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Description	

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A Mathematical Model of Game Refinement and Its Applications to Sports Games

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Abstract

This paper explores a mathematical model of game progress. We claim that a realistic model of the game progress during the in-game period is not linear but exponential. The second derivative value, i.e., acceleration in the sense of dynamics, is derived from the model and we propose to use the value as a measure of game refinement. This is because acceleration of game progress should be related to the emotional impact in our minds, as thrill or engagement in games. We also evaluate well known games using our theory like sport games, that would further be classified by the rule to finish the game. It is expected that the game refinement theory will be widely used as a tool to assess the quality of various types of games as a new game theory.

Keywords: game refinement theory, engagement assessment, sports games, board games, video games

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1. Introduction

Many efforts have been devoted to the study of strategic decision making in the framework of game theory with focus on mathematical models of conflict and cooperation between intelligent rational decision-makers or game-players. Game theory originated in the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games [10], which has been widely recognized as a useful tool in many fields such as economics, political science, psychology, logic and biology.

However, little is known about mathematical theory from the game creator's point of view. It is interesting to know theoretical aspects of increasing attractiveness of games and its sophistication. An early work in this direction has been done by Iida *et al.* [8], in which a measure of game refinement was proposed based on the concept of game outcome uncertainty. A logistic model was constructed in the framework of game refinement theory and applied to many board games including chess variants and Mah Jong [7]. The proposed measure of refinement concerns the information gained on the average branching factor and the average game length for a given game.

It is natural but challenging to apply the game refinement theory to various types of games such as sports games and video games. This encourages us to study a general model of game refinement in the domain of sports games. Although there are some rule changes in the history of every sport games, the core mechanism is still unchanged from the original idea. That is why sports is an interesting subject to game refinement theory.

2. Game Refinement Theory

In this section we construct a game progress model which can be used for various types of games. A measure of game refinement will be derived from the model. Then we consider the gap between board games and other games such as sports games and video games. Moreover, we apply some data from well-known games to confirm the effectiveness of the proposed measure.

2.1. Game progress model

In this study "game progress" has twofold. One is game speed or scoring rate, while another one is game information progress with focus on the game outcome.

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In sports games such as Soccer and Basketball, the scoring rate will be calculated by two factors: (1) goal, i.e., total score and (2) time or steps to achieve the goal. For example, in Basketball the total score is given by the average number of successful shoots, whereas the steps to achieve the goal is estimated by the average number of shoots attempted. Then the game speed of Basketball is given by

$$\frac{average\_number\_of\_successful\_shoots}{average\_number\_of\_shoots}$$

We need to consider a reasonable way to obtain the game speed for various type of games. For some sports games such as Basketball and Soccer, we can obtain statistics of average number of shoots and goals per one game.

For other sports games such as Volleyball and Tennis in which the goal (i.e., score to win) is set in advance, the average number of total points per game may correspond to the time or steps to achieve the goal. For video games such as Pac Man, the steps can be estimated by the average number of movements of Pac Man and the goal is estimated by the scores gained at each stage. In the domain of board games, the steps can be estimated by the average depth of game tree, (i.e., game length) and the goal is estimated by the average branching factor. For the board game case we discuss more detail later on.

Now we consider a model of game information progress. Let G and T be the average number of successful shoots and the average number of shoots per game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in Equation (1).

$$x(t) = \frac{G}{T}\, t \tag{1}$$

However, the game information progress given by Equation (1) is usually not known during the in-game period. The game outcome is uncertain until the very end of game in many games which we call *balanced game* or *seesaw game*. It means that the change of game information becomes greater near the end of game. Thus, the game information progress should be not linear but something like exponential.

Hence, we propose a realistic model of game information progress in Equation (2).

$$x(t) = G(\frac{t}{T})^n \tag{2}$$

Here n stands for a constant parameter which will depend on the perspective of an observer in the game considered. [1]

If one knows the game outcome, for example after the game, or if one can exactly predict in advance the game outcome and its progress, then we have $n = 1$, where $x(t)$ is a linear function of time t. During the in-game period, various values of the parameter n for different observers including players and supporters will be determined. For example, some observers might be optimistic with $0 \leq n < 1$. However, when one feels any challenges to win or achieve the goal, the parameter would be $n > 1$.

Meanwhile, we reasonably assume that the parameter would be $n \geq 2$ in many cases like balanced or seesaw games. Thus, we have the second derivative of $x(t)$ in Equation (2), as shown in Equation (3).

$$x''(t) = \frac{Gn(n-1)}{T^n} t^{n-2} \tag{3}$$

Solving the formula at $t = T$, the equation becomes

$$x''(T) = \frac{G}{T^2} n(n-1)$$

Newton mechanics indicates that the force in the physical world is derived from mass and acceleration. It is assumed in this study that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in our minds, but it is likely that the acceleration of information progress is related to the force in mind. Hence, it is reasonably expected that the larger the value $\frac{G}{T^2}$ is, the more the game becomes exciting due to the uncertainty of game outcome. Thus, we propose to use the value $\frac{G}{T^2}$ or its root square, as shown in Equation (4), as a game refinement value for the game considered.

$$R = \frac{\sqrt{G}}{T} \tag{4}$$

We will consider this game refinement value as R value in the rest of this section.

2.2. Game Progress in board games

Here we consider the gap between board games and other games such as sports games. We assume that the game information progress in sports games is related to game progress in board games. To justify the assumption, we begin the explanation for the case of board games. From Equation (1) we know the game

[1] Note that Equation (2) might be a promising and realistic model of game progress, but its uniqueness (to be exponential) should be investigated more in the future.

information value as a function of game progress or depth of game tree t in board games.

We show, in Figure 1, an illustration of game progress. The whole triangle illustrates a game tree and the middle line illustrates a game progress. As players make their moves, the game progress begins to move down.

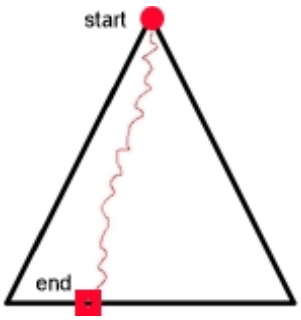

Figure 1. Illustration of game progress through a game tree

At each depth of the game tree, one will choose a move and the game will progress. Calculating the length of middle line in Figure 1 is the key to get the total game progress value. To calculate it, consider Figure 2. In the game tree, we have the horizontal difference between neighboring nodes (say Δl), the difference between levels of tree (say Δt), and distance between them (say d).

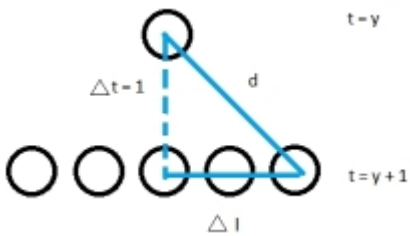

Figure 2. Illustration of distance in game tree

The distance d can be found by using simple Pythagoras theorem, thus resulting in

$$d = \sqrt{\Delta l^2 + 1} \tag{5}$$

B is the branching factor for a decision in a board game. Note that $\Delta l < B$ holds because the maximum horizontal difference between neighboring nodes will be $B - 1$. Assuming that the approximate value of horizontal difference between nodes is $\frac{B}{2}$, then we can make a substitution in Equation (5).

$$d = \sqrt{(\frac{B}{2})^2 + 1}$$

The game progress is the total distance between nodes considered in the game tree. Here, we do not consider Δt^2 because the value $(\Delta t^2 = 1)$ is small compared to B. The game length will be normalized by the average game length D, then the game progress $x(t)$ is given by

$$x(t) = \frac{t}{D} \cdot d = \frac{t}{D}\sqrt{(\frac{B}{2})^2} = \frac{Bt}{2D}$$

In general we have

$$x(t) = c\frac{B}{D}t$$

Where c is a different constant which depends on the game considered. [2] However, we manage to explain how to obtain the game progress value itself. The game progress in the domain of board games forms a linear graph with the maximum value $x(t)$ of B. Assuming $c = 1$, then we have a realistic game progress model for board games, which is given by

$$x(t) = B(\frac{t}{D})^n. \tag{6}$$

Equation (6) shows that our present study comes to the same formula described in the early works [7].

Table 1. Game progress factors and game refinement value for various games

	Time	Goal to achieve	R
Sports games	N	G	$\frac{\sqrt{G}}{N}$
Board games	D	B	$\frac{\sqrt{B}}{D}$

3. Application to Time-limited Sports

In this section we will evaluate the time-limited sports, especially for Soccer and Basketball. Then we compare it with previous research for board games to see if the principle could also be applied to this type of sports.

3.1. Soccer and Basketball

Here we show, in Table 1, a comparison of game progress factors and game refinement value for various type of games. In sports games, G is number of goals and N is the number of attacks. While in board games, B is the branching factor and D is the depth of game tree.

To confirm the effectiveness of proposed measures, some data of games such as Chess and Go [8] from board games and Basketball and Soccer from sports are compared. For Basketball the data were obtained from the NBA website [9], while the data for Soccer were obtained from the UEFA championship [11]. We show, in Table 2, a comparison of game refinement value for various type of games. We suspect when the values

[2] Further investigation is needed to obtain the exact value of c.

Table 2. Measures of game refinement for board games and sports games

Game	B or G	D or N	R
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073

go too high, then observer will get the excitement too fast and when the value goes too low the excitement comes slower thus would not make a game interesting. Sophisticated games should have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games as we can see from sophisticated games that we have observed. We understand that Chess, Go, Soccer and Basketball are both sophisticated with long history.

In the previous study with focus on Chess variants [8], we observed that the proposed measure seems promising to explain the evolutionary changes of game rules. For example, in the Chess history most variants were outsourced and only a few variants survived to the present. The surviving variants went through the sophistication of the game rules to maximize the entertainment impact making the depth of lookahead (i.e., intelligent aspect of games) more critical for the outcome of the game. Experienced players often noted that large and complex games were not attractive at all and interesting and enjoyable games are those with more entertainment impacts. The evolutionary process has produced the present version of Chess which seems a well-balanced search-space complexity and entertainment impact. Modern Chess may be considered a highly matured and optimized Chess-like game. Similarly, through this study we make the same observation for sophisticated sports games such as Basketball and Soccer as well as for board games such as Chess and Go.

4. Application to Score-limited Sports

We consider the application of game refinement theory to score-limited sports such as Badminton, Table Tennis and Volleyball in this section. For each case we need to examine an appropriate model of game progress to derive a game refinement value.

4.1. Badminton

Badminton's official rules are described in the official site of Badminton World Federation [1]. There are two sides of player(s), either singles or doubles. There are 5 types of events: Men's singles, Women's singles, Men's doubles, Women's doubles, and Mixed doubles. Under the new current scoring system, the side which wins the rally scores the point regardless of which side serves. The information length of Badminton game is 21 points, while maintaining a minimum 2-point lead. The first side to win 2 games out of 3 wins the match. This is the g3 x 21h rally point scoring system.

In the past, a g3 x 15h side-out scoring system was used (3 x 11 for women's singles). For this side-out scoring system, only the server can score the point. If the service side loses the rally, no point is awarded, and the service passes to the other side. It can be seen that this could, and often did, result in irregularly long match times. The scoring system was changed in December 2005.

In Badminton, the game progress or scoring rate will be calculated by two factors: winner's score (say W) and total score (say T). Then the game progress of Badminton is given by

$$\frac{average\_winner's\_score}{average\_total\_score}$$

We show, in Table 3, the game refinement values for the old scoring system and the current scoring system. Data was collected from BWF world championship [2][4]. Game refinement values for the two variants is significantly different, indicating the change to game refinement values as affected by the change in scoring. The side-out system gives the higher value 0.121. Under the new system, the game refinement value recedes to a more balanced 0.086. The result implies that the change in the scoring system makes the game more interesting and attractive for observation.

Table 3. Measures of game refinement for Badminton

Scoring system	W	T	R
Side-out	30.070	45.154	0.121
Current	46.336	79.344	0.086

4.2. Table Tennis

Table tennis was first played in 1880s in England as an after dinner activity. It became popular with the introduction of the name 'Ping-Pong' by J. Jacques & Son, and has since undergone a few change of rules and equipment [3]. In this part, we will explain the basic rules of table tennis and the change in equipment, and compare the results from applying game refinement theory on table tennis.

The official rules are available on the official site of ITTF [3]. There are two sides of player(s), either singles or doubles. There are 4 types of events: Men's or Women's singles, and Men's or Women's teams. As with the new badminton and volleyball systems, in table tennis the side which wins the rally scores the

point regardless of service. The information length of game is 11 points, while maintaining a minimum 2-point lead. The match consists of any odd number of games, usually 7.

At first, table tennis rackets were pure wood. Around 1900, the use of layered foam-core rackets, topped with rubber sheeting, dramatically increase the game's speed and added some spin to the game. Prior to the 2000 Summer Olympics, table tennis used a 38 mm ball. The ball size was changed to 40 mm which effectively reduced the game's speed. The scoring system was also changed in September 2001. Until that year, game lengths were 21 points with a 2 point lead, while matches were usually a best-of-3 or -5. Most recently, the doubles event was integrated as a part of team events for 2008 Olympics. Each team features 3 players, 1 singles player and 2 double players, playing a best-of-5 series of 2 singles, follow by 1 doubles, followed by up to 2 more singles. The first side to win 3 wins the match.

We show, in Table 4, the game refinement values for pre-2001 games, post-2001 games, and the team events. Data of Olympic matches from 1988 to present was collected from the records of the ITTF official site [3]. The result indicates that in the case of table tennis, the changes do not significantly affect the game progress. The average game refinement value of 0.076 holds for table tennis since the 1988 Olympics. As for the result of the team event, the game refinement value is lower than for the others, and lower than expected for a well-balanced sophisticated game. We also notice the higher average score for team events. It can be safely assumed that the change to integrate the doubles event into the team events, while lessen the emphasis on less popular doubles event [6], reduces the value of game sophistication beyond the 0.07 – 0.08 window of good balance.

Table 4. Measures of game refinement for Table Tennis

Scoring system	W	T	R
Pre-2001	57.869	101.530	0.075
Post-2001	54.863	96.465	0.077
Team	131.283	232.123	0.049

4.3. Volleyball

In this study we select three important variants from the history of Volleyball: (1) side-out scoring system with 15 points, (2) rally point system with 30 points, and (3) rally point system with 25 points (see Table 5).

We first focus on the current rule, i.e., rally point system with 25 points. We show, in Table 6, statistics on the average point per game in Volleyball games from V-league in Japan [5]. The max point and min point are also shown. Since the average point per game is 44, it is expected that the final score on average is 25 – 19. Likewise, the score in max point case would be 37 – 35 due to deuce and 15 – 6 (15 points) in min point case.

Table 5. Three rule variants of Volleyball

rules	points	set
side-out scoring system	15	best-of-five
rally point system	30	best-of-five
rally point system	25	best-of-five

Table 6. Statistics on point per game in rally point system Volleyball with 25 points (n=486).

	Points
Max points	72
Min points	21
Ave points	44

The game progress in Volleyball can be given by the average number of goals ($G = 25$) and the average number of all scores ($T = 44$). By applying Equation (4), we obtain as the value as follows:

$$R_{25pts} = \frac{\sqrt{25}}{44} = 0.114$$

We next consider the rally point system with 30 points. Since the data for 30 points rally currently is unavailable, we estimate it based on the statistics of 25 points rally point system Volleyball. For this purpose, we assume the same ratio (25 : 19) of winning points and losing points and obtain 30 : 22.8. It means that the average goals G is 30 while the average total points T is 52.8. By applying Equation (4), we obtain as the value as follows:

$$R_{30pts} = \frac{\sqrt{30}}{52.8} = 0.104$$

We show, in Table 7, some statistics of rally point system which include some other cases.

Table 7. Some statistics for rally point system with various goal points.

G	T	R
25	44	0.114
30	53	0.104
35	62	0.095
40	70	0.090
50	88	0.080
60	106	0.073

For the side-out scoring system, we try to simulate to obtain the data since we have no real data. We assume some different *scoring percentage* γ for the side of serving the ball. From the previous results it is estimated that we have the final score 15 : 11.26. The game progress values for some different parameters can be calculated and shown in Table 8.

Table 8. Some statistics for side-out system with 15 points (simulation).

scoring γ (%)	total score	R
60	43.77	0.088
50	52.52	0.074
40	65.65	0.059
37	72.0	0.054
33	86.66	0.045
25	105.04	0.037

We suspect based on our experience that the scoring percentage $\gamma = 50$ holds in the side-out system with 15 points Volleyball when both opposing teams are well balanced. Hence, we take the $R_{25pts} = 0.074$ for representing the game refinement value of 15 points side-out scoring.

We show, in Table 9, the comparison between three Volleyball variants based on the game refinement value.

Table 9. game refinement value for three variants of Volleyball.

variants	points	R
side-out scoring system	15	0.074
rally point system	30	0.104
rally point system	25	0.114

The side-out scoring system (with scoring rate roughly $\gamma = 50$) had been played long time (1947-1999). As we can see from the Volleyball's history and rules changes, the γ should be lower than 40. The game refinement value is different with other sophisticated games such as Chess and Soccer. However, the rule was changed in 1999 to improve game understandability. At the same time, the rule change has made rise in excitement, depicted in higher value of game refinement value.

The study using board games suggests that the game refinement value of sophisticated games are somewhere in the range 0.07 – 0.08 [7]. The higher R value in the current volleyball rules might mean the game become more exciting, but it is not alligned well with our previous classification of sophisticated game. Thus, it can be assumed that this rule changes might not the optimum method to improve the game attractiveness in the *comfortable range*.

5. Conclusions

In this paper we presented a mathematical model based on game progress and game information model with focus on game outcome uncertainty. Game refinement value was derived from the game information progress model. Its second derivative, which is the acceleration in the sense of dynamics, was derived from the model to use the value as a game refinement value. This is because the acceleration of game information progress should relate to the emotional impact such as entertainment and engagement which may correspond to the force in physics. We then investigated a model of game progress in various sport game domains. Applying data from different type of games, we see that this model may only fit for the time-limited sports, such as soccer and basketball. Either this model does not fit for the score limited sports or this type of sport has its own refinement value, which is not 0.07 – 0.08.

This research left some further works that need to be done. More investigation in collecting data of many other games to test and elaborate the model we discussed in this paper will be needed. Automated tasks to collect data from media might also help instead of collecting data from simulation and the internet. Moreover, it is expected to establish the game refinement theory, which will be widely used like classical game theory, not to find the optimal strategy to play a given game, but to assess the sophistication of games or quality of life in game playing.

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References

[1] Badminton world federation's official site. http://www.bwfbadminton.org.
[2] Bwf world championships. http://en.wikipedia.org/wiki/BWF_World_Championships.
[3] International table tennis federation's official site. http://www.ittf.com.
[4] Tournament software badminton. http://www.tournamentsoftware.com/ tournamentlist.aspx?id=2.
[5] V-league official site. http://www.vleague.or.jp/.
[6] ETTA. English table tennis association fs press. http://ettapressoffice.com/history.
[7] H. Iida, H. Takahara, J. Nagashima, Y. Kajihara, and Y. Hashimoto. An application of game-refinement theory to mah jong. In *International Conference on Entertainment Computing (ICEC)*, pages 333–338, 2004.

[8] H. Iida, N. Takeshita, and J. Yoshimura. A metric for entertainment of boardgames: Its implication for evolution of chess variants. *Entertainment Computing Technologies and Applications*, pages 65–72, 2003.

[9] NBA. Nba team statistics. url: http://stats.nba.com. Accessed: 30-10-2013.

[10] J. Neumann. Zur theorie der gesellschaftsspiele. *Mathematische Annalen*, 100(1):295–320, 1928.

[11] UEFA. Uefa 2012/13 team statistics. url: http://www.uefa.com/uefachampionsleague/. Accessed: 8-11-2013.