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Decoding Techniques for Graph-based Random Access High Dense Multiway Multirelay Networks

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Abstract— Densely deployed wireless network is one of the most important solutions for spectrum crunch expected by 2020 with a huge economic impact. This paper proposes decoding schemes for high dense multiway multirelaying (hd-MWMR) systems comprising two multiway relays to serve huge number of users. Due to the nature of huge number, instead of using perfect scheduling, we consider coded random access schemes, where all users transmit their messages without coordination. Although the transmission is uncoordinated, the network structure still has a structure that provide an advantage. The theoretical bound for hd-MWMR systems exploiting two multiway relays is derived. To achieve the bound, we propose simple coding and decoding schemes based on iterative interference cancellation over a sparse graph involving two multiway relays. The results confirm that the second relay helps on the improvement of throughput performances, which is highly required for future wireless networks.

Keywords— Multiway relay, iterative decoding, amplify and forward (AF), graph-based random access, EXIT chart.

I. Introduction

Machine-to-Machine (M2M) communication is expected to grow exponentially in the near future rose by the massive deployment of sensors, RFtags, smart metering, and other devices. Forecasts indicate that 4.9 billion connected things are in use in 2015 and the number will reach 50 billion or even 75 billion for various application categories, e.g., consumer, automotive, generic business, and vertical business applications in 2020. This fact leads to the development of efficient techniques of wireless communications involving very huge number of devices.

Uncoordinated access mechanisms, e.g., random access technique, may lead to resource wastage because of packet collisions are discarded, if the number of concurrent attempts exceeds the multiuser capabilities of the radio access. Random access mechanism is needed to enable successful wireless access. Driven by the M2M applications, we are witnessing the revival of random access mechanism in cellular and satellite networks. The need of efficient random access mechanism is even more pronounced, as they typically assume reception of data from massive number of uncoordinated devices.

Among the random access techniques, coded random access is well suited for M2M communications. We believe that coded random access opens new grounds for designing communications systems that should embrace a high dense networks involving massive number of connected devices.

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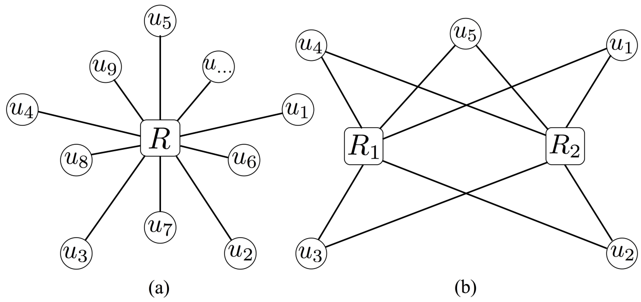

Fig. 1. High dense network structures: (a) General high dense multiway single relaying (hd-MWSR) networks with one relay, (b) high dense multiway multirelaying (hd-MWMR) networks with two relays.

Fig. 1(a) shows a general network structure of high dense multiway single relay (hd-MWSR) networks, where all users or devices[1] are expecting to exchange their informations via a single relay. The potential applications of such networks are satellite networks, cellular networks and even ad-hoc networks for devastated areas due to natural disasters.

This paper looks at the challenges of supporting massive amounts of devices for networks by investigating the advantage of the second relay, called high dense multiway multirelay (hd-MWMR) networks. We are expecting that this new concept can open up new possibilities for enhancing capacity of wireless dense networks. Our considered hd-MWMR is shown in Fig. 1(b) with two relays, R_1 and R_2.

The contributions of this paper are as follows: (i) we derive the theoretical network capacity bound based on the extrinsic information transfer (EXIT) analysis, (ii) we propose a simple decoding technique for hd-MWMR involving two relays.

The rest of the paper is organized as follows. Section II presents system model of the hd-MWMR networks. The decoding strategy and theoretical bound is shown in Sections III and IV, respectively. Section V evaluates the decoding schemes in terms of throughput and packet-loss-rate (PLR). Finally, Section VI concludes this paper.

II. System Model

We assume that the number $(M+1)$ active users is known *a priori*. The channel time is divided into time slots of an equal length and all transmissions are slot-synchronized. We assume that the length of a packet is constant. Each packet is equipped with a pointer to indicate the other packets are transmited in which time-slot.

[1] The terminologies of "user" and "device" are used interchangeably except specified.

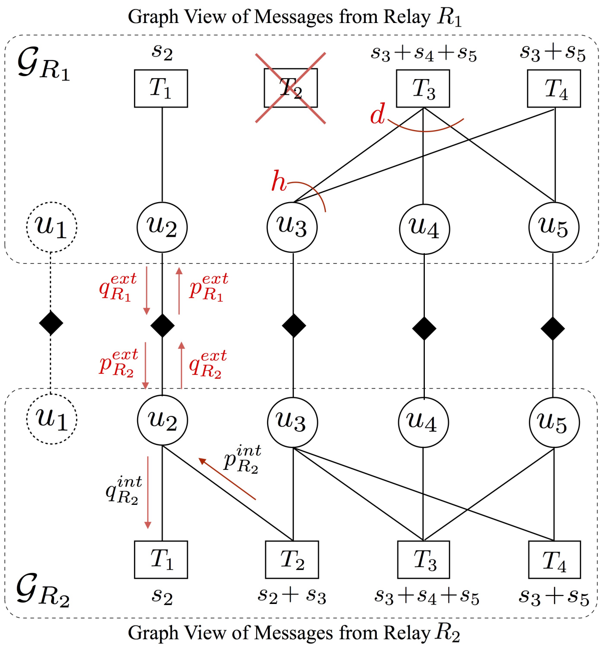

Fig. 2. Factor graphs $\mathcal{G}_{R_1}$ and $\mathcal{G}_{R_2}$ viewed by user u_1 after self-subtraction with its own message.

Future networks requires low latency and low computational complexity schemes. Thus, amplify-and-foward (AF) protocol is considered in this paper, because of its low complexity. Furthermore, AF is well suited for a large number of users since the decoding complexities are all forwarded to each user as the final destination.

Nevertheless, we need to employ successive interference cancellation (SIC) in the decoding process. The SIC can simply be performed by subtracting the resolved messages from the composite signals. Each user detects the messages from other M users after performing self-subtraction with its own message as shown in Fig. 2. The *circles* represent *user nodes* that have degree h, while the *squares* represent slot nodes that have degree d. Fig. 2 shows a factor graph representation of messages received by user u_1 after self-subtraction.

In this paper, we also do not assume multipacket reception (MPR) techniques as in [1], [2] to clearly observe the gain from the multirelays contributions. However, an extension to the case of MPR utilization is rather straightforward.

The channel gains between relays R_1, R_2 and all users are assumed to be perfectly known at both of the relay R_r and the users, e.g., by sending them in a header of every packet.

The hd-MWMR networks involves two types of encoding schemes, i.e., *physical encoding* and *network encoding*. We assume that physical encoding is perfectly performed by using simple but reliable coding schemes, e.g., memory-1 convolutional codes followed by a doped-accumulator, separated by an interleaver [2].

In this paper high signal-to-noise power ratio (SNR) is assumed so that all messages are correctly decodable physiscally. Therefore, the remaining problem is only the network encoding and decoding schemes involving two relays.

We asume half-duplex relaying scheme so that information is exchanged within two phases. First phase is a multiple access channel (MAC) phase, where all users transmit their messages to the relay randomly without coordination. The second phase is a broadcast (BC) phase, where two relays broadcast the received packets[2] to all $(M+1)$ users.

Since this AF protocol is considered, it is convenience to define *pair-of-time slot* (PTS) as [1], [2] comprising time slots of MAC and BC phases denoted as $\mathcal{T}_m$ and $\mathcal{T}_b$, respectively. The difference compared with PTS in [1] is that the additional relay in this paper requires new time slot $\mathcal{T}_b$. Finally, we define one PTS, the t-th PTS, as $T_t = (\mathcal{T}_m, \mathcal{T}_{b_1}, \mathcal{T}_{b_2})_t$ expressing that two parallel MAC phases are followed by two consecutive BC phases.[3]

In this paper, we assume that a *contention period* comprises N PTS for $(M+1)$ users randomly transmitting their messages in the MAC phase. The offered traffic seen by each user is then defined as

$$G = \frac{kM}{N}, \tag{1}$$

where k is a parameter depending on the network encoding scheme. When repetition codes are considered, $k = 1$. Each user randomly picks a code $c_h \backsim (n_h, k)$ from network code sets $\mathcal{C} = \{c_1, c_2, \ldots, c_{n_c}\}$ according to a probability distribution $\Lambda = \{\Lambda_2, \Lambda_3, \ldots, \Lambda_{n_c}\}$, $\sum_{h=2}^{n_c} \Lambda_h = 1$. To obtain a stable network, it should be kept that $h \geq 2$. A code c_h is a linear code with length n_h, dimension k, and rate $R_{n_h} = {}^{k}/_{n_h}$.

Given the distribution Λ, the average rate of the network from the viewpoint of each user is

$$\bar{R}_n = \frac{k}{\bar{n}} = \frac{k}{\sum_{h=2}^{n_c} \Lambda_h n_h}. \tag{2}$$

This paper is aiming to improve the offered traffic G in (1) by the help of the second multiway relay and analyzing suitable code distributions Λ for ideal channels, where both the graphs satisfy $\mathcal{G}_{R_1} \neq \mathcal{G}_{R_2}$.[4]

III. Decoding Strategy

In hd-MWMR networks, although the transmission is randomly uncoordinated, is still visible for analsyis as a network-on-the-graph. Similar to code-of-the-graph structure of low density parity check (LDPC) codes, extrinsic information transfer (EXIT) analysis is also applicable for the hvd-MWMR network structures.

Fig. 2 shows a graph $\mathcal{G} = (\mathcal{G}_{R_1}, \mathcal{G}_{R_1}, \mathcal{E}^e)$ of two connected bipartite graphs $\mathcal{G}_{R_1}$ and $\mathcal{G}_{R_2}$ via external links

[2] Terminologies of "message", "packet" and "information" are used interchangeably except specified.

[3] The additional time slot might be avoided if multiuser detection technique (MUD) is used.

[4] This condition may be achieved in fading channels or the channels for retransmission to the second relay.

$\mathcal{E}^e$. The top bipartite graph $\mathcal{G}_{R_1} = (\mathcal{U}, \mathcal{S}_1, \mathcal{E}_1^i)$ is a graph comprising user nodes $\mathcal{U} = \{u_1, u_2, \ldots, u_M\}$, slot nodes $\mathcal{S}_1 = \{T_1, T_3, \ldots, T_N\}$ and internal edge $\mathcal{E}_1^i$ for messages received via the relay R_1. The lower bipartite graph $\mathcal{G}_{R_2} = (\mathcal{U}, \mathcal{S}_2, \mathcal{E}_2^i)$ is the graph for the messages received via the relay R_2.

The degree distributions of user node at $\mathcal{G}_{R_1}$ and $\mathcal{G}_{R_2}$ shown in Fig. 2 are $\Lambda_{R_1}(x) = \frac{1}{2}x + \frac{1}{2}x^2, \Lambda_{R_2}(x) = \frac{1}{4}x + \frac{1}{2}x^2 + \frac{1}{4}x^3$, respectively. However, in practice the distributions of Λ_{R_1} and Λ_{R_2} are depending on the fading channel conditions. Some slot nodes may be erased if the received power is below a threshold P_{th} as indicated by the loss of T_2 in Fig. 2 so that $\mathcal{S}_1 \neq \mathcal{S}_2$. Matrix for networks shown in Fig. 2 is

$$\mathcal{G}_{R_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \boxed{0 \;\; 0 \;\; 0 \;\; 0} \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \mathcal{G}_{R_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad (3)$$

for the relay R_1 and R_2. The rows represent time slots $\mathcal{S} = \{T_1, T_2, \ldots, T_5\}$, while the columns represent users $\mathcal{U} = \{u_1, u_2, \ldots, u_5\}$. The second row of $\mathcal{G}_{R_1}$ is erased because of the lost of T_2 due to a deep fading effect.

In this paper, we consider repetition code with some distributions

$$\begin{aligned} \Lambda_a(x) &= 0.5x^2 + 0.28x^3 + 0.22x^8, & (4) \\ \Lambda_{b_1}(x) &= x^2, & (5) \\ \Lambda_{b_2}(x) &= x^3, & (6) \\ \Lambda_{b_3}(x) &= x^4. & (7) \end{aligned}$$

Distribution Λ_a has a network rate $R_n^a = 1/\Lambda_a'(1) = 0.278$ packets/slot, which is shown in [3] to have the highest asymptotic threshold $G^* = 0.932$ even though the practical $G = 0.83$ at packet loss rate (PLR) of 10^{-1}. Distributions $\Lambda_{b_1}, \Lambda_{b_2}$, and Λ_{b_3}, are regular repetition codes to be evaluated that have network rates of $R_n^{b_1} = 1/2, R_n^{b_2} = 1/3$, and $R_n^{b_3} = 1/4$ packets/slot, respectively.

The evolution of probability of an edge having unresolved or erasure packet to successful decoding is analysed using EXIT chart. Let's define p as the probability of an edge coming out from a slot node and q as the probability of an edge coming out from a user node. Since two relays are involved, we define *internal* and *external* erasure probabilities, p^{int}, q^{int} and p^{ext}, q^{ext} respectively, as illustrated in Figs. 2 and 3.

Slot nodes are independent of the relay number, therefore, the *internal* erasure probability of an edge emanating from a slot node is [3]

$$p^{int} = 1 - \exp\left\{-q^{int}\frac{G}{\bar{R}_n}\right\}. \quad (8)$$

The exponential function in the p^{int} is coming from the fact of random transmission without coordination that results in the distribution of degree d of the slot nodes is following Poisson distribution by assuming that the $(M+1)$ users are very large as well as the N pair-of-time slots, but keeping throughput G constant.

We consider two types of practical coding schemes, i.e., the repetition codes and maximum distance separable (MDS) codes. The MDS codes are considered if the network code rates $\bar{R}_n > \frac{1}{2}$ are the target of the design.

A. Repetition Codes

Repetition code is the most simple coding scheme as simple as its analysis. When the repetition code is considered, the *internal* erasure probabilities is

$$q^{int} = \sum_{h=2}^{n_c} \lambda_h (p^{int})^{h-1} p^{ext}, \quad (9)$$

where λ_h is the degree of user nodes from *edge perspective* [1], [2]. For external q^{ext}, we can observe from Fig. 2 that an additional edge $\mathcal{E}^e$ exists due to the connection of two relays. Therefore, the external erasure probability is modified as

$$q^{ext} = \sum_{h=2}^{n_c} \lambda_h (p^{int})^h. \quad (10)$$

B. MDS Codes

For a (n_h, k) MDS code, the *internal* erasure probability is expressed as

$$q^{int} = \sum_{\ell=0}^{k-1} \binom{n_h - 1}{\ell} (1 - p^{int})^{\ell} p^{n_h - \ell - 1}, \quad (11)$$

and for *external* erasure probability as

$$q^{ext} = \sum_{\ell=0}^{k-1} \binom{n_h}{\ell} (1 - p^{int})^{\ell} p^{n_h - \ell - 1}. \quad (12)$$

C. Decoding Algorithm

Given the erasure probabilities as described above, this paper proposes a simple joint decoding technique for a big graph involving two multiway relays. To simplify the algorithm, we consider an example of decoding steps shown in Fig. 3 for user u_1.

- ***Step 1:*** Consider one graph, e.g., $\mathcal{G}_{R_1}$ and find a user u_i connected to a time slot with degree $d = 1$. Subtract the received signals of all time slots T_t connected to user u_i with the message of user u_i as shown in Fig. 3(a).
- ***Step 2:*** If no more time slot is found with degree $d = 1$, move to the another graph, e.g., $\mathcal{G}_{R_2}$ and perform the same algorithm as ***Step 1*** as illustrated by Fig. 3(b).
- ***Step 3:*** Repeat ***Step 1*** and ***Step 2*** until all users are resolved as described by Figs. 3(c) and (d).

Because two relays are involved, the decoding speed is increased. Once a decoding process stucks in one graph, the decoding is then moved to another graph, which may not be stuck because of different graph.

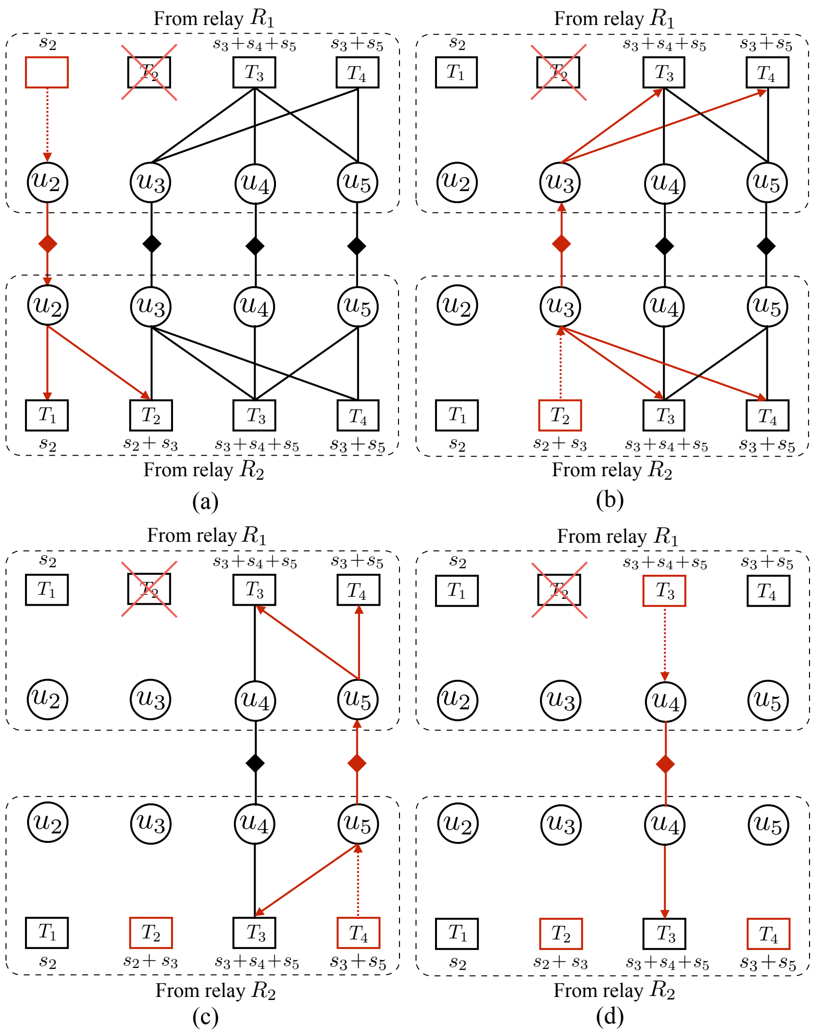

Fig. 3. Decoding steps performed by the user u_1 for networks in Fig. 2 that avoid decoding stuck in one graph.

IV. Theoretical Bound and EXIT Analysis

An edge q^{ext} connecting two graphs causes a rate degration from the original rate $R_o = {}^{k}/_{\bar{n}}$ to

$$R_{new} = \frac{k}{\bar{n}+1} = \frac{kR_o}{k+R_o}, \tag{13}$$

as clearly observed in Fig. 2 that the degree h of each user increases to $(h+1)$ due to the connection to the second relay. With area theorem of EXIT analysis [3] the bound of random access networks with a single relay is obtained from

$$\begin{aligned} A_u + A_s &\leq 1 & (14)\\ R_o + \left\{1 + \left(\frac{R_o}{G}\right) e^{-\frac{G}{R_o}} - \frac{R_o}{G}\right\} &\leq 1, & (15)\end{aligned}$$

where the first term A_u represents the area under the curve of user nodes, while the second terms A_s represents the area under the curve of slot nodes.[5]

[5] We intensively observed that the area under the curve does not *directly* express the rate of networks, especially when the network rate is beyond 1, just as similar case as in LDPC codes. However, we found that the relationship in (15) is valid as long as no overlapping between the curve.

The number of relay does not affect directly the slot nodes erasure probability, thus, the second relay only only changes the erasure probability of the user nodes with new rate R_{new} in (13). By subtitution of R_{new} to the theoretical bound of a multiway single relay networks [2], the new theoretical bound for MWMR networks involving two relays is

$$\frac{kR_o}{k+R_o} + \left(\frac{R_o}{G}\right) e^{-\frac{G}{R_o}} - \frac{R_o}{G} \leq 0. \tag{16}$$

It is clearly observed that the theoretical bound shown in Fig. 4 is higher than the original bound considering only one relay. This new bound can minimize the unachievable region, which is usefull for future dense networks. The drops of lines AB, CD, and the rest shown in the figure is due to the non-smooth change of the network coding rates $\bar{R}_n = {}^{k}/_{(k+1)}$ with $k = \{1, 2, \ldots, 500\}$.

Several tests for some code distributions are performed to verify the accuracy of the new bound of (16). We use $(k, 1)$ repetition codes of $\Lambda_c = x^k$ and $(k+1, k)$ MDS codes Λ_d with $k = \{1, 2, \ldots, 10\}$. It is shown by Fig. 4 that the new bound is confirmed accurate enough

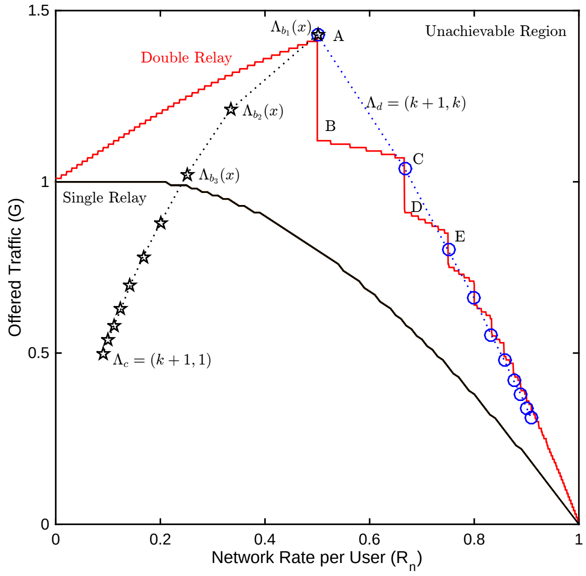

Fig. 4. The new theoretical bound gained from the second multiway relay.

both by repetition codes and MDS codes. The repetition codes achieve maximum G only when $R_n = {}^1\!/\!_2$, while the $(k+1, k)$ MDS codes achieve the bound almost at every point. However, it is important to note here that the new bound in (16) is derived by assuming graphs $\mathcal{G}_{R_1}$ and $\mathcal{G}_{R_2}$ are identical but completely independent and unequal in connection structure.

This section also evaluates the erasure probability evolution using EXIT analysis. Instead of using the mutual information (MI) as usually used in EXIT chart analysis [4], in this paper, we use the equivalent parameter defined as

$$I^{A}_{\mathrm{Slot},R_1} = 1 - q^{int}_{R_1}, \tag{17}$$
$$I^{E}_{\mathrm{Slot},R_1} = 1 - p^{int}_{R_1}, \tag{18}$$

where terms "A" and "E" in I^{A}_{Slot,R_1} and I^{E}_{Slot,R_1} are for *a priori* and *extrinsic* information, respectively. The similar definition is applied for the user nodes as

$$I^{A}_{\mathrm{User},R_1} = 1 - p^{int}_{R_1}, \tag{19}$$
$$I^{E}_{\mathrm{User},R_1} = 1 - q^{int}_{R_1}. \tag{20}$$

The decoding trajectory is used for two purposes: (i) to confirm the accuracy of EXIT analysis, and (ii) to obtain the asymptotic maximum offered traffic G^*. EXIT curves and the corresponding decoding trajectories for distributions Λ_a and Λ_{b_1} are shown in Figs. 5(a) and (b), respectively. The EXIT curves and trajectory are evaluated at graph $\mathcal{G}_{R_1}$ with a projection of $\mathcal{G}_{R_2}$ to $\mathcal{G}_{R_1}$ by performing 100 iterations inside $\mathcal{G}_{R_2}$. The EXIT projection analysis originally used in serial concatenated convolutional codes [5] is also applicable in this analysis. The projected user nodes EXIT curves are shown in dash-dotted lines below the slot nodes EXIT curves.

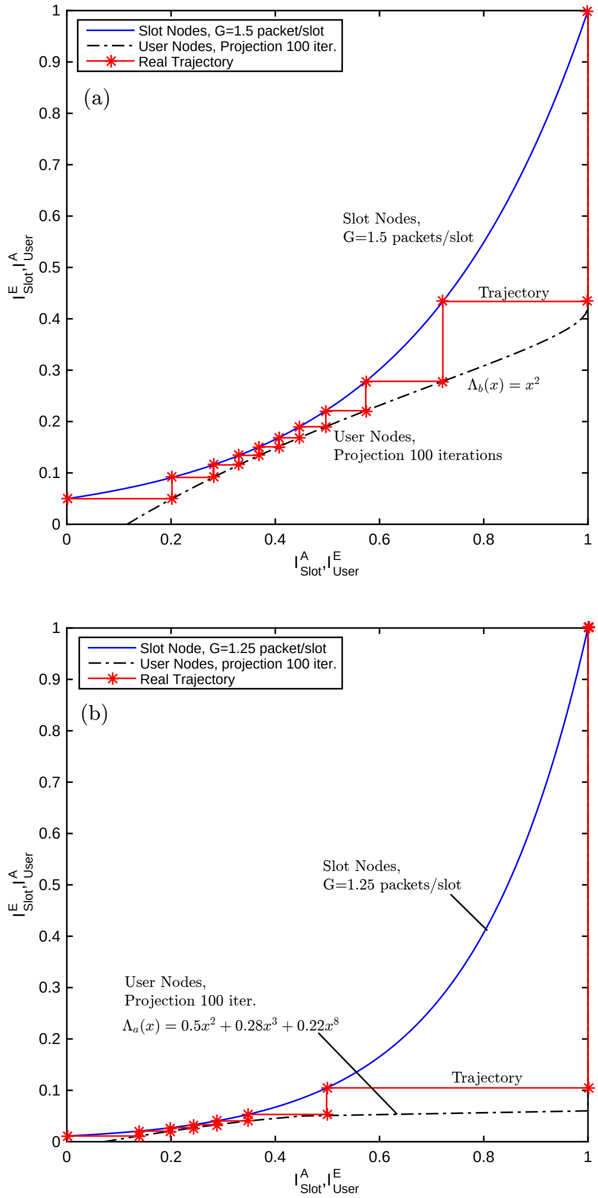

Fig. 5. EXIT projection and decoding trajectories: (a) $\mathcal{G}_{R_1}$ for $\Lambda_{b_1}(x) = x^2$ at $G = 1.5$ packet/slot, and (b) $\mathcal{G}_{R_1}$ for $\Lambda_a(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$ at $G = 1.25$ packet/slot.

Figs. 5(a) and (b) show excellent agreement between trajectory and EXIT curves of both the slot nodes and user nodes. The gap between the user nodes and slot nodes curves indicate the efficiency of the used distribution. Smaller gap is preferable to provide higher thoughput G as shown in Fig. 5(a) with Λ_{b_1}.

Fig. 5(b) shows a bigger gap with Λ_a resulting in smaller $G = 1.25$ packets/slot. The trajectories also confirm that the tunnel between the EXIT curves kept open at $G = 1.25$ packet/slot and $G = 1.5$ packet/slot for Λ_a and Λ_{b_1}, respectively, as predicted by the theoretical bound in Fig. 4 that the maximum throughput is at about $G = 1.5$ packet/slot.

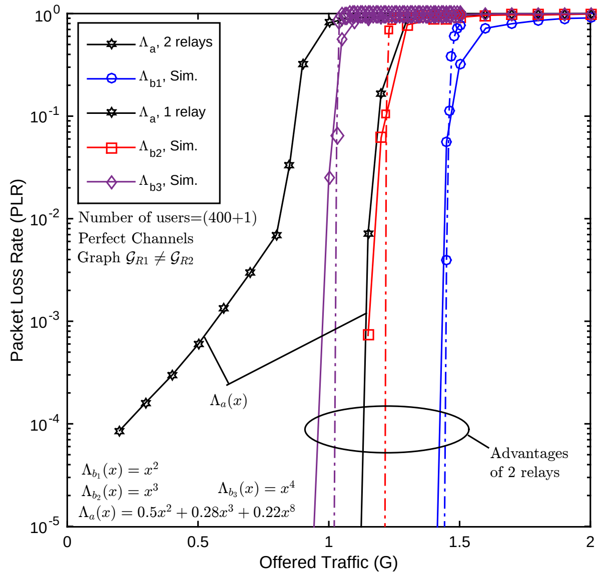

Fig. 6. PLR Performances under ideal channels assumption with $\mathcal{G}_{R_1} \neq \mathcal{G}_{R_2}$.

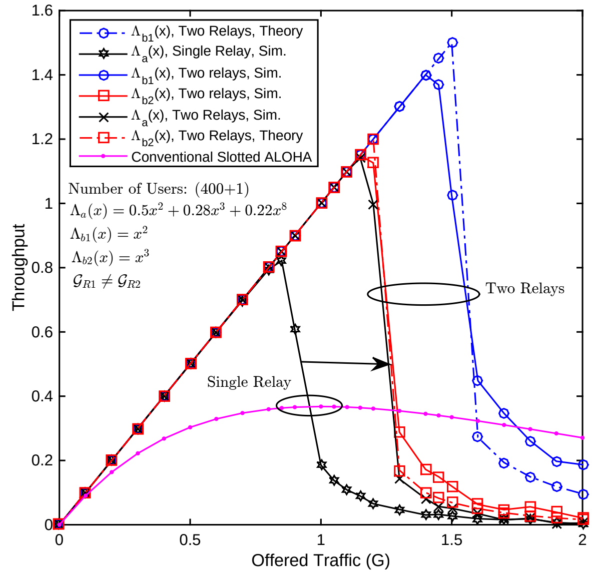

Fig. 7. Throughput performances of multiway multirelays compared with multiway single relay coded random access and slotted ALOHA.

V. Performance Evaluations

The performances of the decoding scheme is evaluated via computer simulations for $(400 + 1)$ users under ideal channels, where the graph $\mathcal{G}_{R_1}$ and $\mathcal{G}_{R_2}$ are completely different, because no help from the second relay if $\mathcal{G}_{R_1} = \mathcal{G}_{R_2}$. Fig. 6 shows the packet-loss-rate (PLR) performances of the distributions $\Lambda_a(x), \Lambda_{b_1}(x), \Lambda_{b_2}(x)$ and $\Lambda_{b_3}(x)$. The dashed-dotted lines are obtained via the trajectory of EXIT analysis, while the solid lines are from massive simulations. All results of computer simulations confirm the prediction by EXIT analyses and the bound derivation in (16).

Fig. 7 shows the throughput performances to investigate in detail the improvement of throughput with two multiway relays, T_{MWMR}. Based on the PLR obtained in Fig. 6, the throughput of the network is calculated as

$$T_{MWMR} = G * (1 - PLR) \tag{21}$$

for $\Lambda_a(x), \Lambda_{b_1}(x)$ and $\Lambda_{b_2}(x)$.

Throughputs of single relay and multiway relay with slotted ALOHA technique are also shown in Fig. 7 as references. The throughput of single relay $G = 0.83$ packets/slot can be improved to $G = 1.18$ packet/slot with two relays using the same $\Lambda_a(x)$. When $\Lambda_{b_1}(x) = x^2$ is used, the throughput is about $G = 1.42$ packet/slot, which is closer to the theorical bound.

The throughput of multiway relay assuming slotted ALOHA technique, T_{ALOHA}, is obtained from

$$T_{ALOHA} = G * e^{-G} \tag{22}$$

with the highest throughput of only around $G = 0.38$ packets/slot. Figs. 6 and 7 clearly show the success of simple decoding technique to gain the benefit of multirelays. Further investigation on fading channels is required by considering, for example, the stopping-set criterion to maintain error-floor of networks under Rayleigh fading channels.

VI. Conclusions

We considered a decoding technique for high dense multiway multirelay networks (hd-MWMR). The theoretical bound for hd-MWMR systems exploiting two multiway relays is derived. We proposed simple encoding and decoding schemes based on iterative interference cancellation over a sparse graph involving two multiway relays. The results via EXIT analysis and computer simulations confirmed that the second relay helps both on (i) the improvement of throughput performances, and (ii) the increase of the decoding speeds, which is highly required for future wireless networks.

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