(l, r)$  of terms that  $l \notin \mathcal{V}$  and  $V(r) \subseteq V(l)$ . A rewrite rule  $(l, r)$  is denoted by  $l \rightarrow r$ . A term rewriting system (TRS)  $\mathcal{R}$  is a set of rewrite rules over the signature  $\mathcal{F}$ .

**Example 2.10** (Continued from Example 2.3). We can define the term rewriting system  $\mathcal{R}$  as below:

$$\begin{aligned} 0 + y &\rightarrow y \\ \mathbf{s}(x) + y &\rightarrow \mathbf{s}(x + y). \end{aligned}$$

## 2.2 Algebraic specification

*Sort* is a set of values. Ordered sorts are sorts with partial relation between them, called *subsort relation*. We use sorts to define a domain and a range of a function. The subsort relation gives the benefit of the sort inheritance, such that if  $A$  is a subsort of  $B$ , then every variable or constant of  $A$  is also a variable or a constant of  $B$ . Moreover, ordered sorts are useful for overloading functions. Since the SQL language has many data types, sets of values, throughout this thesis, we adopt order-sorted term rewriting. In this section we recall the notations for order-sorted terms.

Let  $\mathcal{S}$  be a set of sorts equipped with a subsort relation  $\sqsubseteq$  on  $\mathcal{S}$ .

**Definition 2.11.** Let  $\mathcal{F}$  be a set of pairs  $(f, \tau)$  with a function symbol  $f$  and  $\tau \in \mathcal{S}^+$ . The set  $\mathcal{F}$  is an order-sorted signature if the implication

$$\frac{\tau_1 \sqsubseteq \tau'_1 \quad \cdots \quad \tau_n \sqsubseteq \tau'_n}{\tau_0 \sqsubseteq \tau'_0}$$

holds for all  $(f, \tau_1 \cdots \tau_n \tau_0), (f, \tau'_1 \cdots \tau'_n \tau'_0) \in \mathcal{F}$ . A pair  $(f, \tau_1 \cdots \tau_n \tau_0)$  is also denoted by  $f : \tau_1 \times \cdots \times \tau_n \rightarrow \tau_0$  where  $n$  is the arity of  $f$ .

**Example 2.12.** Let  $\mathcal{S} = \{Int, NeList, List\}$  and  $\sqsubseteq$  its subsort order on relation with  $Int \sqsubseteq NeList \sqsubseteq List$ . The set  $\mathcal{F}$  consisting of

$$\begin{array}{lll} nil : List & cons : List \times List \rightarrow List & head : NeList \rightarrow Int \\ & cons : NeList \times List \rightarrow NeList & tail : NeList \rightarrow List \end{array}$$

forms an order-sorted signature.

We extend terms with sorted terms. Let  $\mathcal{F}$  be a set of order-sorted signatures and let  $\mathcal{V}$  be a set of *variables* as the disjoint union of  $\mathcal{V}^\tau$  for all sorts  $\tau \in \mathcal{S}$ .

**Definition 2.13.** The sort judgment  $t : \tau$  is defined by the next inference rules:

$$\frac{x \in \mathcal{V}^\tau}{x : \tau} \quad \frac{f : \tau_1 \times \cdots \times \tau_n \rightarrow \tau \in \mathcal{F} \quad t_i : \tau_i \text{ for all } i}{f(t_1, \dots, t_n) : \tau} \quad \frac{t : \tau' \quad \tau' \sqsubseteq \tau}{t : \tau}$$

The set  $\{t \mid t : \tau \text{ for some } \tau\}$  is denoted by  $\mathcal{T}(\mathcal{F}, \mathcal{V})$  and its elements are called (well-sorted) terms.

**Example 2.14.** Let  $\mathcal{S} = \{Nat, Bool\}$  and  $\mathcal{F}$  a set of  $\mathcal{S}$ -sorted signature consisting of the following:

$$\begin{array}{lll} eq : Nat \times Nat \rightarrow Bool & + : Nat \times Nat \rightarrow Nat & true : Bool \\ s : Nat \rightarrow Nat & \& : Bool \times Bool \rightarrow Bool & false : Bool \\ 0 : Nat & & \end{array}$$

Let  $\mathcal{V}^{Nat} = \{x, y\}$  and  $\mathcal{V}^{Bool} = \{p, q\}$ . Then sorted terms  $x, y, 0, \mathbf{s}(0), \mathbf{s}(x), \mathbf{s}(y), \mathbf{s}(\mathbf{s}(x))$ , and  $\mathbf{eq}(\mathbf{s}(x), \mathbf{s}(y)), p, \mathbf{true} \ \& \ p, p \ \& \ q$  are members of  $\mathcal{T}(\mathcal{F}, \mathcal{V})$  while  $\mathbf{true} + \mathbf{s}(0), 1 \ \& \ \mathbf{false}$ , and  $\mathbf{eq}(\mathbf{true}, 0)$  are not.

**Definition 2.15.** A substitution  $\sigma$  is a map from  $\mathcal{V}$  to  $\mathcal{T}(\mathcal{F}, \mathcal{V})$  if the following conditions hold:

- If  $x : \tau$  and  $\sigma(x) : \tau'$  are the sort judgments of terms  $x$  and  $\sigma(x)$ , then  $\tau = \tau'$ .
- The domain of  $\sigma$  is finite, where the domain of  $\sigma$  is given by  $\text{dom}(\sigma) = \{x \in V \mid \sigma(x) \neq x\}$ .

We extend substitution definition to a term  $t$  as follow:

$$\sigma(t) = \begin{cases} t' & \text{if } t \text{ is a variable and } \sigma(t) = t' \\ f(\sigma(t_1), \dots, \sigma(t_n)) & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

We write  $t\sigma$  for  $\sigma(t)$ .

**Example 2.16.** Consider the substitution  $\sigma = \{x \mapsto x + z, y \mapsto x\}$ . If  $t = x + (\mathbf{s}(y) + (z + x))$ , then  $t\sigma = (x + z) + (\mathbf{s}(x) + (z + (x + z)))$ .

**Definition 2.17.** An equality is a pair denoted by  $l \approx r$  where  $l, r$  are order-sorted terms which satisfy  $l : \tau$  and  $r : \tau'$ , and then  $\tau = \tau'$ . We call a set of equations  $\mathcal{E}$  an equation system. We define  $\approx_{\mathcal{E}}$  the smallest equivalence relation which  $C[l\sigma] \approx_{\mathcal{E}} C[r\sigma]$  holds for all equations  $l \approx r \in \mathcal{E}$ , contexts  $C$ , and substitution  $\sigma$ .

**Definition 2.18.** An order-sorted rewrite rule is a rewrite rule  $l \rightarrow r$  which satisfies:  $l : \tau$  and  $r : \tau'$ , and then  $\tau' \sqsubseteq \tau$ . We call a set of order-sorted rewrite rules  $\mathcal{R}$  an order-sorted term rewriting system.

**Definition 2.19.** Given an order-sorted rewriting system  $\mathcal{R}$  and an order-sorted equation system  $\mathcal{E}$ . A term  $t$  rewrites to  $t'$  with a rewrite relation  $\rightarrow_{\mathcal{R}/\mathcal{E}}$ , in rewriting modulo equations, if there exists a rewrite rule  $l \rightarrow r \in \mathcal{R}$ , a term  $C$ , a position  $p$ , and a substitution  $\sigma$  such that  $t \approx_{\mathcal{E}} C[\sigma(l)]_p$ , and  $t' \approx_{\mathcal{E}} C[\sigma(r)]_p$ . We write  $t \rightarrow_{\mathcal{R}/\mathcal{E}} t'$  and call it a rewrite step. When  $\mathcal{E}$  is empty, we simply write  $\rightarrow_{\mathcal{R}}$  instead of  $\rightarrow_{\mathcal{R}/\mathcal{E}}$ .

**Example 2.20** (Continued from Example 2.14). *We can define the sorted term rewriting system  $\mathcal{R}$  as below:*

$$\begin{aligned}
& \text{eq}(0, 0) \rightarrow \text{true} \\
& \text{eq}(0, \text{s}(0)) \rightarrow \text{false} \\
& \text{eq}(\text{s}(0), 0) \rightarrow \text{false} \\
& \text{eq}(\text{s}(x), \text{s}(y)) \rightarrow \text{eq}(x, y) \\
& 0 + y \rightarrow y \\
& \text{s}(x) + y \rightarrow \text{s}(x + y). \\
& \text{false} \ \& \ p \rightarrow \text{false} \\
& \text{true} \ \& \ \text{false} \rightarrow \text{false} \\
& \text{true} \ \& \ \text{true} \rightarrow \text{true}
\end{aligned}$$

For instance, computation of  $\text{s}(0) + \text{s}(0) + \text{s}(0)$  is done by the following rewrite steps:

$$\begin{aligned}
& \text{eq}(\text{s}(0) + \text{s}(0), \text{s}(0)) \ \& \ \text{true} \rightarrow_{\mathcal{R}} \text{eq}(0 + \text{s}(\text{s}(0)), \text{s}(\text{s}(0))) \ \& \ \text{true} \\
& \rightarrow_{\mathcal{R}} \text{eq}(\text{s}(\text{s}(0)), \text{s}(\text{s}(0))) \ \& \ \text{true} \\
& \rightarrow_{\mathcal{R}} \text{eq}(\text{s}(0), \text{s}(0)) \ \& \ \text{true} \\
& \rightarrow_{\mathcal{R}} \text{eq}(0, 0) \ \& \ \text{true} \\
& \rightarrow_{\mathcal{R}} \text{true} \ \& \ \text{true} \\
& \rightarrow_{\mathcal{R}} \text{true}.
\end{aligned}$$

**Example 2.21.** *Given a sorted term rewriting system  $\mathcal{R}$  and an equation system  $\mathcal{E}$  as below:*

$$\mathcal{R} = \left\{ \begin{array}{l} x \cdot x \rightarrow x \\ \epsilon \cdot x \rightarrow x \end{array} \right\} \qquad \mathcal{E} = \left\{ \begin{array}{ll} x \cdot y & \approx y \cdot x \\ (x \cdot y) \cdot z & \approx x \cdot (y \cdot z) \end{array} \right\}$$

*Then the rewrite step  $((1 \cdot 2) \cdot 1) \cdot 3 \rightarrow_{\mathcal{R}/\mathcal{E}} (3 \cdot 2) \cdot 1$  holds by the following sequence:  $((1 \cdot 2) \cdot 1) \cdot 3 \approx_{\mathcal{E}} (((1 \cdot 1) \cdot 2) \cdot 3) \rightarrow_{\mathcal{R}} ((1 \cdot 2) \cdot 3) \approx_{\mathcal{E}} (3 \cdot 2) \cdot 1$  while  $((1 \cdot 2) \cdot 1) \cdot 3 \rightarrow_{\mathcal{R}} (3 \cdot 2) \cdot 1$  does not hold.*

# Chapter 3

## K framework

The K framework is an executable framework of the language definition. Formalizing a language in the K framework automatically supplies the K analysis tools. K defines a TRS  $\mathcal{R}$  and an equation system  $\mathcal{E}$  together with their (sorted) signature. In this chapter, we will explain and basic definitions using in the K framework, and show a simple example, called language `SIMPLE`, to show the use in the K framework.

### 3.1 Basic description in K

#### Syntax

We define the syntax of sorts of a language as follows:

$$\tau ::= f(\tau_1, \dots, \tau_n)$$

which stands for

$$f : \tau_1 \times \dots \times \tau_n \rightarrow \tau$$

where  $\tau_1, \dots, \tau_n$  and  $\tau$  are sorts of the language and  $f$  is a function symbol of the language. In the K framework, such a syntax is declared by the keyword `syntax`. We can extend the syntax of sort  $\tau$  by overwriting new BNF definitions. For instance, we want the sort  $\tau$  to have terms  $\tau_1 \dots \tau_n$ , we can overwrite the BNF syntax of sort  $\tau$  as follows:

$$\tau ::= t_1 \dots \tau ::= t_n$$

This is equivalent to  $\tau ::= t_1 \mid \dots \mid t_n$ . In addition, when we define the structure of sort  $\tau$  as follows:

$$\tau ::= \tau_1 \mid \dots \mid \tau_n$$

where  $\tau_1, \dots, \tau_n$  are sorts, this yields subsort relations  $\tau_1 \sqsubseteq \tau, \dots, \tau_n \sqsubseteq \tau$ .

**Definition 3.1.** Let  $\mathcal{W}$  is a set of context variables denoted by  $\{\square_1, \dots, \square_n\}$ . An  $n$ -hole context is a term in  $\mathcal{T}(\mathcal{F}, \mathcal{V} \cup \mathcal{W})$  with the constraint that each hole  $\square \in \mathcal{W}$  appears at most once. Given a substitution  $\sigma = \{\square_1 \mapsto t_1, \dots, \square_n \mapsto t_n\}$  we write  $C[t_1, \dots, t_n]$  for  $C\sigma$ .

We prepare new syntactical notations of rewrite rules.

**Notation 3.2.** Let  $C$  be an  $n$ -hole context. A single step rewrite rule of form  $C[\ell_1, \dots, \ell_n] \rightarrow C[r_1, \dots, r_n]$  is denoted by

$$C \left[ \frac{\ell_1}{r_1}, \dots, \frac{\ell_n}{r_n} \right]$$

In the K framework such a rule is declared as a keyword **rule**.

**Example 3.3.** Consider a rewrite system  $\mathcal{R}$  written by the new syntactical form:

$$\begin{array}{ccc} \frac{\text{eq}(0, 0)}{\text{true}} & \frac{\text{eq}(0, \mathbf{s}(0))}{\text{false}} & \frac{+(0, y)}{y} \\ \\ \text{eq}\left(\frac{\mathbf{s}(x)}{x}, \frac{\mathbf{s}(y)}{y}\right) & \frac{\text{eq}(\mathbf{s}(0), 0)}{\text{false}} & \frac{+(\mathbf{s}(x), y)}{\mathbf{s}(+(x, y))}. \end{array}$$

These rules are corresponding to normal rewrite rules as follows:

$$\begin{aligned} \text{eq}(0, 0) &\rightarrow \text{true} \\ \text{eq}(0, \mathbf{s}(0)) &\rightarrow \text{false} \\ \text{eq}(\mathbf{s}(0), 0) &\rightarrow \text{false} \\ \text{eq}(\mathbf{s}(x), \mathbf{s}(y)) &\rightarrow \text{eq}(x, y) \\ +(0, y) &\rightarrow y \\ +(\mathbf{s}(x), y) &\rightarrow \mathbf{s}(+(x, y)). \end{aligned}$$

The variables in  $\mathcal{W}$  are used to identify the positions where rewriting takes place. The notation above specifies the subterms to be rewritten and their reducts.

The K framework provides several (predefined) sorts together with related rewrite rules.

**Definition 3.4.** A list, map, and bag are defined as follows:

$$\begin{aligned} \text{List} &::= \epsilon_L \mid \text{List} :: \text{List} \mid \tau \\ \text{Map} &::= \epsilon_M \mid \text{Map} :: \text{Map} \mid \text{Binding} \\ \text{Bag} &::= \epsilon_B \mid \text{Bag} * \text{Bag} \mid \tau \end{aligned}$$



where *Binding* ::=  $\tau_1 \mapsto \tau_2$  and  $\tau, \tau_1$ , and  $\tau_2$  are sorts.

A list of sort  $\tau$  is a term of concatenation, denoted by  $::$ , of sorts  $\tau$  in  $\mathcal{T}(\mathcal{F})$  equipped with a term rewriting system  $\mathcal{R}_L$  and a map of binding is a term of concatenation of *Binding*, denoted by  $::$ , equipped with a term rewriting system  $\mathcal{R}_M$  where:

$$\mathcal{R}_M = \{ \epsilon_L :: x \rightarrow x \} \quad \mathcal{R}_M = \{ \epsilon_L :: x \rightarrow x \}$$

Remark that the parametric polymorphism is not supported. Therefore, we have to explicitly declare the sorts of elements, However, if the ordered sort of an element is clear from the context, we will omit the sort information.

A cell of sort  $\tau$  is a term denoted by:

$$\text{Cell} ::= \langle \tau \rangle_{\text{Label}}$$

where *Label* can be any string. A bag of sort  $\tau$  is a term of the AC operator  $*$  of sorts  $\tau$  in  $\mathcal{T}(\mathcal{F})$  equipped with a term rewriting system  $\mathcal{R}_B$  and an equation system  $\mathcal{E}_B$  where:

$$\mathcal{R}_B = \{ \epsilon_B * x \rightarrow x \} \quad \mathcal{E}_B = \left\{ \begin{array}{l} x * y \approx y * x \\ (x * y) * z \approx x * (y * z) \end{array} \right\}$$

We denote

$$\begin{array}{ll} x_1 :: \dots :: x_n :: \epsilon_L & \text{by } [x_1, \dots, x_n], \text{ or } (x_1, \dots, x_n) \\ (x_1 \mapsto y_1) :: \dots :: (x_n \mapsto y_n) :: \epsilon_M & \text{by } \{x_1 \mapsto y_1, \dots, x_n \mapsto y_n\}, \text{ and} \\ x_1 * \dots * x_n * \epsilon_B & \text{by } \{x_1, \dots, x_n\}. \end{array}$$

For any list  $L = [x_1, \dots, x_i, \dots, x_n]$ , we denote  $i$ -th element  $x_i$  of list  $L$  as  $L[i]$ . A parallel product of lists is a function  $\otimes : \text{List} \times \text{List}$  which is defined as follows:

$$[x_1, \dots, x_2] \otimes [y_1, \dots, y_n] = [(x_1, y_1), \dots, (x_n, y_n)]$$

Given a list  $A = [(x_1, y_1), \dots, (x_n, y_n)]$ , we write  $A(x_i)$  for  $y_i$ . Configuration is a bag of cells.

## Computation

Computation is the top sort over all defined sorts in the language definition. Let  $K$  be the sort of the computation, given by

$$K ::= \epsilon_K \mid K \curvearrowright K \mid \diamond$$

Sort  $K$  is the smallest sort with respect to  $\sqsubseteq$  among sorts defined in  $K$ . We can regard the sort  $K$  as a list of any sort, in which  $\curvearrowright$  is the concatenation operator.  $\diamond$  is a reserved constant of the sort  $K$  for pointing a  $K$ -subterm to be executed. Sort  $K$  is equipped with the next rewriting system  $\mathcal{R}$ :

$$\mathcal{R} = \{ \epsilon_K \curvearrowright x \rightarrow x \}$$

## Strictness Attribute

Strictness attribute is an attribute on a function symbol to define its evaluation strategy. For function  $f : \tau_1 \times \dots \times \tau_n$ , evaluation strategy of  $f$  is a list of integers  $i$  where  $1 \leq i \leq n$ . The K framework will automatically generate rewrite rules depending on the strictness attribute. The *strict* attribute is corresponding to non-deterministic evaluation strategy. The attribute *seqstrict* shows an sequential ordering of evaluation among arguments.

**Example 3.5.** We set a function  $_{-}+_{-}$  to evaluate its arguments from the left-to-right manner. We annotate *seqstrict* attribute of  $_{-}+_{-}$ , which is equivalent to the evaluation strategy  $(1, 2)$ . The K framework automatically generates the following four rewrite rules [10].

$$\begin{aligned} a_1 + a_2 &\rightarrow a_1 \curvearrowright (\diamond + a_2) \\ i_1 \curvearrowright (\diamond + a_2) &\rightarrow i_1 + a_2 \\ i_1 + a_2 &\rightarrow a_2 \curvearrowright (i_1 + \diamond) \\ i_2 \curvearrowright (i_1 + \diamond) &\rightarrow i_1 + i_2 \end{aligned}$$

where  $a_1, a_2$  are variables of sort  $K$ , and  $i_1, i_2$  are variables of sort  $Int$ . The evaluation of  $(1 + 2) + (3 + 4)$  is the following rewrite steps:

$$\begin{aligned} (1 + 2) + (3 + 4) &\rightarrow (1 + 2) \curvearrowright (\diamond + (3 + 4)) \\ &\rightarrow 3 \curvearrowright (\diamond + (3 + 4)) \\ &\rightarrow 3 + (3 + 4) \\ &\rightarrow (3 + 4) \curvearrowright (3 + \diamond) \\ &\rightarrow 7 \curvearrowright (3 + \diamond) \\ &\rightarrow 3 + 7 \\ &\rightarrow 10 \end{aligned}$$

## Configurations

A configuration of the K framework represents a state of a program. A configuration contains terms of a program and the environments. In the K definition, we have to specify an initial configuration for the initial state for a run of a program.

**Example 3.6.** The initial configuration of program  $x = 1; y = 2$ ; in the language SIMPLE (see section 3.2) is defined by the following term constructed by three cells.

$$\langle x = 1; y = 2; \rangle_K * \langle \epsilon_M \rangle_{env} * \langle 0 \rangle_{loc}$$

**Notation 3.7.** The  $K$  framework provides notations, defined by ‘ $\_$ ’ and ‘ $\dots$ ’, to represent an anonymous, unnamed, variable in the rewrite rules. Symbol ‘ $\_$ ’ is used when a variable is appeared only in the left-hand side of the rule. Symbol ‘ $\dots$ ’ is used when a variable is appeared both in the left-hand side and the right-hand side of the rule. When we use the abbreviation ‘ $\dots$ ’, we often omit ‘ $,$ ’ (comma) and brackets for the list notation, and ‘ $\curvearrowright$ ’ for the sort  $K$ .

The  $K$  framework represents the empty value  $\epsilon_\tau$  by  $.\tau$  (dot followed by the sort name).

**Example 3.8.** The rewrite rule

$$\left\langle \frac{V = I;}{\epsilon_K} \dots \right\rangle_K \left\langle \dots V \mapsto \frac{i}{I} \dots \right\rangle_{env}$$

in the language **SIMPLE** definition can be defined as

$$\left\langle \frac{V = I;}{\epsilon_K} \curvearrowright k \right\rangle_K \left\langle [e_1, V \mapsto \frac{i}{I}, e_2] \right\rangle_{env}$$

For cells on the top level, we often omit cells that we do not touch.

## 3.2 Example of K

We briefly describe the  $K$  framework by using a simple example to show how we can define a language in  $K$ .

Figure 2.1 shows the definitions in  $K$  of the language **SIMPLE**. There are three parts we have to define: syntax, configuration, and rewrite rules.

### Syntax of SIMPLE

For syntax definition, we define two new sorts which are **Exp**, expression, and **Stmt**, statement. **Exp** is formed by *Int*, Integer, or construct of plus, **Exp** + **Exp**. The plus construct is associated with **strict** attribute which means that its arguments must be evaluated before applying any rule to the construct. **Stmt** is formed by assignment from expression to variable name, *Id*, or is formed by sequencing of statements. In assignment construct, it is associated with **strict(2)** which means *Exp* terms must be evaluated before applying any rule to assignment construct. Additionally, the associativity is associated to the syntax definition, for sequencing statement we associate **left**, which means left associativity.

### Initial configuration of SIMPLE

For the language **SIMPLE**, we define the initial configuration as:

$$\langle \$PGM : K \rangle_K \langle \epsilon_M \rangle_{env}.$$

```

1 module SIMPLE
2   syntax Exp ::= Int
3     | Exp "+" Exp [strict]
4   syntax Stmt ::= Id "=" Exp ";" [strict(2)]
5     | Stmt Stmt [left]
6   syntax KResult ::= Int
7
8   configuration
9     <k> $PGM:K </k>
10    <env> .Map </env>
11
12  rule I1:Int + I2:Int => I1 +Int I2
13  rule <k> V = I:Int ; => . ... </k>
14    <env> ... V |-> (_ => I) ... </env>
15  rule <k> V = I:Int ; => . ... </k>
16    <env> ... . => V |-> I ... </env>
17  rule S1 S2 => S1 ~> S2
18 endmodule

```

Figure 3.1: Definition of the language SIMPLE in K

$K$  cell contains a term (abstract syntax tree) of the input language, denoted by  $\$PGM$ . The  $env$  cell contains an empty map.

### Rules of SIMPLE

In simple language, there are four rewrite rules which are corresponding to the following rewrite rules:

$$\begin{array}{ll}
1 : \frac{I_1 + I_2}{I_1 +_{\text{Int}} I_2} & 3 : \left\langle \frac{V = I; \dots}{\epsilon_k} \right\rangle_K \left\langle \dots V \mapsto \frac{\bar{\phantom{I}}}{I} \dots \right\rangle_{env} \\
2 : \frac{S_1 S_2}{S_1 \curvearrowright S_2} & 4 : \left\langle \frac{V = I; \dots}{\epsilon_k} \right\rangle_K \left\langle \dots \frac{\epsilon_M}{V \mapsto I} \right\rangle_{env}
\end{array}$$

The first rule says that the operator of plus with two integer arguments are rewritten to the primitive operator  $+_{\text{Int}}$  for the addition (on integers), which is predefined function in the K framework. The second rule manages statements  $S_1$  and  $S_2$  to be ordered in terms of sort K. The third rule is an assignment rule which rewrites the assignment statement into empty unit of sort K and change the value in  $env$  map when map already has index  $V$ . The fourth rule is an assignment rule which rewrites the assignment statement into the empty unit of sort K and inserts a new pair of  $V \mapsto I$  to  $env$ .

Suppose that we have a program **TEST** as below:

$$\$PGM \equiv \text{x=3;y=5; x=3 + 5;}$$

If we run the input program using K interpreter then we get the following rewrite steps:

$$\begin{aligned}
& \left\langle [\text{x=3;y=5;x=3 + 5;}] \right\rangle_K \left\langle \epsilon_M \right\rangle_{env} \\
& \rightarrow^+ \left\langle [\text{y=5;x=3 + 5;}] \right\rangle_K \left\langle [x \mapsto 3] \right\rangle_{env} \\
& \rightarrow^+ \left\langle [\text{x=3 + 5;}] \right\rangle_K \left\langle [x \mapsto 3, y \mapsto 5] \right\rangle_{env} \\
& \rightarrow^+ \left\langle [\text{x=8;}] \right\rangle_K \left\langle [x \mapsto 3, y \mapsto 5] \right\rangle_{env} \\
& \rightarrow^+ \left\langle \epsilon_L \right\rangle_K \left\langle [x \mapsto 8, y \mapsto 5] \right\rangle_{env}
\end{aligned}$$

## 3.3 Support environment of the K framework

Based on formal semantics definition, the K framework provides analysis/verification tools [9] which are automatically derived as in figure 3.2.

Once we compile (“kompile” command) the definition in the K framework, it is translated into Maude in which analysis tools are prepared.

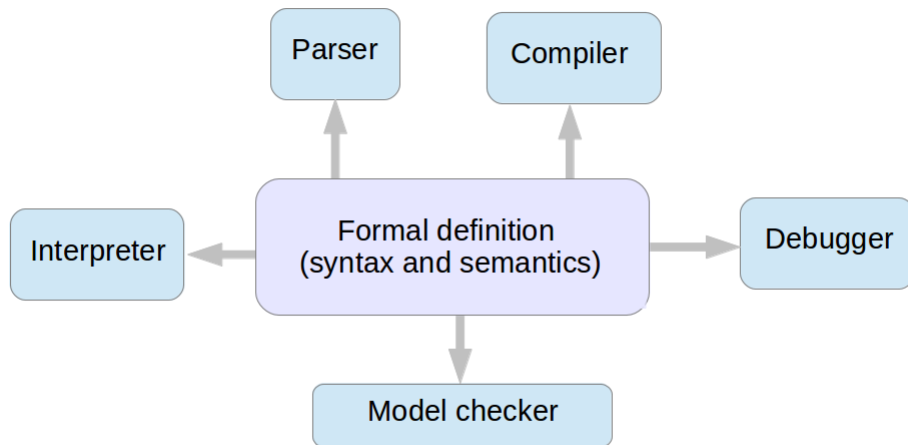


Figure 3.2: Analysis and verification tools [9]

### Parser

The K framework uses SDF for parser generation. SDF generates the abstract-syntax tree from a grammar described in algebraic specification.

### Interpreter

This is an immediate benefit of the language definition in the K framework. The K framework interprets K term by transforming it to Maude and Maude interprets it by rewriting.

### Compiler

The K framework prepares a compiler written in Maude. It transforms K definition into Maude code. It has been replacing from Maude compiler to equivalent transformation in Java which is currently operating in some part of compilation.

### Debugger

The K framework adopts Maude debugger by inserting a break point into K description. It is then translated into Maude code tagged with the break point. Maude debugger traces the execution and stop at each break point until the entire execution is done.

```
pakakorn@ubuntu:~/Desktop/k/Test/SIMPLE$ kompile simple.k
pakakorn@ubuntu:~/Desktop/k/Test/SIMPLE$ krun Test/test.simple
<k>
  .K
</k>
<env>
  x |-> 8
  y |-> 5
</env>
```

Figure 3.3: Result after running the program TEST with SIMPLE language

### Model checker

The K framework supports for concurrent programming which can have non-deterministic behaviours. Maude provides search command to see all possible behaviours and the K framework makes use of it for model checking. The model checker in the K framework adopts linear temporal logic (LTL) benefited from a (model-checker) built-in provided by Maude.

For example, when we can compile this language SIMPLE by using K compiler and run the program by K interpreter, and then we get the following result as Figure 3.3.

# Chapter 4

## SQL semantics

### 4.1 SQL table operations and their treatments

In SQL, all data are stored in the tables. The following are basic elements.

- A *field*, *attribute*, and *column* refers to a datum.
- A *data type* of a field is a domain of values.
- A *record* is a composition of values.
- A *table* is a collection of records.
- A *database* is a set of tables.

Typically table operations include

- Selection: `SELECT fields FROM list_of_tables WHERE predicate ;`  
This statement defines which column to be retrieved in *fields*, which table in *list\_of\_tables*, and the condition for filtering in *predicate*.
- Creation: `CREATE TABLE table_name (list_of_column_definition) ;`  
This statement creates the table with names given by the *table\_name* and the column definitions in *list\_of\_column\_definition* with the data type, the column name, the key assignment, etc.
- Insertion: `INSERT INTO table_name (column_definition) VALUE (values) ;`  
This statement inserts a new record to the *table\_name*. Such a new record has the fields as *list\_of\_column* and its values as *values*.



- Deletion: `DELETE FROM table_name WHERE predicate ;`  
This statement deletes elements in the table *table\_name* such that the predicate *predicate* are satisfied.
- Update: `UPDATE table_name SET assignments WHERE predicate ;`  
This statement updates records of the table *table\_name* with the assignment *assignments* that satisfied *predicate*.

Standard table operations among SQL dialects share similar core semantics described in a textbook except for variations of their syntax. However they vary in details, especially non-standard operations, such as:

- **(1) Treatment of NULL value** The treatment of NULL is one of the most important issues. The differences come from the meaning of this value. This different meanings bring confusion to the definition of semantics.
- **(2) Coercion among types** Coercion are implicitly conversion of types of arguments, e.g., `1 + "1a"` requires the coercion from string to integer.
- **(3) Boolean data types** SQL dialects have different representation of the boolean data type. Some simply uses zero and non-zero like MySQL. Some omits the like Oracle11.
- **(4) Error handling** The ways of error handling can be the following: error constant, explicit error messages, and replacing with possible values. Normally, we can see error handling by printing error messages, but in real SQL dialects, they have their own specific purposes to use the error constant for error representation.
- **(5) Name space** When we want to refer to an specific object, such as a column name in SQL language, we have to identify the name space or path direct to such a column. The name space is designed differently among SQL dialects. This leads difference among SQL dialects.

Among these differences we focus on the coercion, NULL, and the name space by comparison between MySQL and Oracle11.

## 4.2 Coercion

Data type represents a set of values, e.g., 32-bit integer (denoted by INT) and text string (denoted by TEXT) in MySQL. SQL has types of arguments of operations. However, for flexibility, it converts types to fit an operation by the coercion. Coercion consists of rules to convert one type of an object to a new object/value with a different type.

**Example 4.1.** *MySQL executes the statement `SELECT 1 + "1"`. The result of this statement is 2. Basically, operator `+` takes two arguments of integers, but MySQL extends the definition to cover other types. MySQL treats `Int + String` as `Int + Int` by applying implicit type conversion from `String` to `Int`, which converts `"1"` to 1.*

Here we show some possible choices of the coercion.

#### Coercion to boolean type

- Non-zero integers are converted to `TRUE` and 0 is converted to `FALSE`.
- Any string is converted to `FALSE`.
- A string is converted to an integer in some way, then converted to a boolean value.

#### Coercion to integer type

- `FALSE` is converted to 0 and `TRUE` is converted to 1.
- `FALSE` and `TRUE` are converted to *errors*.
- An integer string is converted to a concatenated integer; otherwise an *error*.
- A mixed string is converted to its maximum integer prefix; if it is empty, then 0.
- A string is converted to the concatenation of the ASCII number of each element.

#### Coercion to string type

- A number is converted to an integer string.
- A number is converted to an *error*.

MySQL treats `TRUE` and `FALSE` as aliases of 1 and 0, respectively. We start with examples in Table 4.1 to observe the semantics definition of the operator `+` with the coercion. For testing queries, we use the selection syntax `"SELECT Exp ;"` in MySQL. However, Oracle11 does not allow the selection of boolean-valued expressions *BExp* as an argument of the column, we need to distinguish queries on *BExp* such that `"SELECT 1 FROM DUAL WHERE BExp;"`. Table 4.1, 4.2, and 4.3 compare MySQL and Oracle11 on `+`, `<=`, and `&&`, respectively. Our observation is:

- In MySQL, `TRUE` is considered as 1 and `FALSE` is considered as 0.
- In MySQL, the coercion with `+` and `<=` are different. `+` always tries to coerce arguments into integers. `<=` changes its action depending on the types of arguments. For instance `"Int <= String"` coerces the string to the integer by taking the maximum number prefix. However, `"String <= String"` does not cause coercion; instead, simply overloading `<=` as the lexicographic extension of ASCII numbers.

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	1 + 1 ;	2	2
2	1 + 2 ;	3	3
3	1 + "1" ;	2	2
4	"1" + 1 ;	2	2
5	"1" + "1" ;	2	2
6	"1" + "2" ;	3	3
7	TRUE + 1 ;	2	<i>error</i>
8	FALSE + 1 ;	1	<i>error</i>
9	TRUE + FALSE ;	1	<i>error</i>
10	0 + "a" ;	0	<i>error</i>
11	1 + "a" ;	1	<i>error</i>
12	0 + "1a" ;	1	<i>error</i>
13	1 + "2a" ;	3	<i>error</i>
14	1 + "1a1" ;	2	<i>error</i>
15	0 + "-1" ;	-1	-1
16	0 + "-1a" ;	-1	<i>error</i>
17	"-1a" + "-1a" ;	-2	<i>error</i>
18	TRUE + "-1a" ;	0	<i>error</i>

Table 4.1: Testing queries for coercion of + in MySQL and Oracle

- In Oracle11, we can see that all queries confirm the coercion from an integer to a string. However, the coercion can accept only numbered content and does not accept TRUE and FALSE.
- In Oracle11 all queries result errors. This confirms us that Oracle11 does not allow 1, 0, TRUE, and FALSE for the arguments.

Our hypothesis on their semantics design policy is,

- MySQL adopts Zero and Non-zero to represent FALSE and TRUE.
- In MySQL, the coercion from String to Integer types will return the maximum prefix of numbers; if it is empty, it returns 0.
- In Oracle11, boolean primitive data are kept implicit values such that they are encapsulated for users. Users can use boolean expressions only in conditional expressions and there is no coercion from other types to the boolean type. Oracle11 allows the coercion from the string to the integer types, only when a string is an integer string.

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	1 <= 1 ;	1	1
2	1 <= 0 ;	0	<i>emp tbl</i>
3	1 <= "1" ;	1	1
4	"1" <= 1 ;	1	1
5	1 <= "0" ;	0	<i>emp tbl</i>
6	1 <= "a" ;	0	<i>error</i>
7	1 <= "1a" ;	1	<i>error</i>
8	TRUE <= 0 ;	0	<i>error</i>
9	TRUE <= 1 ;	1	<i>error</i>
10	2 <= TRUE ;	0	<i>error</i>
11	FALSE <= 1 ;	1	<i>error</i>
12	TRUE <= FALSE ;	0	<i>error</i>
13	"1" <= "2" ;	1	1
14	"2" <= "1" ;	0	<i>emp tbl</i>
15	"-1" <= "1" ;	1	1
16	"-2" <= "-1" ;	0	<i>emp tbl</i>
17	"a" <= "b" ;	1	1
18	"b" <= "a" ;	0	<i>emp tbl</i>
19	"a" <= "ab" ;	1	1
20	"bb" <= "ac" ;	0	<i>emp tbl</i>
21	"-" <= "-2" ;	1	1
22	"+1" <= "1" ;	1	1
23	"+2" <= "1" ;	1	1

Table 4.2: Testing queries for coercion of  $\leq$  in MySQL and Oracle

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	1 && 1 ;	1	<i>error</i>
2	1 && 0 ;	0	<i>error</i>
3	1 && "1" ;	1	<i>error</i>
4	"1" && 1 ;	1	<i>error</i>
5	"1" && "1" ;	1	<i>error</i>
6	"1" && "0" ;	0	<i>error</i>
7	TRUE && 1 ;	1	<i>error</i>
8	FALSE && 1 ;	0	<i>error</i>
9	TRUE && FALSE ;	0	<i>error</i>
10	1 && "a" ;	0	<i>error</i>
11	1 && "1a" ;	1	<i>error</i>
12	1 && "1a1" ;	1	<i>error</i>
13	"a" && "a" ;	0	<i>error</i>

Table 4.3: Testing queries for coercion of && in MySQL and Oracle11

### 4.3 Interpretation of NULL value

SQL has a special value NULL, which is interpreted differently among SQL dialects. There are choices on handling NULL:

- NULL is an undefined (unknown) value.
- NULL is an error.
- NULL is the empty string.
- NULL is FALSE.

For example, consider queries in Table 4.4.

Student	
No.	Name
1	NULL
2	Kim
3	Few
4	Nat

When we execute `SELECT * FROM Student`, we cannot decide what is an answer for the row containing NULL value. One might have a question that what name of the student

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	1 + 1 ;	2	2
2	1 + NULL ;	NULL	''
3	NULL + 1 ;	NULL	''
4	NULL + NULL ;	NULL	''
5	NULL    TRUE ;	1	<i>error</i>
6	TRUE    NULL ;	1	<i>error</i>
7	NULL && FALSE ;	0	<i>error</i>
8	FALSE && NULL ;	0	<i>error</i>
9	concat('1', NULL) ;	NULL	'1'

Table 4.4: Testing queries for NULL (undefined) treatment in MySQL and Oracle

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	NULL ;	NULL	''
2	CONCAT(NULL, 1) ;	NULL	'1'
3	CONCAT(NULL, "1") ;	NULL	'1'
4	NULL IS NULL ; ( <i>BExp</i> )	1	1
5	"" IS NULL ; ( <i>BExp</i> )	0	1

Table 4.5: Testing queries for NULL in string expressions in MySQL and Oracle11

number one is. If no answers, one might have a question that why the table has the number one.

MySQL regards NULL as:

- NULL means an unknown or undefined value.
- NULL means an error.

whereas Oracle11 regards it as the empty string. We observe them by examples. The expression of operator +, ||, &&, and concat (OR, AND, and || in Oracle11) to investigate how MySQL and Oracle11 treat (in table 4.4).

In MySQL, we observe that + is strict on NULL as well as other arithmetic and comparison operators. For boolean operations with NULL, MySQL treats it like Kleene's three value logic. They are shown in the queries 5-9 in table 4.4. In Oracle11, NULL is considered as the empty string. The query 9 in Table 4.4 and 4.5 show the contrast.

For MySQL, the queries in Table 4.5 show that even NULL in string behaves as an unknown value and is not equal to the empty string. As NULL is also used as failure of the

No.	<i>Exp</i> in MySQL	result (MySQL)	result (Oracle)
1	1 / 0 ;	NULL	<i>error</i>
2	1 % 0 ;	NULL	1

Table 4.6: Testing queries for NULL as errors in MySQL and Oracle11

No.	Query	result
1	SELECT 1 FROM T JOIN T	<i>error</i>
2	SELECT 1 FROM T JOIN T AS T2	1

Table 4.7: Testing queries for treatment conflict name space in MySQL

evaluation in Table 4.6. In Oracle11, an error is shown by the error messages (“*error*”) and NULL is the empty string. Thus, 1 + NULL contains the coercion on 1 from *Int* to (null) string where MySQL treats it as the sum with an undefined value (then NULL is returned).

The execution of 1/0 and 1%0 (zero-divisor) cause errors, but MySQL returns NULL as a value, while Oracle11 returns an error message.

## 4.4 Name space

MySQL and Oracle11 have the name space depending on how to handle table names. For instance, an operation with the same table, like the self join and product, how to distinguish references to elements in two tables with the same name. Possible choices of the name space are:

- Name conflict is not allowed.
- Identifying an object by prefixes of the database name and the table name (as in MySQL).
- Identifying an object by prefixes of the account name and the table name (as in Oracle).
- Identifying an object by prefixes of the database name, the schema name, and the table name (as in PostgreSQL).

These designs of the name space are to make identification of each object unique.

We observe the behaviour in MySQL in Table 4.7. The query 1 shows that both MySQL and Oracle do not accept self join operation due to the ambiguity of names. However, they solve such a problem by allowing user to give alias as in the query 2.

T1		T2	
A <sub>INT</sub>	B <sub>TEXT</sub>	A <sub>INT</sub>	B <sub>INT</sub>
1	"a"	1	1
2	"b"	2	0

What if we execute `SELECT A FROM T1 JOIN T2`? This query causes an error because both MySQL and Oracle11 cannot identify the attribute `A` whether it is in `T1` or `T2`. We can make the attribute `A` clear by providing a name space as `SELECT T1.A FROM T1 JOIN T2` or `SELECT D.T1.A FROM T1 JOIN T2`, where `D` for a database name. We call such multiple parts of an identifier as a qualifier. Oracle11 does similarly but with a different name space, `SELECT T1.A FROM T1 JOIN T2` or `SELECT U.T1.A FROM T1 JOIN T2`, where `U` for a username.

From our observation, we expect that MySQL accesses one object by identifying the database name and the object name, like `DatabaseName.TableName`. In contrast, Oracle11 accesses one object by identifying the user name and the object name, like `Owner.TableName`. PostgreSQL does with the database name, the schema name, and the table name, like `DatabaseName.SchemaName.TableName`.



# Chapter 5

## KSQL description of standard table operations

We present our formal description of database systems on K-framework, which we named *KSQL*. Typical SQL queries are creation, update, and retrieval of tables in a database. First we formalize tables. In KSQL, we prepare three sorts, *Int*, *Bool*, and *String*. *Int* literal value represents an integer. *Bool* values are represented by zero and one. Our semantics stays at the logical level and ignores the bit-length at the physical level. Here we have the subsort relation  $Bool \sqsubseteq Int$ . *String* values are represented by starting and ending with double quote (""), which contain text in between. We simply call *fields* for terms of sort *Field*, *values* for those of *Val*, *data types* for those of *DataType*, and (*table*) *identifiers* for terms of *Id*. In KSQL, we omit the physical level variations of data types, e.g., Int-16 and Int-32, and then we denote INT for the integer data type and TEXT for the string data type.

### 5.1 Syntax

**Definition 5.1.** A Field element is a tuple of a field and a data type. A Schema is a list of field elements, and a record is a tuple of values. We denote a set of schemas by *Schema* and a set of lists of records by *Record*. We define a table as the triple  $(T, S, R)$  of an identifier *T*, a schema *S*, and a list *R* of records, denoted by  $T[S : R]$ .

**Example 5.2.** Consider the two tables  $T1[S_1 : R_1]$  and  $T2[S_2 : R_2]$  with

$$\begin{aligned} S_1 &= [(A, INT), (B, TEXT)] & S_2 &= [(A, INT), (B, INT)] \\ R_1 &= [(1, "a"), (2, "b"), (3, "c")] & R_2 &= [(1, TRUE), (2, TRUE), (2, FALSE)] \end{aligned}$$

Note that `TRUE` and `FALSE` are aliases of 1 and 0, respectively. These tables are displayed as follows:

T1		T2	
$A_{INT}$	$B_{TEXT}$	$A_{INT}$	$C_{INT}$
1	<code>"a"</code>	1	TRUE
2	<code>"b"</code>	2	TRUE
3	<code>"c"</code>	3	FALSE

A database is a set of tables. Additionally, SQL supports *aliases* for table identifiers in order to solve the name conflict problem.

We formalize the core part of syntax for SQL queries.

**Definition 5.3.** *Queries or statements in KSQL are defined as follows:*

$$\begin{aligned}
 \text{Query} ::= & \text{CREATE TABLE } Id \text{ ( } FieldDcl* \text{ ) ;} \\
 & | \text{INSERT INTO } Id \text{ ( } Field* \text{ ) VALUES ( } Val* \text{ ) ;} \\
 & | \text{SELECT } ProjectionExp \text{ FROM } Id \text{ WHERE } Exp \text{ ;} \\
 & | \text{Query } Query
 \end{aligned}$$

In the creation of table semantics of KSQL, we omit data type checking of values, which will be done at the instruction level. We assume that expression  $Exp$  and  $ProjectionExp$  are defined as follows:

$$\begin{aligned}
 Exp ::= & Id \mid Int \mid String \mid Exp \circ Exp \\
 ProjectionExp ::= & * \mid Field \text{ (, } Field \text{ )}^* \\
 FieldDcl ::= & Field \text{ } DataType \\
 DataType ::= & INT \mid TEXT
 \end{aligned}$$

where  $\circ \in \{+, -, =, <\}$ .  $Field \text{ (, } Field \text{ )}^*$  stands for a non-empty list.

As notational conversion, we use  $T$  for a table identifier,  $n$  for a natural number,  $S$  and  $S'$  for schemas,  $R$  for a list of records,  $r$  for a record, and  $f$  for a field. We start with the syntax of `store` definition, which is a function used in defining the semantics of `CREATE`.

**Definition 5.4.** *The syntax of `store` is given by the next grammar:*

$$K ::= \dots \mid \text{store } Table$$

We define auxiliary functions `doGetTable`, `doCondition`, and `doProjection`, which are used in defining the semantics of `SELECT`.

**Definition 5.5.** *The syntax of `doGetTable`, `doCondition`, and `doProjection` are given by:*

$$\begin{aligned} \text{Table} ::= & \text{doGetTable}( \text{Id} ) \\ & | \text{doCondition}( \text{Table} , \text{Exp} ) \\ & | \text{doProjection}( \text{Table} , \text{ProjectionExp} ) \end{aligned}$$

## 5.2 Configuration

**Definition 5.6.** *Let  $\bar{S}$  be a set of schemas, and let  $\bar{R}$  be a set of lists of records. A (database) configuration is a term of the form:*

$$\text{Configuration} ::= \langle K \rangle_K \langle M_e \rangle_{env} \langle M_s \rangle_{schema} \langle M_r \rangle_{records} \langle Nat \rangle_{loc}$$

where  $M_e : Id \rightarrow Nat$ ,  $M_s : Nat \rightarrow \bar{S}$ , and  $M_r : Nat \rightarrow \bar{R}$ .

**Example 5.7** (Continued from Example 5.2). *We define a configuration for the database consisting of the tables T1 and T2 as below:*

$$\left\{ \begin{array}{l} \langle \epsilon_K \rangle_K \\ \langle [T1 \mapsto 0, T2 \mapsto 1] \rangle_{env} \\ \langle [0 \mapsto [(A, INT), (B, TEXT)], 1 \mapsto [(A, INT), (C, INT)]] \rangle_{schema} \\ \langle [0 \mapsto [(1, "a"), (2, "b"), (3, "c")], 1 \mapsto [(1, 1), (2, 1), (3, 0)]] \rangle_{records} \\ \langle 2 \rangle_{loc} \end{array} \right.$$

## 5.3 Rewrite rules

Before we define the semantics of the table creation, we need to define the interpretation of `store`. The `store` function moves a table content from a cell of  $K$  into the cells of  $env$ ,  $schema$ , and  $records$  in the configuration.

**Definition 5.8.** *The semantics of store is given by the following two rules:*

$$\begin{array}{l}
1 : \left\langle \frac{\text{store } T[S : R]}{\epsilon_K} \dots \right\rangle_K \\
\left\langle \dots T \mapsto n \dots \right\rangle_{env} \\
\left\langle \dots n \mapsto \frac{-}{S} \dots \right\rangle_{schema} \\
\left\langle \dots n \mapsto \frac{-}{R} \dots \right\rangle_{records} \\
\\
2 : \left\langle \frac{\text{store } T[S : R]}{\epsilon_K} \dots \right\rangle_K \\
\left\langle \dots \frac{\epsilon_M}{T \mapsto n} \right\rangle_{env} \\
\left\langle \dots \frac{\epsilon_M}{n \mapsto S} \right\rangle_{schema} \\
\left\langle \dots \frac{\epsilon_M}{n \mapsto R} \right\rangle_{records} \\
\left\langle n \mapsto n + 1 \right\rangle_{loc}
\end{array}$$

There are two rewrite rules for the `store` evaluation. The first rule is applied when we already have an identifier  $T$  in the *env* cell. The second rule is applied when we have no identifier  $T$  in the *env* cell. We will use `store` to define the semantics of `CREATE`. The `CREATE` query is used to create a table together with the declaration of a list of fields.

**Definition 5.9.** *The evaluation of CREATE is defined as follows:*

$$\left\langle \frac{\text{CREATE TABLE } T(Fd) ;}{T[\text{createSchema}(Fd) : \epsilon_L] \curvearrowright \text{store } \diamond} \dots \right\rangle_K$$

where `createSchema` is a function defined as below:

$$\text{createSchema}(Fd) = [f \mid f d \in Fd \text{ for some data type } d]$$

Here we use the comprehension notation  $[\dots \mid \dots]$ . The K framework does not support it but we can easily translate such a notation to corresponding recursive definitions in the K framework. We briefly explain the evaluation of the table creation. The evaluation of the table creation creates a structure of the table  $T$  containing fields from a list  $Fd$  of field declarations, and then use `store` to store the table in the configuration. An insertion inserts a new record to the configuration.

**Definition 5.10.** *The evaluation of INSERT is defined as follows:*

$$\begin{array}{l}
\left\langle \frac{\text{INSERT INTO } T(S) \text{ VALUES}(Vs);}{\epsilon_K} \dots \right\rangle_K \\
\left\langle \dots L \mapsto \frac{[R]}{[[Vs], R]} \dots \right\rangle_{records} \\
\left\langle \dots T \mapsto L \dots \right\rangle_{env} \\
\left\langle \dots L \mapsto S \dots \right\rangle_{schema}
\end{array}$$

where we assume that list  $Vs$  of values and the schema  $S$  have the same number of elements.

When inserting a new record, KSQL first find the location in the *env* cell and then store a schema and a new record of values in *schema* and *records* cells respectively.

The selection is the most complicated. As its syntax indicates, it consists of three ingredients. It begins with the table retrieval part, followed by the condition and the projection part. Their semantics is defined by auxiliary functions `doGetTable`, `doCondition`, and `doProjection`.

**Definition 5.11.** *The semantics of `doGetTable` is defined as follows:*

$$\left\langle \frac{\text{doGetTable}(T)}{T[S : R]} \dots \right\rangle_K$$

$$\left\langle \dots T \mapsto L \dots \right\rangle_{env}$$

$$\left\langle \dots L \mapsto S \dots \right\rangle_{schema}$$

$$\left\langle \dots L \mapsto R \dots \right\rangle_{records}.$$

Before we define `doCondition`, we need two more auxiliary functions `eval` and `filter`. The function `eval` is to evaluate an expression using data in a record, which is specific to a schema. The function `filter` is to filter a list of records, which satisfies the condition for the schema.

**Definition 5.12.** *For an identifier  $I$ , integers  $n$  and  $m$ , a string  $s$ , and a value  $v$ , we inductively define the `eval` function as follows:*

$$\text{eval} : \text{Schema} \times \text{Record} \times \text{Exp} \rightarrow \text{Exp}$$

$$\text{eval}(S, r, I) = (S \otimes r)(I)$$

$$\text{eval}(S, r, n) = n$$

$$\text{eval}(S, r, s) = s$$

$$\text{eval}(S, r, n + m) = \text{eval}(S, r, n) + \text{eval}(S, r, m)$$

$$\text{eval}(S, r, n - m) = \text{eval}(S, r, n) - \text{eval}(S, r, m)$$

$$\text{eval}(S, r, n = m) = \text{eval}(S, r, n) = \text{eval}(S, r, m)$$

$$\text{eval}(S, r, n < m) = \text{eval}(S, r, n) < \text{eval}(S, r, m)$$

**Definition 5.13.** *The definition of `filter` for a schema  $S$ , a list  $R$  of records, and an expression  $E$  is defined by:*

$$\text{filter} : \text{Schema} \times \text{Records} \times \text{Exp} \rightarrow \text{Records}$$

$$\text{filter}(S, R, E) = [ r \in R \mid \text{eval}(S, r, E) = 1 ]$$

**Definition 5.14.** *The semantics of doCondition is defined as follows:*

$$\left\langle \frac{\text{doCondition}(T[S : R], E)}{T[S : \text{filter}(S, R, E)]} \dots \right\rangle_K$$

The last auxiliary function is doProjection, and we need one more auxiliary function project, which extracts attributes that we need.

**Definition 5.15.** *The definition of project, which generates a new schema S' from a schema S and a list R of records, is as follows:*

$$\begin{aligned} \text{project} &: \text{Schema} \times \text{Records} \times \text{Schema} \rightarrow \text{Records} \\ \text{project}(S, R, S') &= [(S \otimes r)(f) \mid f \in S' \mid r \in R] \end{aligned}$$

**Definition 5.16.** *The semantics of doProjection is defined as follows:*

$$1 : \left\langle \frac{\text{doProjection}(T[S : R], *)}{T[S : R]} \dots \right\rangle_K \quad 2 : \left\langle \frac{\text{doProjection}(T[S : R], S')}{\text{project}(S, R, S')} \dots \right\rangle_K$$

The first rule contains \* (which means all of attributes) and returns a table without changes. The second rule projects only needed attributes from the table, and generates S'.

The selection consists of (1) get the table content from the table name (doGetTable), (2) filter its records that satisfy the conditions (doCondition), and (3) project needed attributes (doProjection).

**Definition 5.17.** *The semantics of SELECT is defined as follows:*

$$\left\langle \frac{\text{SELECT } P \text{ FROM } T \text{ WHERE } E ;}{\text{doGetTable}(T) \curvearrow \text{doCondition}(\diamond, E) \curvearrow \text{doProjection}(\diamond, P)} \dots \right\rangle_K$$

In addition to the definition above, we need structural rules. Each of which instantiates a value to  $\diamond$  and changes the sequence of queries into the list of K terms.

**Definition 5.18.** *The structural rules for the instantiation of a table to  $\diamond$  are defined as follows:*

$$\begin{aligned} &\left\langle \frac{T[S : R] \curvearrow \text{store } \diamond}{\text{store } T[S : R]} \dots \right\rangle_K \\ &\left\langle \frac{T[S : R] \curvearrow \text{doCondition}(\diamond, E)}{\text{doCondition}(T[S : R], E)} \dots \right\rangle_K \\ &\left\langle \frac{T[S : R] \curvearrow \text{doProjection}(\diamond, P)}{\text{doProjection}(T[S : R], P)} \dots \right\rangle_K \end{aligned}$$

**Definition 5.19.** *The structural rule to transform a term of a sequence of queries into a list of K terms is defined as follows:*

$$\left\langle \frac{Q_1 Q_2}{Q_1 \curvearrowright Q_2} \dots \right\rangle_K$$

The rule above gets a term formed by  $Q_1$  and  $Q_2$  and then rewrite it into a K term  $Q_1 \curvearrowright Q_2$ . The K framework will try to rewrite the first argument of K terms until it reaches normal form, then it unifies the first term to the next term.

**Example 5.20.** *Let  $Q = S_1, \dots, S_5$  with for convenience, we first introduce statement variables  $S_1, \dots, S_5$  where*

```

S1 = CREATE TABLE T1(A INT, B TEXT) ;
S2 = INSERT INTO T1(A, B) VALUES(1, "a") ;
S3 = INSERT INTO T1(A, B) VALUES(2, "b") ;
S4 = INSERT INTO T1(A, B) VALUES(3, "c") ;
S5 = SELECT * FROM T1 WHERE A > 1 ;

```

*The query Q is evaluated as follows:*

$$\begin{aligned}
& \langle S_1 S_2 S_3 S_4 S_5 \rangle_K C_1 \quad \text{where } C_1 = \langle \epsilon_M \rangle_{env} \langle \epsilon_M \rangle_{schema} \langle \epsilon_M \rangle_{records} \langle 0 \rangle_{loc} \\
& \rightarrow^* \langle S_1 \curvearrowright S_2 \curvearrowright S_3 \curvearrowright S_4 \curvearrowright S_5 \rangle_K C_1 \\
& \rightarrow^* \langle T_1 \curvearrowright store \diamond \curvearrowright S_2 \curvearrowright S_3 \curvearrowright S_4 \curvearrowright S_5 \rangle_K C_1 \\
& \rightarrow^* \langle store T_1 \curvearrowright S_2 \curvearrowright S_3 \curvearrowright S_4 \curvearrowright S_5 \rangle_K C_1 \\
& \rightarrow^* \langle S_2 \curvearrowright S_3 \curvearrowright S_4 \curvearrowright S_5 \rangle_K C_2 \quad \text{where } C_2 = D \langle 0 \mapsto \epsilon_L \rangle_{records} \\
& \quad \text{and } D = \langle [T_1 \mapsto 0] \rangle_{env} \langle 0 \mapsto [(A, INT), (B, TEXT)] \rangle_{schema} \langle 1 \rangle_{loc} \\
& \rightarrow^* \langle S_3 \curvearrowright S_4 \curvearrowright S_5 \rangle_K C_3 \quad \text{where } C_3 = D \langle 0 \mapsto [(1, "a")] \rangle_{records} \\
& \rightarrow^* \langle S_4 \curvearrowright S_5 \rangle_K C_4 \quad \text{where } C_4 = D \langle 0 \mapsto [(1, "a"), (2, "b")] \rangle_{records} \\
& \rightarrow^* \langle S_5 \rangle_K C_5 \quad \text{where } C_5 = D \langle 0 \mapsto [(1, "a"), (2, "b"), (3, "c")] \rangle_{records} \\
& \rightarrow^* \langle doGetTable(T_1) \curvearrowright doCondition(\diamond, A > 1) \curvearrowright doProjection(\diamond, *) \rangle_K C_5 \\
& \rightarrow^* \langle T_2 \curvearrowright doCondition(\diamond, A > 1) \curvearrowright doProjection(\diamond, *) \rangle_K C_5 \\
& \rightarrow^* \langle doCondition(T_2, A > 1) \curvearrowright doProjection(\diamond, *) \rangle_K C_5 \\
& \rightarrow^* \langle T_2 \curvearrowright doProjection(\diamond, *) \rangle_K C_5 \\
& \rightarrow^* \langle T_3 \curvearrowright doProjection(\diamond, *) \rangle_K C_5 \\
& \rightarrow^* \langle doProjection(T_3, *) \rangle_K C_5 \\
& \rightarrow^* \langle T_3 \rangle_K C_5
\end{aligned}$$

T <sub>1</sub>	
A <sub>INT</sub>	B <sub>TEXT</sub>

T <sub>2</sub>	
A <sub>INT</sub>	B <sub>TEXT</sub>
1	"a"
2	"b"
3	"c"

T <sub>3</sub>	
A <sub>INT</sub>	B <sub>TEXT</sub>
2	"b"
3	"c"



# Chapter 6

## KSQL descriptions of coercion, NULL, and name space in MySQL

### 6.1 Coercion in arithmetic and boolean operations

We define the semantics of the coercion in the K framework as observed in the section 4. First we recall the structure of expressions.

$$Exp ::= \dots \mid Exp \circ Exp \mid \text{NOT } Exp$$

where  $\circ \in \{+, -, *, /, \%, =, >=, >, <=, <, !=, ||, \&\&\}$  and  $Exp$  is an expression, which refers to an integer or a string value.

The K framework provides functions that take one literal value and convert it from some type to the string and vice versa.

**Definition 6.1.** *The K framework provides functions for types conversion as follow:*

$$\begin{aligned} \text{tokenToString} &: Token \rightarrow String \\ \text{parseToken} &: \tau \times String \rightarrow Token \end{aligned}$$

where  $Token$  is a set of literal values (string and integer). The function `tokenToString` converts the value into the corresponded string and `parseToken` returns the literal value corresponds to the sort  $\tau$  such that its value is described in the  $String$ .

By combining the two functions above, we construct a new function as follows:

$$\mathcal{C} : \tau_1 \times \tau_2 \times Token_{\tau_1} \rightarrow Token_{\tau_2}$$

which convert the value  $v$  of the sort  $\tau_1$ , corresponding to the value of the sort  $\tau_2$ . We also denote  $\mathcal{C}_{\tau_1 \rightarrow \tau_2}(v)$  for  $\mathcal{C}(\tau_1, \tau_2, v)$ . We call  $\mathcal{C}$  a coercion function and we denote the coercion function for an operator  $f$  by  $\mathcal{C}^f$ .

In MySQL, TRUE and FALSE are aliases of 1 and 0, respectively.

**Definition 6.2.** The coercion function from *Int* to *Bool* is defined as follows:

$$\mathcal{C}_{Int \rightarrow Bool}^\circ(i) = \begin{cases} 0 & \text{if } i = 0 \\ 1 & \text{otherwise} \end{cases}$$

where  $i$  is an integer.

Conversion from *String* to *Int* is more complicated.

**Definition 6.3.** The coercion function from *String* to *Int* is defined as follows:

$$\mathcal{C}_{String \rightarrow Int}^\circ(s) = \begin{cases} n & \text{if } s \text{ is } \overbrace{[+-]?[0-9]^+}^n \wedge [0-9]?(.*) \\ 0 & \text{otherwise} \end{cases}$$

where  $n$  is an integer.

Now, we define the semantics of arithmetic and boolean operators with the coercion.

**Definition 6.4.** Let  $i, i_1$ , and  $i_2$  be integers, and let  $s, s_1$ , and  $s_2$  be strings. The semantics of the arithmetic operator  $\circ$  is defined as:

$$\begin{array}{ll} 1 : \frac{i_1 \circ i_2}{m} & 2 : \frac{i \circ s}{i \circ \mathcal{C}_{String \rightarrow Int}^\circ(s)} \\ 3 : \frac{s \circ i}{\mathcal{C}_{String \rightarrow Int}^\circ(s) \circ i} & 4 : \frac{s_1 \circ s_2}{\mathcal{C}_{String \rightarrow Int}^\circ(s_1) \circ \mathcal{C}_{String \rightarrow Int}^\circ(s_2)} \end{array}$$

where  $\circ \in \{+, -, *, /, \%\}$  and  $m$  is the value after execution of the standard arithmetic on integers  $i_1$  and  $i_2$ .

**Definition 6.5.** Let  $b, b_1$ , and  $b_2$  be boolean values, let  $i, i_1$ , and  $i_2$  be integers, and let  $s, s_1$ , and  $s_2$  be strings. The semantics of the logical operator  $\circ$  is defined as:

$$\begin{array}{l}
1 : \frac{b_1 \circ b_2}{m} \\
3 : \frac{i \circ b}{\mathcal{C}_{Int \rightarrow Bool}^\circ(i) \circ b} \\
7 : \frac{i \circ s}{i \circ \mathcal{C}_{String \rightarrow Int}^\circ(s)} \\
7 : \frac{s_1 \circ s_2}{\mathcal{C}_{String \rightarrow Int}^\circ(s_1) \circ \mathcal{C}_{String \rightarrow Int}^\circ(s_2)} \\
2 : \frac{b \circ i}{b \circ \mathcal{C}_{Int \rightarrow Bool}^\circ(i)} \\
4 : \frac{i_1 \circ i_2}{\mathcal{C}_{Int \rightarrow Bool}^\circ(i_1) \circ \mathcal{C}_{Int \rightarrow Bool}^\circ(i_2)} \\
5 : \frac{s \circ i}{\mathcal{C}_{String \rightarrow Int}^\circ(s) \circ i}
\end{array}$$

where  $\circ \in \{\&\&, ||\}$  and  $m$  is the value after execution of the standard boolean operation on  $b_1$  and  $b_2$ .

Next we formalize the semantics of comparison operators.

**Definition 6.6.** Let  $i, i_1$ , and  $i_2$  be integers and let  $s, s_1$ , and  $s_2$  be strings. The semantics of comparison operator  $\diamond$  is defined as follows:

$$\begin{array}{l}
\frac{i_1 \diamond i_2}{m} \\
\frac{s \diamond i}{\mathcal{C}_{String \rightarrow Int}^\circ(s) \diamond i} \\
\frac{i \diamond s}{i \diamond \mathcal{C}_{String \rightarrow Int}^\circ(s)} \\
\frac{s_1 \diamond s_2}{s_1 \diamond_{lex} s_2}
\end{array}$$

where  $\diamond \in \{=, <, \leq, >, \geq, \neq\}$ ,  $m$  is 1 if  $i_1 \diamond i_2$  holds; 0, otherwise, and  $\diamond_{lex}$  is the string comparison corresponds to the lexicographical ordering on strings.

The type coercion function  $\mathcal{C}_{String \rightarrow Int}^\circ$  is defined as the same as  $\mathcal{C}_{String \rightarrow Int}^\circ$ . Table 6.1, 6.2, and 6.3 show testing results of KSQL and MySQL.

## 6.2 Treatment of NULL value

**Definition 6.7.** NULL is a constant of the sort Val.

As we have observed behaviour of NULL in Section 4. The NULL is of the bottom data type, and NULL follows to strict semantics for functions (except for boolean operations).

**Definition 6.8.** For each operator  $f : \tau_1 \times \cdots \times \tau_{NULL} \times \cdots \times \tau_n \rightarrow \tau$ , the semantics of NULL is defined as follows:

$$\frac{f(e_1, \dots, NULL, \dots, e_n)}{NULL},$$

where  $e_1, \dots, e_n$  are expressions and  $\tau_{NULL}$ .

No.	query	result (MySQL)	result (KSQL)
1	SELECT 1 + 1 ;	2	2
2	SELECT 1 + 2 ;	3	3
3	SELECT 1 + "1" ;	2	2
4	SELECT "1" + 1 ;	2	2
5	SELECT "1" + "1" ;	2	2
6	SELECT "1" + "2" ;	3	3
7	SELECT TRUE + 1 ;	2	2
8	SELECT FALSE + 1 ;	1	1
9	SELECT TRUE + FALSE ;	1	1
10	SELECT 0 + "a" ;	0	0
11	SELECT 1 + "a" ;	1	1
12	SELECT 0 + "1a" ;	1	1
13	SELECT 1 + "2a" ;	3	3
14	SELECT 1 + "1a1" ;	2	2
15	SELECT 0 + "-1" ;	-1	-1
16	SELECT 0 + "-1a" ;	-1	-1
17	SELECT "-1a" + "-1a" ;	-2	-2
18	SELECT TRUE + "-1a" ;	0	0

Table 6.1: Comparison of the coercion of + between MySQL and KSQL

Number	Query	result (MySQL)	result (KSQL)
1	SELECT 1 <= 1 ;	1	1
2	SELECT 1 <= 0 ;	0	0
3	SELECT 1 <= "1" ;	1	1
4	SELECT "1" <= 1 ;	1	1
5	SELECT 1 <= "0" ;	0	0
6	SELECT 1 <= "a" ;	0	0
7	SELECT 1 <= "1a" ;	1	1
8	SELECT TRUE <= 0 ;	0	0
9	SELECT TRUE <= 1 ;	1	1
10	SELECT 2 <= TRUE ;	0	0
11	SELECT FALSE <= 1 ;	1	1
12	SELECT TRUE <= FALSE ;	0	0
13	SELECT "1" <= "2" ;	1	1
14	SELECT "2" <= "1" ;	0	0
15	SELECT "-1" <= "1" ;	1	1
16	SELECT "-2" <= "-1" ;	0	0
17	SELECT "a" <= "b" ;	1	1
18	SELECT "b" <= "a" ;	0	0
19	SELECT "a" <= "ab" ;	1	1
20	SELECT "bb" <= "ac" ;	0	0
21	SELECT "-" <= "-2" ;	1	1

Table 6.2: Comparison of the coercion of  $\leq$  between MySQL and KSQL

No.	Query	result (MySQL)	result (KSQL)
1	SELECT 1 && 1 ;	1	1
2	SELECT 1 && 0 ;	0	0
3	SELECT 1 && "1" ;	1	1
4	SELECT "1" && 1 ;	1	1
5	SELECT "1" && "1" ;	1	1
6	SELECT "1" && "0" ;	0	0
7	SELECT TRUE && 1 ;	1	1
8	SELECT FALSE && 1 ;	0	0
9	SELECT TRUE && FALSE ;	0	0
10	SELECT 1 && "a" ;	0	0
11	SELECT 1 && "1a" ;	1	1
12	SELECT 1 && "1a1" ;	1	1
13	SELECT "a" && "a" ;	0	0

Table 6.3: Comparison of the coercion of && between MySQL and KSQL

The semantics of NULL in MySQL is two fold.

- Description of an error, which obeys to the strict semantics (Definition 6.8)
- The bottom constant in a three-valued logic (Definition 6.9)

In MySQL, if a function is a boolean operation, NULL behaves as the bottom constant. Otherwise, NULL is an error constant.

**Definition 6.9.** *The semantics of boolean operators*

$$\text{NOT} : \text{Bool} \quad \&\& : \text{Bool} \times \text{Bool} \rightarrow \text{Bool} \quad || : \text{Bool} \times \text{Bool} \rightarrow \text{Bool}$$

are defined as follows:

&&	TRUE	NULL	FALSE		TRUE	NULL	FALSE	NOT
TRUE	TRUE	NULL	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
NULL	NULL	NULL	FALSE	NULL	TRUE	NULL	NULL	NULL
FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	NULL	FALSE	TRUE

Lastly we have two special rules for zero-divisor in / and % operators.

**Definition 6.10.** *The semantics of zero-divisors of operators / and % are defined as*

$$e / 0 \rightarrow \text{NULL} \quad e \% 0 \rightarrow \text{NULL}$$

The other errors are defined similarly using NULL representation. However, we cannot generalize NULL semantics for each operator, because of conflicts of the specification above.

The tables 6.4, 6.5, and 6.6 show testing results on NULL of MySQL and KSQL.

No.	Query	result (MySQL)	result (KSQL)
1	SELECT 1 + 1 ;	2	2
2	SELECT 1 + NULL ;	NULL	NULL
3	SELECT NULL + 1 ;	NULL	NULL
4	SELECT NULL + NULL ;	NULL	NULL
5	SELECT NULL    TRUE ;	1	1
6	SELECT TRUE    NULL ;	1	1
7	SELECT NULL && FALSE ;	0	0
8	SELECT FALSE && NULL ;	0	0

Table 6.4: Comparison of NULL (undefined) treatment between MySQL and KSQL

No.	MySQL query	result (MySQL)	result (KSQL)
1	SELECT NULL ;	NULL	NULL
2	SELECT CONCAT(NULL, 1) ;	NULL	NULL
3	SELECT CONCAT(NULL, "1") ;	NULL	NULL
4	SELECT NULL IS NULL ;	1	1
5	SELECT "" IS NULL ;	0	0

Table 6.5: Comparison of NULL in string expressions between MySQL and KSQL

No.	Query	result (MySQL)	result (KSQL)
1	SELECT 1 / 0 ;	NULL	NULL
2	SELECT 1 % 0 ;	NULL	NULL

Table 6.6: Comparison of NULL as errors between MySQL and KSQL

## 6.3 Name space

KSQL supports the name space. We keep a field name in the form *TableName.FieldName* and the schema is the set of those form. The semantics supports the field name extension.

**Definition 6.11.** *The evaluation of CREATE is defined as follows:*

$$\left\langle \frac{\text{CREATE TABLE } T(Fd) ;}{T[\text{createSchema}(T, Fd) : \epsilon_L] \curvearrowright \text{store } \diamond} \cdots \right\rangle_K$$

where  $T$  is a table identifier and `createSchema` is a function defined below:

$$\text{createSchema}(T, Fd) = [T.f \mid f d \in Fd \text{ for some data type } d]$$

MySQL allows a user to identify the field by either by a field name or by a table name with the field name. We define a function to unify them to the latter form, i.e., *TableName.FieldName*.

**Definition 6.12.** *The function uniform is defined as follows:*

$$\text{uniform}(T, S) = [T.f \mid f \in S \vee T.f \in S]$$

Corresponding to this extension, each operation is redefined.

**Definition 6.13.** *The semantics of SELECT is redefined as follows:*

$$\left\langle \frac{\text{SELECT } P \text{ FROM } T \text{ WHERE } E ;}{\text{doGetTable}(T) \curvearrowright \text{doCondition}(\diamond, E) \curvearrowright \text{doProjection}(\diamond, \text{uniform}(T, P))} \cdots \right\rangle_K$$

Other rules are redefined in a similar way.

## 6.4 Limits and difficulties in KSQL

### Coercion

To support the type coercion, we need to define many rewrite rules. For instance the semantics of  $+$  and  $\leq$  operators require four rewrite rules for each. This is to support the coercion from string to integer.

Our implementation of the coercion in KSQL is based on extensive case analysis. If  $f$  has  $n$ -arguments, the direct encoding of the coercion from sort  $\tau_i$  to  $\tau'_i$  ( $1 \leq i \leq n$ ) is to prepare

$$f(t_1, \dots, t_n) \rightarrow f(\mathcal{C}_{\tau_1 \rightarrow \tau'_1}^f(t_1), \dots, \mathcal{C}_{\tau'_n \rightarrow \tau'_n}^f(t_n))$$

(if  $\mathcal{C}_{\tau_i \rightarrow \tau'_i}^f(t_i) \neq t_i$  for some  $i$ ), and becomes  $2^n - 1$  rules.



Furthermore, if we have  $m$  different sorts, Each combination requires such rewrite rules and the total number of rewrite rules is  $(m - 1)(2^n - 1)$ .

Furthermore, if there are  $p$  operators  $f_1, \dots, f_p$  in a language, each of them has  $n_1, \dots, n_p$  arguments respectively. Then the number of rewrite rules becomes

$$\sum_{i=1}^p (m - 1)(2^{n_i} - 1).$$

For the coercion of current KSQL, there are 13 operators and 2 sorts. Each operator has 2 arguments, thus we need 39 rules (instead of 13 rules).

This exponential growth prohibits to apply our current method to a practical language. There are several possibility to reduce the number of rules. The first idea is to separate operators into groups of operators, which have the same type of argument. For example, arithmetic operators and string operators have certain similar structure in each category. By defining new structure of expressions as below:

$$\begin{aligned} Op &::= + \mid - \mid * \mid / \mid \% \\ AExp &::= AExp Op AExp \end{aligned}$$

The type of these operators' arguments is the integer type. Then, we can simply define semantics of the coercion as the rules 2, 3, 4 in Definition 6.4 by replacing  $\circ$  with  $Op$ . This can reduce the number of rules from 15 to 3.

The second idea would be to use inheritance of the ordered sort, which we have not investigate so far.

## Treatment of NULL

As we have seen that the semantics to support NULL requires many rewrite rules.

Some operators IS NULL, IS NOT NULL, or IFNULL, explicitly accept NULL as an argument. Except for such operators, the n-arguments require n-rules. Suppose that there are  $p$  operators  $f_1, \dots, f_p$  with  $n_1, \dots, n_p$  arguments, respectively. Then the number of rewrite rules is

$$\sum_{i=1}^p n_i.$$

Additionally, we have to define rules for error handling. It is difficult to exactly observe all behaviours of the error handling.

Although the number of rewrite rules for NULL treatment affect less (since it is not exponential). Still it is difficult because MySQL has many error-handling solutions. Further investigation will be required.

## Logical and physical model of data types

For integer and string data types, KSQL supports unbounded bits of integers and strings. In fact, MySQL has variation of data types. For example, the integer data type varies TINYINT, SMALLINT, MEDIUMINT, INT, and BIGINT for 8-bit, 16-bit, 32-bit, 64-bit, and 128-bit integers, respectively. Current implementation of KSQL does not support such detailed variation of data types. This is because when we insert a datum into a field of integer, MySQL solves overflow and underflow of such an integer by bounding its value, which is not supported by current KSQL.

## Uniqueness of primary key

Oracle11 and MySQL do not allow the duplication of the primary key. When the primary key has duplicated values, Oracle11 and MySQL return errors. In current implementation of KSQL, each record has a product type and thus allows duplicated records. This can be simply solved by defining a function type. For instance, consider a record that contains three integer columns, A, B, and C. Suppose that the primary key contains only the attribute A. Then, a record (1, 2, 3) is a relation represented by (1, [2, 3]) instead of a list [1, 2, 3]. We can define a type of records as  $Int \rightarrow (Int \times Int)$  instead of  $Int \times Int \times Int$ .

# Chapter 7

## Related work

There are several works to give formal semantics of practical languages. We overview such works in three views.

- Formal semantics of SQL [8]
- Formal semantics of other programming languages: PHP [11], DATALOG [1], self-modifying x86 [2].
- Executable formal semantics on K framework: JAVA [4], Verilog [6], Scheme [5], and C [3].

### 7.1 Formal semantics of SQL language

The semantics is based on first-order-logic (FOL) and this leads to the problem of the NULL definition.

**Negri, Pelagatti, and Sbattella (1991)** gave a formal semantics of SQL queries [8]. The work starts with the translation of SQL to a formal model consisting of a set of rules in first order logic. The formal model is called *Extended Three Valued Predicate Calculus* (E3VPC). However, their semantics is still missing the real semantics, e.g. NULL. First, an undefined value NULL in MySQL is in Kleene's three valued logic beyond the standard FOL. Secondly, their semantics focuses on the selection query only, while ours semantics additionally define creation, deletion, update, and insertion.

### 7.2 Other formal semantics

**PHP** Tozawa, Tatsubori, Onodera and Minamide (2009) gave the definition of a copy-on-write semantics of PHP language [11]. The semantics is used to solve copy-on-assignment,

which causes copy overhead. Copy-on-write model is formalized by using graph rewriting.

**DATALOG** Alpuente, Feliu, Joubert, and Villanueva (2010) formalized the definition of DATALOG in a rewriting logic using Maude [1]. DATALOG is also a relational query language, beyond RDB, like SQL. It is a subset of Prolog (Prolog with only predicates and constants) and allow recursive expressions. Their focus is more on standard operations, but not on boundary cases, like error handling and coercion.

**x86** Bonfante, Marion, and Reynaud-Plantey (2009) gave the formal semantics for self-modifying x86 programs [2]. A self-modifying binary program can be constructively rewritten to a non-modifying program.

### 7.3 Executable formal semantics on K framework

There are several programming languages being defined on the K framework. They motivated us to describe a core SQL language in the K framework.

**Java** Farzan, Chen, Meseguer, and Rosu (2004) gave the semantic of a program analysis framework for Java [4]. The semantics can be applied to model-checker provided by K framework.

**Scheme** Meredith, Hills, and Rosu (2007) defined an equational semantics of Scheme [5]. The semantics includes the support for macros.

**Verilog** Meredith, Katelman, Meseguer, and Rosu (2010) gave a formal executable semantics of Verilog [6]. Their semantics is used to emulate programs and search on its behaviours.

**C** Ellison and Rosu (2012) gave of an executable formal semantics of C [3]. Contributions of this work are not only the formal semantics itself, but also illustrating a way to discover bugs and the use of K framework for defining non-deterministic behaviours of a C program.

# Chapter 8

## Conclusion

In this thesis, we investigated the formal semantics of the core of SQL dialects, specifically MySQL and Oracle11. The former is implemented in the K framework as KSQL.

We found semantics differences on the coercion of types, the interpretation of NULL, the name space management, the error handling, and the logical/physical model of data types. Among the differences, we focus on the first three parts. We observe differences between MySQL and Oracle11:

- Consider the coercion from a string to an integer. Oracle11 returns an error if a string contains non-digit characters, while MySQL tries to conceal this problem.
- MySQL interprets NULL as an undefined value for boolean operations. However, NULL is also used as the return of an error like zero divisor. Surprisingly, MySQL further accepts NULL as an explicit argument, e.g., NULL IS NULL, ISNULL, IS NOT NULL, and IFNULL. We feel that these behaviour would lead inconsistency. Oracle11 has more reasonable behaviour. It interprets NULL as the empty string. An error is reported by an error message, not resulting NULL as an output.
- Oracle11 and MySQL adopt different name space conversion. The name space is the matter when the table with the same name appears repeatedly. For instance, self-join and self-product are such cases. Both MySQL and Oracle refuse them.

Our work is just to open possibility of formal semantics of SQL. There are several obstructions as future work.

- Current implementation of coercion is based on an exhaustive case analysis, which explodes with an exponentially many number of rewrite rules. (NULL requires similar case analysis, but the number of rules does not grow exponentially.) It can be reduced by unifying similar cases. Other possibility is to use the inheritance among ordered sorts.

- Currently, a table is defined to have a product type, and the uniqueness of the primary key is not guaranteed. This can be solved by applying a function type for a table, instead of the product type.
- Currently, KSQL ignores physical models (bit-length) of data types, which affects the semantics of JOIN operator. This is the reason why current KSQL does not implement JOIN.

# Appendices

# Appendix A

## SQL semantics in K-framework

### A.1 Expression Syntax

```
1 module EXP-SYNTAX
2   syntax Boolean ::= "TRUE" | "FALSE"
3   syntax Null ::= "NULL"
4   syntax Val ::= Int | Bool | Float | String | Null | Boolean
5   syntax Vals ::= List{Val, ","}
6   syntax Exp ::= Val | Id
7   syntax KResult ::= #Int | #Bool | Id | #String | #Float | Null
8   syntax Exp ::= "(" Exp ")" [bracket]
9     > "-" Exp [strict]
10    > Exp "*" Exp [strict, left]
11    | Exp "/" Exp [strict, left]
12    | Exp "DIV" Exp [strict, left]
13    | Exp "MOD" Exp [strict, left]
14    | Exp "%" Exp [strict, left]
15    > Exp "+" Exp [strict, left]
16    | Exp "-" Exp [strict, left]
17   syntax Exp ::= "(" Exp ")" [bracket]
18     > "!" Exp [strict]
19     > Exp "=" Exp [strict, non-assoc]
20     | Exp ">=" Exp [strict, non-assoc]
21     | Exp ">" Exp [strict, non-assoc]
22     | Exp "<=" Exp [strict, non-assoc]
23     | Exp "<" Exp [strict, non-assoc]
24     | Exp "!=" Exp [strict, non-assoc]
25     | Exp "<>" Exp [strict, non-assoc]
26     | Exp "IS" "TRUE" [strict]
27     | Exp "IS" "FALSE" [strict]
28     | Exp "IS" "NULL" [strict]
29     | Exp "IS" "UNKNOWN" [strict]
30     | Exp "IS" "NOT" "TRUE" [strict]
31     | Exp "IS" "NOT" "FALSE" [strict]
32     | Exp "IS" "NOT" "NULL" [strict]
33     | Exp "IS" "NOT" "UNKNOWN" [strict]
34     > "NOT" Exp [strict]
35     > Exp "&&" Exp [strict(1), left]
36     | Exp "AND" Exp [strict(1), left]
37     > Exp "||" Exp [strict, left]
38     | Exp "OR" Exp [strict, left]
```



```

39  syntax Exp ::= "CONCAT" "(" Vals ")" [strict]
40  | "ELT" "(" Int "," Strings ")" [strict]
41  | "FIELD" "(" String "," Strings ")" [strict]
42  | "INSERT" "(" String "," Int "," Int "," Strings ")" [strict]
43  | "INSTR" "(" String "," String ")" [strict]
44  | "LENGTH" "(" String ")" [strict]
45  | "LOCATE" "(" String "," String ")" [strict]
46  | "LOCATE" "(" String "," String "," Int ")" [strict]
47  | "TRIM" "(" String ")" [strict]
48  | "LTRIM" "(" String ")" [strict]
49  | "RTRIM" "(" String ")" [strict]
50  | "POSITION" "(" String "IN" String ")" [strict]
51  | "REPEAT" "(" String "," Int ")" [strict]
52  | "REPLACE" "(" String "," String "," String ")" [strict]
53  | "LEFT" "(" String "," Int ")" [strict]
54  | "RIGHT" "(" String "," Int ")" [strict]
55  | "SPACE" "(" Int ")" [strict]
56  | "SUBSTRING" "(" String "," Int ")" [strict]
57  | "SUBSTRING" "(" String "FROM" Int ")" [strict]
58  | "SUBSTRING" "(" String "," Int "," Int ")" [strict]
59  | "SUBSTRING" "(" String "FROM" Int "FOR" Int ")" [strict]
60  syntax Ints ::= List{Int,","} [strict]
61  syntax Strings ::= List{String,","} [strict]
62  syntax Exps ::= List{Exp,","} [strict]
63  syntax Ids ::= List{Id,","}
64  syntax Bool ::= in( String , Strings)
65  syntax Int ::= getOrd(String) [strict]
66  syntax String ::= intToChar(Int) [function]
67  syntax Int ::= charToInt(String) [function]
68  syntax Int ::= strcmp(String,String) [function]
69  syntax Int ::= cmpChar(String,String) [function]
70  syntax Int ::= cmpChar(String,String) [function]
71  syntax Int ::= convertsi(String) [function]
72  syntax Int ::= firstLetterAt(String) [function]
73 endmodule

```

## A.2 Expression Semantics

```

1 module EXP
2   imports EXP-SYNTAX
3   rule TRUE:Boolean => 1 [anywhere]
4   rule FALSE:Boolean => 0 [anywhere]
5   rule true:Boolean => 1
6   rule false:Boolean => 0
7   rule ! 0 => 1
8   rule ! I:Int => 0 when I =Int 0
9   rule ! NULL => NULL
10  rule - I:Int => 0 -Int I
11  rule I1:Int * I2:Int => I1 *Int I2
12  rule I1:Int / I2:Int => I1 /Int I2 when I2 /=K 0
13  rule I1:Int / I2:Int => NULL when I2 ==K 0
14  rule I1:Int DIV I2:Int => I1 /Int I2 when I2 /=K 0
15  rule I1:Int DIV I2:Int => NULL when I2 ==K 0
16  rule I1:Int MOD I2:Int => I1 %Int I2 when I2 /=K 0
17  rule I1:Int MOD I2:Int => NULL when I2 ==K 0
18  rule I1:Int % I2:Int => I1 %Int I2 when I2 /=K 0
19  rule I1:Int % I2:Int => NULL when I2 ==K 0
20  rule I1:Int + I2:Int => I1 +Int I2
21  rule I1:Int - I2:Int => I1 -Int I2

```

```

22 rule I1:Int >= I2:Int => 1 when I1 >=Int I2
23 rule I1:Int >= I2:Int => 0 when I1 <Int I2
24 rule I1:Int > I2:Int => 1 when I1 >Int I2
25 rule I1:Int > I2:Int => 0 when I1 <=Int I2
26 rule I1:Int <= I2:Int => 1 when I1 <=Int I2
27 rule I1:Int <= I2:Int => 0 when I1 >Int I2
28 rule I1:Int < I2:Int => 1 when I1 <Int I2
29 rule I1:Int < I2:Int => 0 when I1 >=Int I2
30
31 rule S:String + I:Int => convertsi(S) + I [anywhere]
32 rule I:Int + S:String => I + convertsi(S) [anywhere]
33 rule S1:String + S2:String => convertsi(S1) + convertsi(S2) [anywhere]
34 rule S:String - I:Int => convertsi(S) - I [anywhere]
35 rule I:Int - S:String => I - convertsi(S) [anywhere]
36 rule S1:String - S2:String => convertsi(S1) - convertsi(S2) [anywhere]
37 rule S:String * I:Int => convertsi(S) * I [anywhere]
38 rule I:Int * S:String => I * convertsi(S) [anywhere]
39 rule S1:String * S2:String => convertsi(S1) * convertsi(S2) [anywhere]
40 rule S:String / I:Int => convertsi(S) / I [anywhere]
41 rule I:Int / S:String => I / convertsi(S) [anywhere]
42 rule S1:String / S2:String => convertsi(S1) / convertsi(S2) [anywhere]
43 rule S:String DIV I:Int => convertsi(S) DIV I [anywhere]
44 rule I:Int DIV S:String => I DIV convertsi(S) [anywhere]
45 rule S1:String DIV S2:String => convertsi(S1) DIV convertsi(S2) [anywhere]
46 rule S:String % I:Int => convertsi(S) % I [anywhere]
47 rule I:Int % S:String => I % convertsi(S) [anywhere]
48 rule S1:String % S2:String => convertsi(S1) % convertsi(S2) [anywhere]
49
50 rule F1:Float * F2:Float => F1 *Float F2
51 rule I1:Int * F2:Float => Int2Float(I1) * F2
52 rule F1:Float * I2:Int => F1 * Int2Float(I2)
53
54 rule F1:Float / F2:Float => F1 /Float F2 when F2 /=K 0
55 rule F1:Float / F2:Float => NULL when F2 ==K 0
56 rule F1:Float DIV F2:Float => F1 /Float F2 when F2 /=K 0
57 rule F1:Float DIV F2:Float => NULL when F2 ==K 0
58
59 rule I1:Int / F2:Float => Int2Float(I1) / F2
60 rule F1:Float / I2:Int => F1 / Int2Float(I2)
61 rule I1:Int DIV F2:Float => Int2Float(I1) DIV F2
62 rule F1:Float DIV I2:Int => F1 DIV Int2Float(I2)
63
64 rule F1:Float MOD F2:Float => F1 %Float F2 when F2 /=K 0
65 rule F1:Float MOD F2:Float => NULL when F2 ==K 0
66 rule F1:Float % F2:Float => F1 %Float F2 when F2 /=K 0
67 rule F1:Float % F2:Float => NULL when F2 ==K 0
68
69 rule I1:Int MOD F2:Float => Int2Float(I1) MOD F2
70 rule F1:Float MOD I2:Int => F1 MOD Int2Float(I2)
71 rule I1:Int % F2:Float => Int2Float(I1) % F2
72 rule F1:Float % I2:Int => F1 % Int2Float(I2)
73
74 rule F1:Float + F2:Float => F1 +Float F2
75 rule I1:Int + F2:Float => Int2Float(I1) + F2
76 rule F1:Float + I2:Int => F1 + Int2Float(I2)
77 rule F1:Float - F2:Float => F1 -Float F2
78 rule I1:Int - F2:Float => Int2Float(I1) - F2
79 rule F1:Float - I2:Int => F1 - Int2Float(I2)
80 rule F1:Float >= F2:Float => 1 when F1 >=Float F2
81 rule F1:Float >= F2:Float => 0 when F1 <Float F2
82 rule I1:Int >= F2:Float => Int2Float(I1) >= F2

```

```

83 rule F1:Float >= I2:Int => F1 >= Int2Float(I2)
84 rule F1:Float > F2:Float => 1 when F1 >Float F2
85 rule F1:Float > F2:Float => 0 when F1 <=Float F2
86 rule I1:Int > F2:Float => Int2Float(I1) > F2
87 rule F1:Float > I2:Int => F1 > Int2Float(I2)
88 rule F1:Float <= F2:Float => 1 when F1 <=Float F2
89 rule F1:Float <= F2:Float => 0 when F1 >Float F2
90 rule I1:Int <= F2:Float => Int2Float(I1) <= F2
91 rule F1:Float <= I2:Int => F1 <= Int2Float(I2)
92 rule F1:Float < F2:Float => 1 when F1 <Float F2
93 rule F1:Float < F2:Float => 0 when F1 >=Float F2
94 rule I1:Int < F2:Float => Int2Float(I1) < F2
95 rule F1:Float < I2:Int => F1 < Int2Float(I2)
96
97 rule S:String = I:Int => convertsi(S) = I [anywhere]
98 rule I:Int = S:String => I = convertsi(S) [anywhere]
99 rule S:String < I:Int => convertsi(S) < I [anywhere]
100 rule I:Int < S:String => I < convertsi(S) [anywhere]
101 rule S:String <= I:Int => convertsi(S) <= I [anywhere]
102 rule I:Int <= S:String => I <= convertsi(S) [anywhere]
103 rule S:String > I:Int => convertsi(S) > I [anywhere]
104 rule I:Int > S:String => I > convertsi(S) [anywhere]
105 rule S:String >= I:Int => convertsi(S) >= I [anywhere]
106 rule I:Int >= S:String => I >= convertsi(S) [anywhere]
107 rule S:String != I:Int => convertsi(S) != I [anywhere]
108 rule I:Int != S:String => I != convertsi(S) [anywhere]
109 rule S:String <> I:Int => convertsi(S) <> I [anywhere]
110 rule I:Int <> S:String => I <> convertsi(S) [anywhere]
111
112 rule I1:Int = I2:Int => 1 when I1 =Int I2 [anywhere]
113 rule I1:Int = I2:Int => 0 when I1 /=Int I2 [anywhere]
114 rule I1:Int != I2:Int => 1 when I1 /=Int I2 [anywhere]
115 rule I1:Int != I2:Int => 0 when I1 =Int I2 [anywhere]
116 rule F1:Float = F2:Float => 1 when F1 ==Float F2 [anywhere]
117 rule F1:Float = F2:Float => 0 when F1 /=Float F2 [anywhere]
118 rule F1:Float != F2:Float => 1 when F1 /=Float F2 [anywhere]
119 rule F1:Float != F2:Float => 0 when F1 ==Float F2 [anywhere]
120 rule S1:String >= S2:String => 1 when strcmp(S1,S2) >=Int 0 [anywhere]
121 rule S1:String >= S2:String => 0 when strcmp(S1,S2) =Int -1 [anywhere]
122 rule S1:String > S2:String => 1 when strcmp(S1,S2) =Int 1 [anywhere]
123 rule S1:String > S2:String => 0 when strcmp(S1,S2) <=Int 0 [anywhere]
124 rule S1:String < S2:String => 1 when strcmp(S1,S2) =Int -1 [anywhere]
125 rule S1:String < S2:String => 0 when strcmp(S1,S2) >=Int 0 [anywhere]
126 rule S1:String <= S2:String => 1 when strcmp(S1,S2) <=Int 0 [anywhere]
127 rule S1:String <= S2:String => 0 when strcmp(S1,S2) =Int 1 [anywhere]
128 rule S1:String = S2:String => 1 when strcmp(S1,S2) =Int 0 [anywhere]
129 rule S1:String = S2:String => 0 when strcmp(S1,S2) /=Int 0 [anywhere]
130 rule S1:String != S2:String => 1 when strcmp(S1,S2) /=Int 0 [anywhere]
131 rule S1:String != S2:String => 0 when strcmp(S1,S2) =Int 0 [anywhere]
132 rule (N1 <> N2) => N1 != N2 [anywhere]
133
134 rule NULL + _ => NULL
135 rule _ + NULL => NULL
136 rule NULL - _ => NULL
137 rule _ - NULL => NULL
138 rule NULL * _ => NULL
139 rule _ * NULL => NULL
140 rule NULL / _ => NULL
141 rule _ / NULL => NULL
142 rule NULL DIV _ => NULL
143 rule _ DIV NULL => NULL

```

```

144 rule NULL MOD _ => NULL
145 rule _ MOD NULL => NULL
146 rule NULL % _ => NULL
147 rule _ % NULL => NULL
148 rule NULL <= _ => NULL
149 rule _ <= NULL => NULL
150 rule NULL < _ => NULL
151 rule _ < NULL => NULL
152 rule NULL = _ => NULL
153 rule _ = NULL => NULL
154 rule NULL != _ => NULL
155 rule _ != NULL => NULL
156 rule NULL <> _ => NULL
157 rule _ <> NULL => NULL
158 rule NULL >= _ => NULL
159 rule _ >= NULL => NULL
160 rule NULL > _ => NULL
161 rule _ > NULL => NULL
162
163 rule I:Int IS TRUE => 1 when I /=K 0
164 rule 0 IS TRUE => 0
165 rule NULL IS TRUE => 0
166 rule A IS TRUE => 0 when A ==Int 0
167 rule I:Int IS FALSE => 0 when I /=K 0
168 rule 0 IS FALSE => 1
169 rule NULL IS FALSE => 0
170 rule A IS FALSE => 1 when A ==Int 0
171 rule I:Int IS UNKNOWN => 0 when I /=K 0
172 rule 0 IS UNKNOWN => 0
173 rule NULL IS UNKNOWN => 1
174 rule A IS UNKNOWN => 1 when A ==K NULL
175 rule I:Int IS NULL => 0 when I /=K 0
176 rule 0 IS NULL => 0
177 rule NULL IS NULL => 1
178 rule A IS NULL => 0 when A ==K NULL
179
180 rule I:Int IS NOT TRUE => 0 when I /=K 0
181 rule 0 IS NOT TRUE => 1
182 rule NULL IS NOT TRUE => 1
183 rule A IS NOT TRUE => 0 when A ==Int 0
184 rule I:Int IS NOT FALSE => 1 when I /=K 0
185 rule 0 IS NOT FALSE => 0
186 rule NULL IS NOT FALSE => 1
187 rule A IS NOT FALSE => 1 when A ==Int 0
188 rule I:Int IS NOT UNKNOWN => 1 when I /=K 0
189 rule 0 IS NOT UNKNOWN => 1
190 rule NULL IS NOT UNKNOWN => 0
191 rule A IS NOT UNKNOWN => 0 when A ==K NULL
192 rule I:Int IS NOT NULL => 1 when I /=K 0
193 rule 0 IS NOT NULL => 1
194 rule NULL IS NOT NULL => 0
195 rule A IS NOT NULL => 0 when A ==K NULL
196
197 rule 0 && 0 => 0
198 rule 0 && 1 => 0
199 rule 0 && NULL => 0
200 rule I:Int && 0 => 0 when I /=K 0
201 rule I:Int && 1 => 1 when I /=K 0
202 rule I:Int && NULL => NULL when I /=K 0
203 rule NULL && 0 => 0
204 rule NULL && I:Int => NULL when I /=K 0

```

```

205 rule NULL && NULL => NULL
206 rule I1:Int && S2:String => I1 && convertsi(S2)
207 rule S1:String && I2:Int => convertsi(S1) && I2
208 rule S1:String && S2:String => convertsi(S1) && convertsi(S2)
209
210 rule 0 || 0 => 0
211 rule 0 || I:Int => 1 when I /=K 0
212 rule 0 || NULL => NULL
213 rule I:Int || 0 => 1 when I /=K 0
214 rule I:Int || I => 1 when I /=K 0
215 rule I:Int || NULL => 1
216 rule NULL || 0 => NULL
217 rule NULL || I:Int => 1 when I /=K 0
218 rule NULL || NULL => NULL
219 rule I1:Int || S2:String => I1 || convertsi(S2)
220 rule S1:String || I2:Int => convertsi(S1) || I2
221 rule S1:String || S2:String => convertsi(S1) || convertsi(S2)
222
223 rule I:Int && B => B when I /=K 0
224 rule 0 && B => 0
225 rule I:Int || B => 1 when I /=K 0
226 rule 0 || B => B
227 rule I:Int AND B => B when I /=K 0
228 rule 0 AND B => 0
229 rule I:Int OR B => 1 when I /=K 0
230 rule 0 OR B => B
231 rule I1 AND I2 => I1 && I2
232 rule I1 OR I2 => I1 || I2
233
234 rule NOT 0 => 1
235 rule NOT I:Int => 0 when I /=Int 0
236 rule NOT NULL => NULL
237
238
239 rule B1:Bool && B2:Bool => B1 andBool B2
240 rule B1:Bool AND B2:Bool => B1 andBool B2
241 rule B1:Bool || B2:Bool => B1 orBool B2
242 rule B1:Bool OR B2:Bool => B1 orBool B2
243
244 rule CONCAT(.Vals) => ""
245 rule CONCAT(S:String) => S
246 rule CONCAT(S:String, Vs:Vals) => CONCAT(Vs) ~> S +String HOLE
247 rule S2:String ~> S1 +String HOLE => S1 +String S2 [structural]
248 rule NULL ~> S1 +String HOLE => NULL [structural]
249
250 rule CONCAT(NULL, Vs:Vals) => NULL
251 rule CONCAT(I:Int, Vs:Vals) => CONCAT(Int2String(I),Vs)
252 rule CONCAT(F:Float, Vs:Vals) => CONCAT(Float2String(F),Vs)
253
254
255 rule ELT(1, S:String, Ss:Strings) => S
256 rule ELT(N:Int, S:String, Ss:Strings) => ELT(N -Int 1, Ss)
257
258 rule FIELD(S:String, Ss:Strings) => 0 when notBool in(S,Ss)
259 rule FIELD(S:String, .Strings) => 0
260 rule FIELD(S:String, S2:String, Ss:Strings) => 1 when S ==String S2
261 rule FIELD(S:String, S2:String, Ss:Strings) => 1 + FIELD(S,Ss) when notBool (S
  ==String S2)
262
263 rule INSERT(S1:String, P:Int, L:Int, S2:String) => (substrString(S1, 0, P -Int 1)
  +String S2) +String (substrString(S1, (P -Int 1) +Int L, lengthString(S1)))

```

```

264 rule INSTR(S1:String, S2:String) => findString(S1,S2,3) +Int 1
265 rule LENGTH(S:String) => lengthString(S)
266 rule LOCATE(Sub:String, S:String) => findString(S,Sub,0) +Int 1
267 rule LOCATE(Sub:String, S:String, P:Int) => findString(S,Sub,P) +Int 1
268 rule POSITION( Sub:String IN S:String) => findString(S,Sub,0) +Int 1
269 rule TRIM(S:String) => trim(S)
270 rule LTRIM(S:String) => ltrim(S)
271 rule RTRIM(S:String) => rtrim(S)
272 rule REPEAT(S:String,0) => ""
273 rule REPEAT(S:String, N:Int) => REPEAT(S, (N -Int 1)) ~> S +String HOLE
274 rule S2:String ~> S +String HOLE => S +String S2 [structural]
275 rule REPLACE(S:String, FromS:String, ToS:String) => replaceAll(S,FromS,ToS)
276 rule LEFT(S:String, L:Int) => substrString(S,0,L)
277 rule RIGHT(S:String, L:Int) => substrString(S,lengthString(S) -Int L, lengthString(S))
278 rule SPACE(0) => ""
279 rule SPACE(N:Int) => SPACE(N -Int 1) ~> HOLE +String " "
280 rule S:String ~> HOLE +String S2 => S +String S2 [structural]
281 rule SUBSTRING(S:String, StartP:Int) => substrString(S, StartP -Int 1,
lengthString(S))
282 rule SUBSTRING(S:String FROM StartP:Int) => substrString(S, StartP -Int 1,
lengthString(S))
283 rule SUBSTRING(S:String , StartP:Int , Len:Int) => substrString(S, StartP -Int 1,
(StartP -Int 1) +Int Len)
284 rule SUBSTRING(S:String FROM StartP:Int FOR Len:Int) => substrString(S, StartP -Int
1, (StartP -Int 1) +Int Len)
285
286 rule in(S:String , .Strings ) => false [anywhere]
287 rule in(S:String , S2:String , Ss:Strings) => true when S ==String S2 [anywhere]
288 rule in(S:String , S2:String , Ss:Strings) => in(S, Ss) when S !=String S2 [anywhere]
289
290 rule strcmp("", "") => 0
291 rule strcmp("", S:String) => -1 when lengthString(S) >Int 0
292 rule strcmp(S:String, "") => 1 when lengthString(S) >Int 0
293 rule strcmp(S1:String, S2:String) => strcmp(substrString(S1,1,lengthString(S1)),
substrString(S2,1,lengthString(S2))) when ordChar(substrString(S1,0,1)) ==Int
ordChar(substrString(S2,0,1))
294 rule strcmp(S1:String, S2:String) => cmpChar(substrString(S1,0,1),
substrString(S2,0,1)) when ordChar(substrString(S1,0,1)) !=Int
ordChar(substrString(S2,0,1))
295
296 rule cmpChar(S1:String, S2:String) => -1 when ordChar(S1) <Int ordChar(S2)
297 rule cmpChar(S1:String, S2:String) => 0 when ordChar(S1) ==Int ordChar(S2)
298 rule cmpChar(S1:String, S2:String) => 1 when ordChar(S1) >Int ordChar(S2)
299
300 rule intToChar(I:Int) => chrChar(I)
301 rule charToInt(S:String) => ordChar(S)
302
303 rule convertsi(S:String) => 0 when notBool (#isDigit(substrString(S,0,1)) orBool
(substrString(S,0,1) ==String "+" orBool substrString(S,0,1) ==String "-"))
304 rule convertsi(S:String) => 0 when (substrString(S,0,1) ==String "+" orBool
substrString(S,0,1) ==String "-") andBool (notBool #isDigit(substrString(S,1,2)))
305 rule convertsi(S:String) => String2Int(substrString(S,1,(1 +Int
firstLetterAt(substrString(S,1,lengthString(S)))))) when substrString(S,0,1)
==String "+" andBool #isDigit(substrString(S,1,2))
306 rule convertsi(S:String) => String2Int(substrString(S,0,(1 +Int
firstLetterAt(substrString(S,1,lengthString(S)))))) when substrString(S,0,1)
==String "-" andBool #isDigit(substrString(S,1,2))
307 rule convertsi(S:String) => String2Int(substrString(S,0,1 +Int
firstLetterAt(substrString(S,1,lengthString(S)))))) when #isDigit(substrString(S,0,1))
308
309 rule firstLetterAt("") => 0

```

```

310 rule firstLetterAt(S:String) => (1 +Int
    firstLetterAt(substrString(S,1,lengthString(S)))) when #isDigit(substrString(S,0,1))
311
312 rule firstLetterAt(S:String) => 0 when notBool #isDigit(substrString(S,0,1))
313
314 endmodule

```

## A.3 Table Syntax

```

1
2 module TABLE-SYNTAX
3   imports EXP
4   syntax DataType ::= "INT" | "BOOL" | "TEXT" | "FLOAT"
5   ///**** Representation ****///
6   syntax #TableElement ::= "e(" Vals ")" [strict]
7   syntax TableElement ::= #TableElement
8   syntax TableElements ::= List{TableElement,","} [strict]
9   syntax #Record ::= "r[" TableElements "]" [strict]
10  syntax Record ::= #Record
11  syntax #Field ::= "f(" String "," DataType "," Bool "," Bool ")"
12  syntax Field ::= #Field
13  syntax Fields ::= List{Field,","} [strict]
14  syntax #Schema ::= "s[" Fields "]" [strict]
15  syntax Schema ::= #Schema
16  syntax #Table ::= Id "[" Schema ":" Record "]" [strict]
17  syntax Val ::= FieldRep
18  syntax FieldRep ::= FieldRep1 | FieldRep2
19  syntax FieldRep1 ::= Id | " " Id " "
20  syntax FieldRep1s ::= List{FieldRep1,","}
21  syntax FieldRep2 ::= Id "." Id
22  syntax Collumn ::= FieldRep
23  syntax Collumns ::= List{FieldRep,","}
24  syntax KResult ::= #Record | #Field | #Schema | #Table | #TableElement
25
26  ///**** Main function ****///
27  syntax Table ::= #Table
28                | Table "union" Table [strict]
29                | Table "intersect" Table [strict]
30                | Table "difference" Table [strict]
31                | Table "cartesian" Table [strict]
32                | Table "x" Table [strict]
33
34  syntax Table ::= union(Table,Table) [strict]
35  syntax Table ::= difference(Table,Table) [strict]
36  syntax Table ::= intersect(Table,Table) [strict]
37  syntax Table ::= catesian(Table,Table) [strict]
38  syntax Record ::= union(Record,Record) [strict]
39  syntax Record ::= difference(Record,Record) [strict]
40  syntax Record ::= intersect(Record,Record) [strict]
41  syntax Record ::= catesian(Record,Record) [strict]
42  syntax Table ::= join(Table,Table) [strict]
43  syntax Table ::= join(Table,Table,Exp) [strict(1,2)]
44  syntax Table ::= leftJoin(Table,Table,Exp) [strict(1,2)]
45  syntax Table ::= rightJoin(Table,Table,Exp) [strict(1,2)]
46  syntax Table ::= select(Table,Exp) [strict(1)]
47  syntax Record ::= leftJoin2(Schema,Schema,Record,Record,Exp)
48  syntax Record ::= unionLeftJoin(Schema,Schema,TableElement,Record)
49  syntax Record ::= rightJoin2(Schema,Schema,Record,Record,Exp)
50  syntax Record ::= unionRightJoin(Schema,Schema,TableElement,Record)

```

```

51
52////**** Auxiliary function ****////
53 syntax Id ::= "tmp"
54 syntax Int ::= getIndex(Schema,String) [strict]
55 syntax Int ::= getIndex2(Schema,String) [strict]
56 syntax Exp ::= getValue(TableElement,Exp) [strict]
57 syntax Record ::= filter(Schema,Record,Exp) [strict(1,2)]
58 syntax SelectElement ::= s(Int,TableElement) [strict]
59 syntax KResult ::= SelectElement
60 syntax Exp ::= eval(Schema,TableElement,Exp) [strict(1,2)]
61 syntax Field ::= getFieldFromSchema(Collumn,Schema) [strict]
62 syntax Table ::= project(Table,Schema) [strict]
63 syntax Record ::= project2(Record,Schema,Schema) [strict]
64 syntax TableElement ::= project3(TableElement,Schema,Schema) [strict]
65 syntax Val ::= getValue(Schema,TableElement,String) [strict]
66 syntax Table ::= rename(Table,Id) [strict]
67 syntax Table ::= changeFieldNameCorrespondToTable(Table) [strict]
68 syntax Field ::= rename(Field,String) [strict]
69 syntax Fields ::= concatFieldName(Schema,Id) [strict]
70 syntax String ::= changeFieldRepIntoStringName( FieldRep ) [strict]
71 syntax String ::= changeFieldRepIntoStringName( Id , Id ) [strict]
72 syntax Schema ::= excludeFields(Schema,FieldRep1s)
73 syntax Schema ::= excludeField(Schema,FieldRep1)
74
75 syntax TableElement ::= addNullElementOnBottom(TableElement,Int) [strict]
76 syntax TableElement ::= addNullElementOnTop(TableElement,Int) [strict]
77 syntax Int ::= numberOfElementInRecord(Record) [strict]
78 syntax TableElement ::= addElementOnBottom(TableElement,Val) [strict]
79 syntax TableElement ::= addElementOnTop(TableElement,Val) [strict]
80 syntax K ::= if(Int,K,K)
81 syntax Schema ::= addElement(Field,Schema) [strict]
82 syntax Schema ::= concat(Schema, Schema) [strict]
83 syntax Record ::= concat(Record, Record) [strict]
84 syntax TableElement ::= append(TableElement,TableElement) [strict]
85 syntax TableElement ::= addTopElement(Val , TableElement) [strict]
86 syntax Record ::= addElement(TableElement,Record) [strict]
87 syntax Record ::= appendElementToRecord(TableElement,Record) [strict]
88 syntax Int ::= numberOfFields(Schema) [strict]
89 syntax Bool ::= checkUnionCompatible( Schema , Schema) [strict]
90 syntax Bool ::= isTableElementEqual( TableElement,TableElement) [strict]
91 syntax Bool ::= consistOf(Record,TableElement) [strict]
92 syntax Bool ::= in( String , Ids)
93 syntax K ::= num(Ids)
94 endmodule

```

## A.4 Table Semantics

```

1 module TABLE
2   imports TABLE-SYNTAX
3
4   ////**** Main function ****////
5   rule T1:Table cartesian T2:Table => catesian(T1, T2) [anywhere]
6   rule T1:Table union T2:Table => union(T1,T2) [anywhere]
7   rule T1:Table difference T2:Table => difference(T1,T2) [anywhere]
8   rule T1:Table intersect T2:Table => intersect(T1,T2) [anywhere]
9
10  // Union
11  rule union(Id1:Id[S1:Schema : R1:Record] , Id2:Id[S2:Schema : R2:Record]) =>
    union(R1,R2) ~> tmp[S1 : HOLE] when checkUnionCompatible(S1,S2) [structural]

```



```

12
13 rule union(R,r[.TableElements]) => R [anywhere,structural]
14 rule union(r[.TableElements],R) => R [anywhere,structural]
15 rule union(r[T:TableElement,Ts:TableElements],R) => union(r[Ts],R) when
    consistOf(R,T) [anywhere]
16 rule union(r[T1:TableElement,
    Ts1:TableElements],r[T2:TableElement,Ts2:TableElements]) => union(r[Ts1],r[T2,Ts2])
    ~> addElement(T1,HOLE) when notBool
    consistOf(r[T2:TableElement,Ts2:TableElements],T1) [anywhere]
17
18 // Difference
19 rule difference(Id1:Id[S1:Schema : R1:Record] , Id2:Id[S2:Schema : R2:Record]) =>
    difference(R1,R2) ~> tmp[S1 : HOLE ] when checkUnionCompatible(S1,S2)
20
21 rule difference(r[.TableElements],R) => r[.TableElements] [structural]
22 rule difference(r[T:TableElement,Ts:TableElements],R) => difference(r[Ts],R) when
    consistOf(R,T)
23 rule difference(r[T:TableElement,Ts:TableElements],R) => difference(r[Ts],R) ~>
    addElement(T, HOLE ) when notBool consistOf(R,T)
24
25 // Intersect
26 rule intersect(Id1:Id[S1:Schema : R1:Record] , Id2:Id[S2:Schema : R2:Record]) =>
    intersect(R1,R2) ~> tmp[S1: HOLE ] when checkUnionCompatible(S1,S2)
27 rule intersect(r[.TableElements],R) => r[.TableElements] [structural]
28 rule intersect(r[T:TableElement,Ts:TableElements],R) => intersect(r[Ts],R) when
    notBool consistOf(R,T)
29 rule intersect(r[T1:TableElement,
    Ts1:TableElements],r[T2:TableElement,Ts2:TableElements]) => intersect(r[Ts1],r[Ts2])
    ~> addElement(T1, HOLE ) when consistOf(r[T2:TableElement,Ts2:TableElements],T1)
30
31 // Catesian
32 rule catesian(r[.TableElements],_) => r[.TableElements] [structural]
33 rule catesian(r[E1:TableElement, Es1:TableElements],r[Es2:TableElements]) =>
    concat(appendElementToRecord(E1,r[Es2]),catesian(r[Es1],r[Es2]))
34 rule catesian(T1:Id[S1:Schema : R1:Record],T2:Id[S2:Schema : R2:Record]) =>
    tmp[concat(S1,S2) : catesian(R1,R2)]
35 rule T1:Table cartesian T2:Table =>
    catesian(changeFieldNameCorrespondToTable(T1),changeFieldNameCorrespondToTable(T2))
36
37 // Renaming
38 rule rename(T1:Id[S:Schema : R:Record], T2:Id) => T2[S : R] [anywhere]
39 rule rename(f(_,T,B1,B2),S2:String) => f(S2,T,B1,B2) [anywhere]
40 rule concatFieldName(s[.Fields],T:Id) => .Fields [anywhere]
41 rule concatFieldName(s[f(F:String,DT:DataType,B1:Bool,B2:Bool),Fs:Fields],T:Id) =>
    rename(f(F,DT,B1,B2),(#tokenToString(T) +String ".") +String F),
    concatFieldName(s[Fs],T) [anywhere]
42
43 // cross join
44 rule join(T1:Table, T2:Table) => T1 cartesian T2
45 rule join(T1:Table, T2:Table, E:Exp) => T1 cartesian T2 ~> select( HOLE , E)
46 rule T:#Table ~> select(HOLE , E) => select(T,E) [structural]
47
48 // left join
49 rule leftJoin(T1:Id[ S1:Schema : R1] , T2:Id[ S2:Schema : R2:Record ] , E:Exp) =>
50 tmp[ concat(S1,S2) : leftJoin2(S1,S2, R1,R2,E) ]
51 rule leftJoin2(S1:Schema,S2:Schema, r[ .TableElements] , R:Record, E:Exp) => r[
    .TableElements ]
52 rule leftJoin2(S1:Schema,S2:Schema, r[TE1:TableElement, TEs:TableElements],
    R:Record, E:Exp) => filter(concat(S1,S2),catesian( r[ TE1 ], R), E) ~>
    unionLeftJoin(S1,S2,TE1,HOLE) ~> union(HOLE,leftJoin2(S1,S2,r[TEs],R,E))
53 rule Result:#Record ~> unionLeftJoin(S1:Schema,S2:Schema,T:TableElement,HOLE) =>

```

```

unionLeftJoin(S1,S2,T,Result) [structural]
54 rule Result:#Record ~>
union(HOLE,leftJoin2(S1:Schema,S2:Schema,R1:Record,R2:Record,E:Exp)) =>
union(Result,leftJoin2(S1,S2,R1,R2,E)) [structural]
55 rule unionLeftJoin(S1:Schema,S2:Schema,T:TableElement,Result:Record) =>
numberOfElementInRecord(Result) = 0 ~> if(HOLE, r[
addNullElementOnBottom(T,numberOfFields(S2)) ], Result)
56
57 // rightjoin
58 rule rightJoin(T1:Id[ S1:Schema : R1] , T2:Id[ S2:Schema : R2:Record ], E:Exp) =>
59 tmp[ concat(S1,S2) : rightJoin2(S1,S2, R1,R2,E) ]
60 rule rightJoin2(S1:Schema,S2:Schema, R:Record ,r[ .TableElements], E:Exp) => r[
.TableElements ]
61 rule rightJoin2(S1:Schema,S2:Schema, R:Record, r[TE1:TableElement,
TEs:TableElements], E:Exp) => filter(concat(S1,S2),catesian( R, r[ TE1 ]), E) ~>
unionRightJoin(S1,S2,TE1,HOLE) ~> union(HOLE,rightJoin2(S1,S2,R,r[TEs],E))
62 rule Result:#Record ~> unionRightJoin(S1:Schema,S2:Schema,T:TableElement,HOLE) =>
unionRightJoin(S1,S2,T,Result) [structural]
63 rule Result:#Record ~>
union(HOLE,rightJoin2(S1:Schema,S2:Schema,R1:Record,R2:Record,E:Exp)) =>
union(Result,rightJoin2(S1,S2,R1,R2,E)) [structural]
64 rule unionRightJoin(S1:Schema,S2:Schema,T:TableElement,Result:Record) =>
numberOfElementInRecord(Result) = 0 ~> if(HOLE, r[
addNullElementOnTop(T,numberOfFields(S2)) ], Result)
65
66////**** Auxiliary function ****////
67
68 rule getIndex(s[ .Fields ], S2:String) => NULL [anywhere]
69 rule getIndex(s[f(S1:String,_,_,_) , Fs:Fields] , S2:String) => 0 when S1 ==String S2
[anywhere]
70 rule getIndex(s[f(S1:String,_,_,_) , Fs:Fields] , S2:String) => (getIndex(s[Fs],S2) +
1) when notBool S1 ==String S2 [anywhere]
71 rule getIndex2(s[ .Fields ], S2:String) => NULL [anywhere]
72 rule getIndex2(s[f(S1:String,_,_,_) , Fs:Fields] , S2:String) => 0 when
substrString(S1,(findChar(S1, ".",0) +Int 1),lengthString(S1)) ==String S2 [anywhere]
73 rule getIndex2(s[f(S1:String,_,_,_) , Fs:Fields] , S2:String) => (getIndex2(s[Fs],S2)
+ 1) when substrString(S1,(findChar(S1, ".",0) +Int 1),lengthString(S1)) /=String S2
[anywhere]
74
75 rule excludeFields(S:Schema, .FieldRep1s) => S [anywhere]
76 rule excludeFields(S:Schema, F:FieldRep1,Fs:FieldRep1s) =>
excludeFields(excludeField(S,F),Fs) [anywhere]
77 rule changeFieldRepIntoStringName( I1:Id . I2:Id ) => ((#tokenToString(I1) +String
".") +String #tokenToString(I2)) [anywhere]
78 rule changeFieldRepIntoStringName( T:Id , F:Id ) => ((#tokenToString(T) +String
".") +String #tokenToString(F)) [anywhere]
79
80 rule excludeField(s[.Fields] , ' I:Id ') => s[ .Fields ] [anywhere]
81 rule excludeField(s[ f(FN:String,D:DataType,B1:Bool,B2:Bool) , Fs:Fields] , ' I:Id ')
=> excludeField(s[Fs], ' I ') when substrString(FN,(findChar(FN, ".",0) +Int
1),lengthString(FN)) ==String #tokenToString(I) [anywhere]
82 rule excludeField(s[ f(FN:String,D:DataType,B1:Bool,B2:Bool) , Fs:Fields] , ' I:Id ')
=> addElement(f(FN,D,B1,B2),excludeField(s[Fs], ' I ')) when
substrString(FN,(findChar(FN, ".",0) +Int 1),lengthString(FN)) /=String
#tokenToString(I) [anywhere]
83 rule excludeField(s[.Fields] , I:Id ) => s[ .Fields ] [anywhere]
84 rule excludeField(s[ f(FN:String,D:DataType,B1:Bool,B2:Bool) , Fs:Fields] , I:Id )
=> excludeField(s[Fs], ' I ') when substrString(FN,(findChar(FN, ".",0) +Int
1),lengthString(FN)) ==String #tokenToString(I) [anywhere]
85 rule excludeField(s[ f(FN:String,D:DataType,B1:Bool,B2:Bool) , Fs:Fields] , I:Id ) =>
addElement(f(FN,D,B1,B2),excludeField(s[Fs], ' I ')) when

```

```

      substrString(FN,(findChar(FN,".",0) +Int 1),lengthString(FN)) /=String
      #tokenToString(I) [anywhere]
86
87 rule getValue(_, NULL) => NULL
88 rule getValue(e(V:Val, _), 0) => V
89 rule getValue(e(_, Vs:Vals), I:Int) => getValue(e(Vs), I -Int 1)
90 rule getValue(s[.Fields], e(Vs:Vals), S2:String) => NULL [structural]
91 rule getValue(s[f(S1:String, D:DataType, B1, B2), Fs:Fields], e(V:Val, Vs:Vals), S2:String)
    => V when S1 ==String S2
92 rule getValue(s[f(S1:String, D:DataType, B1, B2), Fs:Fields], e(V:Val, Vs:Vals), S2:String)
    => getValue(s[Fs], e(Vs), S2) when S1 /=String S2
93 rule select(I:Id[ S:Schema : R:Record ], E:Exp) => filter(S,R,E) ~> I[ S : HOLE ]
94 rule R:Record ~> T[ S : HOLE ] => T[ S : R ] [structural]
95 rule r[ Ts:TableElements ] ~> s(0, T:TableElement) => r[Ts]
96 rule r[ Ts:TableElements ] ~> s(I:Int, T:TableElement) => r[T, Ts] when I /=Int 0
97 rule r[ .TableElements ] ~> s(I:Int, T:TableElement) => r[T] when I /=Int 0
98 rule r[ .TableElements ] ~> s(0, T:TableElement) => r[ .TableElements ]
99
100 rule filter( S, r[ .TableElements ] , E ) => r[ .TableElements ] [structural]
101 rule filter( S:Schema, r[ T1:TableElement, Ts:TableElements ], E:Exp ) =>
    eval(S, T1, E) ~> s(HOLE, T1) ~> filter(S, r[Ts], E)
102 rule I:Int ~> s(HOLE, T:TableElement) => s(I, T)
103 rule S:SelectElement ~> R:Record => R ~> S [structural]
104
105 rule project(T:Id[ S1:Schema : R:Record ], S2:Schema) => T[ S2 : project2( R, S1, S2) ]
106 rule project2(r[ .TableElements ], S1, S2) => r[ .TableElements ]
107 rule project2(r[ T:TableElement, Ts:TableElements ], S1:Schema, S2:Schema) =>
    addElement(project3(T, S1, S2), project2(r[Ts], S1, S2))
108 rule project3(T:TableElement, S:Schema, s[ .Fields ]) => e(.Vals)
109 rule project3(T:TableElement, S:Schema, s[ f(FN:String, _, _, _), Fs:Fields ]) =>
    getValue(T, getIndex(S, FN)) ~> addElementOnTop(project3(T, S, s[Fs]), HOLE)
110 rule NULL ~> addElementOnTop(T, HOLE) => addElementOnTop(T, NULL)
111 rule V:Val ~> addElementOnTop(T, HOLE) => addElementOnTop(T, V)
112
113 rule eval(_, T, NULL) => NULL
114 rule eval(_, T, B:Bool) => B
115 rule eval(_, T, I:Int) => I
116 rule eval(_, T, S:String) => S
117 rule eval(S:Schema, T:TableElement, I:Id) => getValue(T, getIndex2(S, #tokenToString(I)))
    ~> eval(S, T, HOLE)
118 rule eval(S:Schema, T:TableElement, ' I:Id ') =>
    getValue(T, getIndex2(S, #tokenToString(I))) ~> eval(S, T, HOLE)
119 rule eval(S:Schema, T:TableElement, F:FieldRep) =>
    getValue(T, getIndex(S, changeFieldRepIntoStringName(F))) ~> eval(S, T, HOLE)
120 rule V:Val ~> eval(S, T, HOLE) => eval(S, T, V) [structural]
121 rule eval(S, T, - E:Exp) => - eval(S, T, E)
122 rule eval(S, T, E1:Exp * E2:Exp) => eval(S, T, E1) * eval(S, T, E2)
123 rule eval(S, T, E1:Exp / E2:Exp) => eval(S, T, E1) / eval(S, T, E2)
124 rule eval(S, T, E1:Exp DIV E2:Exp) => eval(S, T, E1) DIV eval(S, T, E2)
125 rule eval(S, T, E1:Exp MOD E2:Exp) => eval(S, T, E1) MOD eval(S, T, E2)
126 rule eval(S, T, E1:Exp % E2:Exp) => eval(S, T, E1) % eval(S, T, E2)
127 rule eval(S, T, E1:Exp + E2:Exp) => eval(S, T, E1) + eval(S, T, E2)
128 rule eval(S, T, E1:Exp - E2:Exp) => eval(S, T, E1) - eval(S, T, E2)
129 rule eval(S, T, ! E:Exp) => ! eval(S, T, E)
130 rule eval(S, T, E1:Exp = E2:Exp) => eval(S, T, E1) = eval(S, T, E2)
131 rule eval(S, T, E1:Exp >= E2:Exp) => eval(S, T, E1) >= eval(S, T, E2)
132 rule eval(S, T, E1:Exp > E2:Exp) => eval(S, T, E1) > eval(S, T, E2)
133 rule eval(S, T, E1:Exp <= E2:Exp) => eval(S, T, E1) <= eval(S, T, E2)
134 rule eval(S, T, E1:Exp < E2:Exp) => eval(S, T, E1) < eval(S, T, E2)
135 rule eval(S, T, E1:Exp != E2:Exp) => eval(S, T, E1) != eval(S, T, E2)
136 rule eval(S, T, E1:Exp <> E2:Exp) => eval(S, T, E1) <> eval(S, T, E2)

```

```

137 rule eval(S,T, E:Exp IS TRUE) => eval(S,T, E) IS TRUE
138 rule eval(S,T, E:Exp IS FALSE) => eval(S,T, E) IS FALSE
139 rule eval(S,T, E:Exp IS NULL) => eval(S,T, E) IS NULL
140 rule eval(S,T, E:Exp IS UNKNOWN) => eval(S,T, E) IS UNKNOWN
141 rule eval(S,T, E:Exp IS NOT TRUE) => eval(S,T, E) IS NOT TRUE
142 rule eval(S,T, E:Exp IS NOT FALSE) => eval(S,T, E) IS NOT FALSE
143 rule eval(S,T, E:Exp IS NOT NULL) => eval(S,T, E) IS NOT NULL
144 rule eval(S,T, E:Exp IS NOT UNKNOWN) => eval(S,T, E) IS NOT UNKNOWN
145 rule eval(S,T, NOT E:Exp ) => NOT eval(S,T, E)
146 rule eval(S,T, E1:Exp && E2:Exp) => eval(S,T, E1) && eval(S,T,E2)
147 rule eval(S,T, E1:Exp AND E2:Exp) => eval(S,T, E1) AND eval(S,T,E2)
148 rule eval(S,T, E1:Exp || E2:Exp) => eval(S,T, E1) || eval(S,T,E2)
149 rule eval(S,T, E1:Exp OR E2:Exp) => eval(S,T, E1) OR eval(S,T,E2)
150 rule eval(S,T, CONCAT(Vs:Vals)) => CONCAT(Vs)
151 rule eval(S,T, ELT(I:Int,Ss:Strings)) => ELT(I,Ss)
152 rule eval(S,T, FIELD(S1:String,Ss:Strings)) => FIELD(S1,Ss)
153 rule eval(S,T, INSERT(S1:String,I1:Int,I2:Int,Ss:Strings)) => INSERT(S1,I1,I2,Ss)
154 rule eval(S,T, INSTR(S1:String,S2:String)) => INSTR(S1,S2)
155 rule eval(S,T, LENGTH(S1:String)) => LENGTH(S1)
156 rule eval(S,T, LOCATE(S1:String,S2:String)) => LOCATE(S1,S2)
157 rule eval(S,T, LOCATE(S1:String,S2:String,I:Int)) => LOCATE(S1,S2,I)
158 rule eval(S,T, TRIM(S1:String)) => TRIM(S1)
159 rule eval(S,T, LTRIM(S1:String)) => LTRIM(S1)
160 rule eval(S,T, RTRIM(S1:String)) => RTRIM(S1)
161 rule eval(S,T, POSITION(S1:String IN S2:String)) => POSITION(S1 IN S2)
162 rule eval(S,T, REPEAT(S1:String, I:Int)) => REPEAT(S1,I)
163 rule eval(S,T, LEFT(S1:String, I:Int)) => LEFT(S1,I)
164 rule eval(S,T, RIGHT(S1:String, I:Int)) => RIGHT(S1,I)
165 rule eval(S,T, SPACE(I:Int)) => SPACE(I)
166 rule eval(S,T, SUBSTRING(S1:String,I:Int)) => SUBSTRING(S1,I)
167 rule eval(S,T, SUBSTRING(S1:String FROM I:Int)) => SUBSTRING(S1 FROM I)
168 rule eval(S,T, SUBSTRING(S1:String, I1:Int, I2:Int)) => SUBSTRING(S1,I1,I2)
169 rule eval(S,T, SUBSTRING(S1:String FROM I1:Int FOR I2:Int)) => SUBSTRING(S1 FROM I1
FOR I2)
170
171 rule getFieldFromSchema(I:Id,s[ f(FN2,T:DataType,B1:Bool,B2:Bool), Fs:Fields]) =>
f(FN2,T:DataType,B1:Bool,B2:Bool) when #tokenToString(I) ==String
substrString(FN2,(findChar(FN2, "." ,0) +Int 1),lengthString(FN2)) [anywhere]
172 rule getFieldFromSchema(' I:Id ',s[ f(FN2,T:DataType,B1:Bool,B2:Bool), Fs:Fields]) =>
f(FN2,T:DataType,B1:Bool,B2:Bool) when #tokenToString(I) ==String
substrString(FN2,(findChar(FN2, "." ,0) +Int 1),lengthString(FN2)) [anywhere]
173 rule getFieldFromSchema(F:FieldRep1,s[ f(FN2,T:DataType,B1:Bool,B2:Bool), Fs:Fields])
=> f(FN2,T:DataType,B1:Bool,B2:Bool) when changeFieldRepIntoStringName(F) ==String
FN2 [anywhere]
174 rule getFieldFromSchema(C:Column,s[ f(FN2,T:DataType,B1:Bool,B2:Bool), Fs:Fields])
=> getFieldFromSchema(C,s[Fs]) [anywhere]
175
176 rule changeFieldNameCorrespondToTable(T:Id[S:Schema : R:Record]) =>
T:Id[s[concatFieldName(S,T)] : R] when T !=K tmp [anywhere]
177 rule changeFieldNameCorrespondToTable(T:Id[S:Schema : R:Record]) => T:Id[S : R] when
T ==K tmp [anywhere]
178
179 rule addNullElementOnBottom(T:TableElement,0) => T [anywhere]
180 rule addNullElementOnBottom(T:TableElement,I:Int) =>
addNullElementOnBottom(append(T,e(NULL)), I -Int 1) [anywhere]
181
182 rule addNullElementOnTop(T:TableElement,0) => T [anywhere]
183 rule addNullElementOnTop(T:TableElement,I:Int) =>
addNullElementOnTop(addTopElement(NULL,T), I -Int 1) [anywhere]
184
185 rule numberOfElementInRecord(r[ .TableElements ]) => 0 [anywhere]

```

```

186 rule   numberOfElementInRecord(r[ T:TableElement , Ts:TableElements]) => 1 +
      numberOfElementInRecord(r[Ts]) [anywhere]
187 rule   addElementOnBottom(T:TableElement ,V:Val) => append(T,e(V)) [anywhere]
188 rule   addElementOnTop(e( Vs:Vals ),V:Val) => e(V,Vs) [anywhere]
189 rule   if(I:Int,K1,K2) => K1 when I /=Int 0 [anywhere]
190 rule   if(0,K1,K2) => K2 [anywhere]
191 rule   I:#Int ~> if(HOLE,K1:K,K2:K) => if(I,K1,K2) [anywhere]
192 rule   addElement(T:TableElement ,r[Ts:TableElements]) => r[T,Ts] [anywhere]
193 rule   addElement(F:Field,s[Fs:Fields]) => s[F,Fs] [anywhere]
194 rule   concat(s[.Fields] , S:Schema) => S [anywhere]
195 rule   concat(s[F1:Field,Fs1:Fields],s[Fs2:Fields]) => concat(s[Fs1],s[Fs2]) ~>
      addElement(F1, HOLE ) [anywhere]
196 rule  concat(r[.TableElements] , R:Record) => R [anywhere]
197 rule  concat(r[T1:TableElement ,Tb1:TableElements],r[Tb2:TableElements]) =>
      concat(r[Tb1],r[Tb2]) ~> addElement(T1, HOLE ) [structural] [anywhere]
198 rule  S:#Schema ~> addElement(F:Field , HOLE ) => addElement( F, S)
      [anywhere,structural]
199 rule  R:#Record ~> tmp [S : HOLE ] => tmp [S : R] [anywhere,structural]
200 rule  R:#Record ~> addElement( T1:TableElement , HOLE ) => addElement(T1,R)
      [anywhere,structural]
201 rule  append(e(.Vals),e(Vs:Vals)) => e(Vs) [anywhere]
202 rule  append(e(V:Val , Vs1:Vals) ,e(Vs2:Vals)) => append(e(Vs1),e(Vs2)) ~>
      addTopElement(V , HOLE ) [anywhere]
203 rule  addTopElement(V:Val ,e(.Vals)) => e(V) [anywhere]
204 rule  addTopElement(V:Val ,e(Vs:Vals)) => e(V,Vs) [anywhere]
205 rule  T:#TableElement ~> addTopElement(V:Val , HOLE) => addTopElement( V , T)
      [structural]
206 rule  addElement(E:TableElement ,r[Es:TableElements]) => r[E,Es]
207 rule  appendElementToRecord(E1,r[.TableElements]) => r[.TableElements] [structural]
208 rule  appendElementToRecord(E1:TableElement ,r[E2:TableElement ,Es:TableElements]) =>
      addElement(append(E1,E2),appendElementToRecord(E1,r[Es]))
209 rule  numberOfFields(s[.Fields]) => 0 [anywhere,structural]
210 rule  numberOfFields(s[_ , Fs:Fields]) => 1 +Int numberOfFields(s[Fs]) [anywhere]
211 rule  checkUnionCompatible(s[.Fields] , s[.Fields]) => true [anywhere,structural]
212 rule  checkUnionCompatible(s[.Fields] , s[_]) => false [anywhere,structural]
213 rule  checkUnionCompatible(s[_] , s[.Fields]) => false [anywhere,structural]
214 rule  checkUnionCompatible(s[f(S1:String,T1:DataType ,_ ,_) ,
      Fs1:Fields],s[f(S2:String,T2:DataType ,_ ,_) , Fs2:Fields]) =>
      checkUnionCompatible(s[Fs1],s[Fs2]) when S1 ==K S2 andBool T1 ==K T2 [anywhere]
215 rule  checkUnionCompatible(s[f(S1:String,T1:DataType ,_ ,_) ,
      _],s[f(S2:String,T2:DataType ,_ ,_) , _]) => false when S1 /=K S2 orBool T1 /=K T2
      [anywhere]
216 rule  isTableElementEqual(e(.Vals),e(.Vals)) => true [anywhere,structural]
217 rule  isTableElementEqual(e(V1:Val ,Vs1:Vals),e(V2:Val ,Vs2:Vals)) => false when V1 /=K
      V2 [anywhere]
218 rule  isTableElementEqual(e(V1:Val ,Vs1:Vals),e(V2:Val ,Vs2:Vals)) =>
      isTableElementEqual(e(Vs1),e(Vs2)) when V1 ==K V2 [anywhere]
219 rule  consistOf(r[.TableElements] , E2) => false [anywhere,structural]
220 rule  consistOf(r[E1:TableElement ,Ts:TableElements] , E2) => true when
      isTableElementEqual(E1,E2) [anywhere]
221 rule  consistOf(r[E1:TableElement ,Ts:TableElements] , E2) =>
      consistOf(r[Ts:TableElements] , E2) when notBool isTableElementEqual(E1,E2)
      [anywhere]
222 rule  in( S:String , .Ids ) => false [anywhere]
223 rule  in(S:String , I1:Id , Is:Ids) => true when S ==String #tokenToString(I1)
      [anywhere]
224 rule  in(S:String , I1:Id , Is:Ids) => in(S, Is) when S /=String #tokenToString(I1)
      [anywhere]
225 rule  num(.Ids) => 0
226 rule  num(_ , Xs:Ids) => num(Xs) +Int 1
227 endmodule

```

## A.5 SQL Syntax

```
1 module SQL-SYNTAX
2   imports TABLE
3   syntax Table ::= Stm
4   syntax Stm ::= CreateStm | InsertStm | SelectStm | UpdateStm | DeleteStm | DropStm
5   syntax Stms ::= Stm | Stms Stms [left,structural]
6
7   syntax Table ::= doGetTableExp(TableExp) [strict]
8   syntax Table ::= doConditionExp(Table,ConditionExp) [strict]
9   syntax Table ::= doProjectionExp(Table,ProjectionExp) [strict]
10
11  // Store
12  syntax K ::= "store" Table
13
14  // Create
15  syntax CreateStm ::= "CREATE" "TABLE" Id "(" FieldDcls ")" ";"
16  | "CREATE" "TABLE" Id "(" FieldDcls "," CreateOptionList ")" ";"
17  syntax ProjectionExp ::= "*" | AsClauseOrCollumns
18  syntax AsClauseOrCollumn ::= Collumn | AsClause
19  syntax AsClauseOrCollumns ::= List{AsClauseOrCollumn,","}
20  syntax AsClause ::= Collumn "AS" Collumn
21  syntax ConditionExp ::= "WHERE" Exp
22
23  syntax FieldDcl ::= Id DataType
24  syntax FieldDcls ::= List{FieldDcl,","}
25  syntax CreateOption ::= "PRIMARY" "KEY" "(" Ids ")"
26  syntax CreateOptionList ::= List{CreateOption,","}
27  syntax K ::= doCreateOption( CreateOptionList , Table)
28  syntax Schema ::= createSchemaFromCollumns(AsClauseOrCollumns,Schema) [strict]
29  syntax Fields ::= makeField( FieldDcls ) [strict]
30  syntax Table ::= setPrimaryKey( Ids , Table ) [strict]
31  syntax Schema ::= setPrimaryKey( Ids , Schema ) [strict]
32
33  // Select
34  syntax SelectStm ::= "SELECT" Exp ";"
35  | "SELECT" ProjectionExp TableExp ";"
36  | "SELECT" ProjectionExp TableExp ConditionExp ";"
37
38  // Insert
39  syntax InsertStm ::= "INSERT" "INTO" Id "(" Ids ")" "VALUES" "(" Vals ")" ";"
40
41  // Delete
42  syntax DeleteStm ::= "DELETE" TableExp ConditionExp ";"
43  syntax Table ::= doDeleteRecords(Table,Exp) [strict(1)]
44  syntax Record ::= deleteAllWhere(Schema,Record,Exp) [strict(1,2)]
45  syntax Record ::= delete(TableElement,Record) [strict]
46
47  // Drop
48  syntax DropStm ::= "DROP" "TABLE" Ids ";"
49  syntax K ::= dropTable(Ids) [strict]
50
51  // Join
52  syntax Table ::= getTableFromId(Id)
53  syntax Table ::= getTableFromIds(Ids)
54
55  syntax TableExp ::= "FROM" Ids [strict]
56  | "FROM" JoinExp [strict]
57
58  syntax JoinExp ::= Ids "JOIN" Ids [strict(1,2)]
```

```

59 | Ids "JOIN" Ids JoinCondition [strict(1,2)]
60 | Ids "INNER" "JOIN" Ids [strict(1,2)]
61 | Ids "INNER" "JOIN" Ids JoinCondition [strict(1,2)]
62 | Ids "CROSS" "JOIN" Ids [strict]
63 | Ids "CROSS" "JOIN" Ids JoinCondition [strict]
64 | Ids "LEFT" "JOIN" Ids JoinCondition [strict(1,2)]
65 | Ids "LEFT" "OUTER" "JOIN" Ids JoinCondition [strict(1,2)]
66 | Ids "RIGHT" "JOIN" Ids JoinCondition [strict(1,2)]
67 | Ids "RIGHT" "OUTER" "JOIN" Ids JoinCondition [strict(1,2)]
68 | Ids "NATURAL" "JOIN" Ids [strict(1,2)]
69 | Ids "NATURAL" "LEFT" "JOIN" Ids [strict(1,2)]
70 | Ids "NATURAL" "LEFT" "OUTER" "JOIN" Ids [strict(1,2)]
71 | Ids "NATURAL" "RIGHT" "JOIN" Ids [strict(1,2)]
72 | Ids "NATURAL" "RIGHT" "OUTER" "JOIN" Ids [strict(1,2)]
73 syntax Table ::= joinUsing(Table,Table,Exp,FieldRep1s) [strict(1,2,4)]
74 syntax Table ::= leftJoinUsing(Table,Table,Exp,FieldRep1s) [strict(1,2,4)]
75 syntax Table ::= rightJoinUsing(Table,Table,Exp,FieldRep1s) [strict(1,2,4)]
76 syntax Table ::= naturalJoin(Table,Table) [strict]
77 syntax Table ::= naturalLeftJoin(Table,Table) [strict]
78 syntax JoinCondition ::= OnClause | UsingClause
79 syntax Table ::= naturalLeftJoin(Table,Table) [strict]
80 syntax Table ::= naturalRightJoin(Table,Table) [strict]
81
82 syntax OnClause ::= "ON" Exp
83 syntax UsingClause ::= "USING" "(" Collumns ")"
84 syntax Exp ::= changeCommonCollumnToEqualExp(Id, Id, FieldRep1s)
85 syntax KResult ::= FieldRep | FieldEquality
86 syntax FieldEquality ::= FieldRep "=" FieldRep
87 syntax Ids ::= commonField(Schema,Schema) [strict]
88 syntax Bool ::= hasCommonField(Schema,Field) [strict]
89
90 // Update
91 syntax UpdateStm ::= "UPDATE" Id "SET" AssignValues "WHERE" Exp ";" [strict(1,2)]
92 syntax #AssignValue ::= Id "=" Val
93 | String "=" Val
94 syntax AssignValue ::= #AssignValue
95 | Id "=" Exp [strict]
96 | String "=" Exp [strict]
97 syntax KResult ::= #AssignValue | AsClause
98 syntax AssignValues ::= List{AssignValue,","}
99 syntax Table ::= doUpdateValues(Table,Exp,AssignValues) [strict(1,3)]
100 syntax Record ::= updateAllWhere(Schema, Record, Exp, AssignValues) [strict(1,2,4)]
101 syntax TableElement ::= update(Schema, TableElement, AssignValues) [strict]
102 syntax TableElement ::= update2(Schema, TableElement, AssignValue) [strict]
103 syntax Field ::= changeFieldNameTo(Field,String) [strict]
104 endmodule

```

## A.6 SQL Semantics

```

1 module SQL
2   imports SQL-SYNTAX
3
4   // Configuration
5   configuration <T color = "red">
6     <k> $PGM:K </k>
7     <env color = "blue"> .Map </env>
8     <store color = "green">
9       <schema color = "pink"> .Map </schema>
10    <record color = "orange"> .Map </record>

```

```

11         </store>
12         <nextloc color = "yellow"> 0 </nextloc>
13     </T>
14
15 //**** Main function ****//
16
17 // Delete
18 rule DELETE FROM I:Id WHERE E:Exp ; => getTableFromId(I) ~> doDeleteRecords(HOLE,E)
19     ~> store HOLE
20 rule T:#Table ~> doDeleteRecords(HOLE,E) => doDeleteRecords(T,E)
21 rule doDeleteRecords(I:Id[ S:Schema : R:Record ], E:Exp) => I[ S :
22     deleteAllWhere(S,R,E)]
23 rule deleteAllWhere(S,r[ .TableElements ], E) => r[ .TableElements ] [structural]
24 rule deleteAllWhere(S,r[ T:TableElement, Ts:TableElements ], E:Exp) => eval(S,T,E) ~>
25     delete(T,deleteAllWhere(S,r[ Ts ], E))
26 rule true ~> delete(T,D) => D
27 rule false ~> delete(T,D) => addElement(T,D)
28
29 // Update
30 rule UPDATE I:Id SET A:AssignValue WHERE E:Exp ; => getTableFromId(I) ~>
31     doUpdateValues(HOLE, E, A) ~> store HOLE
32 rule T:#Table ~> doUpdateValues(HOLE,E,A) => doUpdateValues(T,E,A)
33 rule doUpdateValues(I:Id[ S:Schema : R:Record ],E:Exp,As:AssignValues) => I[ S :
34     updateAllWhere(S,R,E,As)]
35 rule updateAllWhere(S,r[ .TableElements ],E,As:AssignValues) => r[ .TableElements ]
36     [structural]
37 rule updateAllWhere(S,r[ T:TableElement, Ts:TableElements],E:Exp,As:AssignValues) =>
38     eval(S,T,E) ~> update(S,T,As) ~> addElement(HOLE, updateAllWhere(S,r[Ts],E,As))
39     [structural]
40 rule true ~> update(S,T,As) ~> addElement(HOLE, U) => addElement(update(S,T,As),U)
41     [structural]
42 rule false ~> update(S,T,As) ~> addElement(HOLE, U) => addElement(T,U) [structural]
43 rule update(S:Schema, T:TableElement, .AssignValues) => T
44 rule update(S:Schema, T:TableElement, A:AssignValue, As:AssignValues) =>
45     update(S,update2(S,T,A),As)
46 rule update2( _, e(.Vals), A ) => e(.Vals) [structural]
47 rule update2(s[ f(FName:String,_,_,_), Fs:Fields], e(V:Val, Vs:Vals), F2Name:Id =
48     VNew:Val) => e(VNew,Vs) when FName ==String #tokenToString(F2Name)
49 rule update2(s[ f(FName:String,_,_,_), Fs:Fields], e(V:Val, Vs:Vals), F2Name:Id =
50     VNew:Val) => addTopElement(V,update2(s[Fs], e(Vs), F2Name = VNew)) when FName
51     /=String #tokenToString(F2Name)
52
53 // Drop
54 rule DROP TABLE Ts:Ids ; => dropTable(Ts)
55 rule dropTable( .Ids ) => .
56 rule <k> dropTable(I1:Id, Ids) => dropTable(Id) </k>
57     <env> ... ((I1 => NULL) |-> L:Int) ... </env>
58     <store>
59         <schema> ... (L |-> (S => NULL)) ... </schema>
60         <record> ... (L |-> (R => NULL)) ... </record>
61     </store>
62 // Get Table
63 rule <k> getTableFromId(I:Id) => I[ S : R ] ... </k>
64     <env> ... (I |-> L:Int) ... </env>
65     <store>
66         <schema> ... (L |-> S) ... </schema>
67         <record> ... (L |-> R) ... </record>
68     </store>
69 rule getTableFromIds( I:Id ) => getTableFromId(I) [structural, anywhere]
70 rule getTableFromIds(I1:Id, Is:Ids) => join(getTableFromId(I1),getTableFromIds(Is))
71     [anywhere]

```



```

58
59 // Store
60 rule <k> I:Id[ S:#Schema : R:#Record] ~> store HOLE => . ... </k>
61   <env> ... I |-> L ... </env>
62   <store>
63     <schema> ... L |-> ( _ => S ) ... </schema>
64     <record> ... L |-> ( _ => R ) ... </record>
65   </store>
66 rule <k> I:Id[ S:#Schema : R:#Record] ~> store HOLE => . ... </k>
67   <env> ... . => I |-> L </env>
68   <store>
69     <schema> ... . => L |-> S </schema>
70     <record> ... . => L |-> R </record>
71   </store>
72   <nextloc> L:Int => L +Int 1 </nextloc>
73
74 rule store T:Table => T ~> store HOLE [structural]
75
76 // Create
77 rule S1:Stms S2:Stms => S1 ~> S2 [structural]
78 rule CREATE TABLE TNAME:Id ( FDcls:FieldDcls ) ; =>
79   changeFieldNameCorrespondToTable(TNAME[s[ makeField( FDcls ) ] : r[ .TableElements ]
80   ]) ~> store HOLE
81 rule CREATE TABLE Id ( FDcls:FieldDcls , Opt:CreateOptionList ) ; =>
82   changeFieldNameCorrespondToTable(doCreateOption(Opt,Id[s[ makeField( FDcls ) ] :
83   r[.TableElements ]])) ~> store HOLE
84
85 rule makeField(.FieldDcls) => .Fields [structural,anywhere]
86 rule makeField(I:Id T:DataType, Dcls:FieldDcls) => f(#tokenToString(I) , T
87   ,false,false) , makeField(Dcls) [anywhere]
88
89 rule doCreateOption( .CreateOptionList , T) => T
90 rule doCreateOption(PRIMARY KEY (KIds:Ids), Opt:CreateOptionList , T:Table) =>
91   doCreateOption(Opt, setPrimaryKey(KIds,T))
92
93 // Insert
94 rule <k> (INSERT INTO I:Id(Fs:Ids) VALUES (Vs:Vals) ; => .) ... </k>
95   <env> ... I |-> L ... </env>
96   <store>
97     <schema> ... L |-> S ... </schema>
98     <record> ... (L |-> (r[ Es ] => concat(r[Es],r[e(Vs)]))) ... </record>
99   </store>
100
101 // Select
102 rule SELECT E:Exp ; => E
103 rule <k> SELECT P:ProjectionExp T:TableExp ;
104   => doGetTableExp(T) ~> doProjectionExp( HOLE , P ) ... </k>
105 rule <k> SELECT P:ProjectionExp T:TableExp C:ConditionExp ; => doGetTableExp(T) ~>
106   doConditionExp( HOLE , C ) ~> doProjectionExp( HOLE , P ) ... </k>
107 rule T:#Table ~> doConditionExp(HOLE,C) => doConditionExp( T , C ) [structural]
108 rule T:#Table ~> doProjectionExp( HOLE , P) => doProjectionExp(T,P) [structural]
109 rule doConditionExp(T:Table , WHERE E:Exp ) => select(T,E) [anywhere]
110 rule doProjectionExp(T:Table, * ) => T [anywhere]
111 rule doProjectionExp(T:Id[ S:Schema : R:Record], As:AsClauseOrCollumns) =>
112   project((T[ S : R ]),createSchemaFromCollumns( As,S )) [anywhere]
113
114 //**** Auxiliary function ****//
115 rule changeFieldNameTo(f(FN1,D:DataType ,B1:Bool ,B2:Bool),FN2:String) =>
116   f(substrString(FN1,0,(findChar(FN1,".",0) +Int 1)) +String FN2,D,B1,B2) [anywhere]
117 rule createSchemaFromCollumns( .AsClauseOrCollumns , S:Schema) => s[ .Fields ]
118   [anywhere,structural]

```

```

109 rule createSchemaFromCollumns(C:Collumn,Cs:AsClauseOrCollumns,S:Schema) =>
110     addElement( getFieldFromSchema(C,S) , createSchemaFromCollumns(Cs,S)) [anywhere]
111 rule createSchemaFromCollumns(C1:Collumn AS I:Id, Cs:AsClauseOrCollumns,S:Schema) =>
112     addElement( changeFieldNameTo(getFieldFromSchema(C1,S), #tokenToString(I) ) ,
113         createSchemaFromCollumns(Cs,S)) [anywhere]
114 rule createSchemaFromCollumns(C1:Collumn AS ' I:Id ', Cs:AsClauseOrCollumns,S:Schema)
115     => addElement( changeFieldNameTo(getFieldFromSchema(C1,S), #tokenToString(I) ) ,
116         createSchemaFromCollumns(Cs,S)) [anywhere]
117 rule setPrimaryKey( KIds:Ids, TName:Id[ S : R] ) => TName[setPrimaryKey(KIds, S) :
118     R] [anywhere]
119 rule setPrimaryKey( KIds:Ids , s[.Fields]) => s[.Fields] [anywhere]
120 rule setPrimaryKey( KIds:Ids , s[f( S:String , T:DataType , B1, B2 ) , Fs:Fields] )
121     => addElement( f(S,T,true,B2) , setPrimaryKey(KIds, s[Fs])) when in(S,KIds)
122     [anywhere]
123 rule setPrimaryKey( KIds:Ids , s[f( S:String , T:DataType , B1, B2 ) , Fs:Fields] )
124     => addElement( f(S,T,B1,B2) , setPrimaryKey(KIds, s[Fs])) when notBool in(S,KIds)
125     [anywhere]
126 rule doGetTableExp(FROM Is:Ids) => getTableFromIds(Is)
127 rule doGetTableExp(FROM Is1:Ids JOIN Is2:Ids) =>
128     join(getTableFromIds(Is1),getTableFromIds(Is2))
129 rule doGetTableExp(FROM Is1:Ids JOIN Is2:Ids ON E:Exp) =>
130     join(getTableFromIds(Is1),getTableFromIds(Is2),E)
131 rule doGetTableExp(FROM Is1:Id JOIN Is2:Id USING(Fs:FieldRep1s)) =>
132     joinUsing(getTableFromIds(Is1),getTableFromIds(Is2),
133         changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
134 rule doGetTableExp(FROM Is1:Ids INNER JOIN Is2:Ids) =>
135     join(getTableFromIds(Is1),getTableFromIds(Is2))
136 rule doGetTableExp(FROM Is1:Ids INNER JOIN Is2:Ids ON E:Exp) =>
137     join(getTableFromIds(Is1),getTableFromIds(Is2),E)
138 rule doGetTableExp(FROM Is1:Id INNER JOIN Is2:Id USING(Fs:FieldRep1s)) =>
139     joinUsing(getTableFromIds(Is1),getTableFromIds(Is2),
140         changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
141 rule joinUsing(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema : R2:Record] , E:Exp ,
142     Fs:FieldRep1s) => join(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema : R2:Record] ,
143     E:Exp) ~> project(HOLE,concat(S1,excludeFields(S2,Fs)))
144 rule T:#Table ~> project(HOLE,S) => project(T,S) [structural]
145 rule doGetTableExp(FROM Is1:Ids LEFT JOIN Is2:Ids ON E:Exp) => leftJoin(
146     getTableFromIds(Is1) , getTableFromIds(Is2),E)
147 rule doGetTableExp(FROM Is1:Id LEFT JOIN Is2:Id USING(Fs:FieldRep1s)) =>
148     leftJoinUsing( getTableFromIds(Is1),
149         getTableFromIds(Is2),changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
150 rule doGetTableExp(FROM Is1:Ids LEFT OUTER JOIN Is2:Ids ON E:Exp) => leftJoin(
151     getTableFromIds(Is1) , getTableFromIds(Is2),E)
152 rule doGetTableExp(FROM Is1:Id LEFT OUTER JOIN Is2:Id USING(Fs:FieldRep1s)) =>
153     leftJoinUsing( getTableFromIds(Is1),
154         getTableFromIds(Is2),changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
155 rule leftJoinUsing(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema : R2:Record] ,
156     E:Exp , Fs:FieldRep1s) => leftJoin(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema :
157     R2:Record] , E:Exp) ~> project(HOLE,concat(S1,excludeFields(S2,Fs)))
158 rule doGetTableExp(FROM Is1:Ids RIGHT JOIN Is2:Ids ON E:Exp) => rightJoin(
159     getTableFromIds(Is1),getTableFromIds(Is2),E)
160 rule doGetTableExp(FROM Is1:Id RIGHT JOIN Is2:Id USING(Fs:FieldRep1s)) =>
161     rightJoinUsing(
162         getTableFromIds(Is1),getTableFromIds(Is2),changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
163 rule doGetTableExp(FROM Is1:Ids RIGHT OUTER JOIN Is2:Ids ON E:Exp) => rightJoin(

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141     getTableFromIds(Is1),getTableFromIds(Is2),E)
142 rule doGetTableExp(FROM Is1:Id RIGHT OUTER JOIN Is2:Id USING(Fs:FieldRep1s)) =>
143     rightJoinUsing(
144         getTableFromIds(Is1),getTableFromIds(Is2),changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
145 rule rightJoinUsing(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema : R2:Record] ,
146     E:Exp , Fs:FieldRep1s) => rightJoin(T1:Id[S1:Schema : R1:Record] , T2:Id[S2:Schema :
147     R2:Record] , E:Exp) ~> project(HOLE,concat(excludeFields(S1,Fs),S2))
148 rule doGetTableExp(FROM Is1:Ids CROSS JOIN Is2:Ids) => catesian(
149     getTableFromIds(Is1), getTableFromIds(Is2))
150 rule doGetTableExp(FROM Is1:Ids CROSS JOIN Is2:Ids ON E:Exp) => join(
151     getTableFromIds(Is1), getTableFromIds(Is2),E)
152 rule doGetTableExp(FROM Is1:Id CROSS JOIN Is2:Id USING(Fs:FieldRep1s)) => joinUsing(
153     getTableFromIds(Is1), getTableFromIds(Is2),
154     changeCommonCollumnToEqualExp(Is1,Is2,Fs),Fs)
155 rule doGetTableExp(FROM Is1:Id NATURAL JOIN Is2:Id) =>
156     naturalJoin(getTableFromIds(Is1),getTableFromIds(Is2))
157 rule naturalJoin(T1:Id[ S1:Schema : R1:Record ], T2:Id[ S2:Schema : R2:Record ]) =>
158     joinUsing(T1[ S1 : R1],T2[ S2 : R2
159     ],changeCommonCollumnToEqualExp(T1,T2,commonField(S1,S2)),commonField(S1,S2))
160 rule doGetTableExp(FROM Is1:Id NATURAL LEFT JOIN Is2:Id) =>
161     naturalLeftJoin(getTableFromIds(Is1),getTableFromIds(Is2))
162 rule doGetTableExp(FROM Is1:Id NATURAL LEFT OUTER JOIN Is2:Id) =>
163     naturalLeftJoin(getTableFromIds(Is1),getTableFromIds(Is2))
164 rule naturalLeftJoin(T1:Id[ S1:Schema : R1:Record ], T2:Id[ S2:Schema : R2:Record
165     ]) => leftJoinUsing(T1[ S1 : R1],T2[ S2 : R2
166     ],changeCommonCollumnToEqualExp(T1,T2,commonField(S1,S2)),commonField(S1,S2))
167 rule doGetTableExp(FROM Is1:Id NATURAL RIGHT JOIN Is2:Id) =>
168     naturalRightJoin(getTableFromIds(Is1),getTableFromIds(Is2))
169 rule doGetTableExp(FROM Is1:Id NATURAL RIGHT OUTER JOIN Is2:Id) =>
170     naturalRightJoin(getTableFromIds(Is1),getTableFromIds(Is2))
171 rule changeCommonCollumnToEqualExp( T1:Id, T2:Id, .FieldRep1s ) => true [anywhere]
172 rule changeCommonCollumnToEqualExp( T1:Id, T2:Id, F1:Id , Fs:FieldRep1s ) =>
173     (((T1.F1 = T2.F1):Exp) && (changeCommonCollumnToEqualExp(T1:Id,T2:Id,
174     Fs:FieldRep1s))) [anywhere]
174 rule changeCommonCollumnToEqualExp( T1:Id, T2:Id , ' F1:Id ' , Fs:FieldRep1s ) =>
175     (((T1.F1 = T2.F1):Exp) && (changeCommonCollumnToEqualExp(T1:Id,T2:Id,
176     Fs:FieldRep1s))) [anywhere]
177 rule naturalLeftJoin(T1:Id[ S1:Schema : R1:Record ], T2:Id[ S2:Schema : R2:Record
178     ]) => leftJoinUsing(T1[ S1 : R1],T2[ S2 : R2
179     ],changeCommonCollumnToEqualExp(T1,T2,commonField(S1,S2)),commonField(S1,S2))
180 rule doGetTableExp(FROM Is1:Id NATURAL RIGHT JOIN Is2:Id) =>
181     naturalRightJoin(getTableFromIds(Is1),getTableFromIds(Is2))
182 rule doGetTableExp(FROM Is1:Id NATURAL RIGHT OUTER JOIN Is2:Id) =>
183     naturalRightJoin(getTableFromIds(Is1),getTableFromIds(Is2))
184 rule changeCommonCollumnToEqualExp( T1:Id, T2:Id , ' F1:Id ' , Fs:FieldRep1s ) =>
185     (((T1.F1 = T2.F1):Exp) && (changeCommonCollumnToEqualExp(T1:Id,T2:Id,
186     Fs:FieldRep1s))) [anywhere]
187 rule commonField(s[.Fields],S2:Schema) => .Ids [anywhere]
188 rule commonField(s[f(FN1:String,D,B1,B2), Fs:Fields],S2:Schema) =>

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hasCommonField(S2,f(FN1:String,D,B1,B2)) ~>
if(HOLE,String2Id(substrString(FN1,(findChar(FN1,".",0) +Int 1),lengthString(FN1)))
, commonField(s[Fs],S2),commonField(s[Fs],S2)) [anywhere]
174
175 rule hasCommonField(s[ .Fields ], _ ) => false [anywhere]
176 rule hasCommonField(s[f(FN1:String,D,B1,B2), Fs:Fields],f(FN2:String,_,_,_)) => true
when substrString(FN1,(findChar(FN1,".",0) +Int 1),lengthString(FN1)) ==String
substrString(FN2,(findChar(FN2,".",0) +Int 1),lengthString(FN2)) [anywhere]
177 rule hasCommonField(s[f(FN1:String,D,B1,B2), Fs:Fields],f(FN2:String,D2,B3,B4)) =>
hasCommonField(s[Fs],f(FN2,D2,B3,B4)) when substrString(FN1,(findChar(FN1,".",0)
+Int 1),lengthString(FN1)) !=String substrString(FN2,(findChar(FN2,".",0) +Int
1),lengthString(FN2)) [anywhere]
178 endmodule

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