Title	Caterpillar Graphにおける独立点集合遷移問題についての研究
Author(s)	山田，武
Citation	
Issue Date	2016-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/13616
Rights	
Description	Supervisor:上原　隆平，情報科学研究科，修士

Reconfiguration Problem for Independent Set on a Caterpillar Graph

Takeshi Yamada (1310710)
School of Information Science,
Japan Advanced Institute of Science and Technology

February 6, 2016

Keywords: reconfiguration problem, independent set, Sliding Token Problem, caterpillar graph

Recently, reconfiguration problems have attracted the attention in the field of theoretical computer science. The problem arises when we wish to find a step-by-step transformation between two feasible solutions of a problem such that intermediate results are also feasible and each step conforms to a fixed reconfiguration rule, that is, an adjacency relation defined on feasible solutions of the original problem. This kind of reconfiguration problem has been studied extensively for several well-known problems, including independent set, satisfiability, set cover, clique, matching, and so on. From the viewpoint of theoretical computer science, one of the most important problems is the 3SAT problem. Beside the 3SAT problem, the independent set problem is another representative and important problem in theoretical computer science.

Reconfiguration problems for independent set have been studied under different reconfiguration rules, as follows.

Token Sliding (TS rule): This rule corresponds to Sliding Token,we can slide a single token only along an edge of a graph.

Token Jumping (TJ rule): A single token can jump to any vertex (including a non-adjacent one) if it results in an independent set.

Token Addition and Removal (TAR rule): We can either add or remove a single token at a time if it results in an independent set of cardinality at least a given threshold. Therefore, under the TAR rule, independent sets in the sequence may not have the same cardinality.

The Sliding Token Problem was introduced by Hearn and Demaine as a one-player game, which can be seen as a reconfiguration problem for independent set. Recall that an independent set of a graph G is a vertex subset of G in which no two vertices are adjacent. Suppose that we are given two independent sets II_b and II_r of a graph such that $|\mathrm{II}_b| = |\mathrm{II}_r|$, and imagine that a token is placed on each vertex in II_b. Then, the Sliding Token Problem is to determine whether there exists a sequence $<\mathrm{II}_1, \mathrm{II}_2, \cdots, \mathrm{II}_\ell>$ of independent sets of G such that

(a) $\mathrm{II}_1 = \mathrm{II}_b$, $\mathrm{II}_\ell = \mathrm{II}_r$, and $|\mathrm{II}_i| = |\mathrm{II}_b| = |\mathrm{II}_r|$ for all i, $1 \leqq i \leqq \ell$;and

(b) for each i, $2 \leqq i \leqq \ell$, there is an edge $\{u, v\}$ in G such that $\mathrm{II}_{i-1} \setminus \mathrm{II}_i = \{u\}$ and $\mathrm{II}_i \setminus \mathrm{II}_{i-1} = \{v\}$, that is, II_i can be obtained from II_{i-1} by sliding exactly one token on a vertex $u \in \mathrm{II}_{i-1}$ to its adjacent vertex v along $\{u, v\} \in E$.

The reconfiguration problems tend to be PSPACE - complete in general, and some polynomial time algorithms are shown in restricted cases. We gave linear time algorithms to solve the Sliding Token

Problems for proper interval graph and trivially perfect graph before.

We think the Sliding Token Problem for caterpillar graph in this paper. Let $G=(S\cup L,E)$ be a caterpillar graph with spine S which induces the path $(s_1,\cdots,s_m)$, and leaf set L. We assume that $m\geqq 2$, $\deg(s_1)\geqq 2$, and $\deg(s_m)\geqq 2$.

First, we show that we can assume that each spine vertex has at most one leaf without loss of generality. For any given caterpillar $G=(S\cup L,E)$ and two independent sets Π_b and Π_r on G, there is a linear time reduction from them to another caterpillar $G^*=(S^*\cup L^*,E^*)$ and two independent sets $\Pi_b{}^*$ and $\Pi_r{}^*$ such that (1) G, Π_b, and Π_r are a yes-instance of the Sliding Token Problem if and only if G^*, $\Pi_b{}^*$, and $\Pi_r{}^*$ are a yes-instance of the Sliding Token Problem, (2) the maximum degree of G^* is at most 3, and (3) $\deg(s_1)=\deg(s_m)=2$, where $m=|S^*|$. In other words, the Sliding Token Problem for caterpillar graph is sufficient to consider only caterpillar graphs of maximum degree 3.

Then, we introduce a key notion of the Sliding Token Problem for caterpillar graph that is named locked path. Let G and Π be a caterpillar graph and an independent set of G, respectively. A path $p=(p_1,p_2,\ldots,p_k)$ on G is locked by Π if and only if (a) k is odd and greater than 2, (b) $\Pi\cap P=\{p_1,p_3,p_5,\cdots,p_k\}$, (c) $\deg(p_1)=\deg(p_k)=1$, and $\deg(p_3)=\deg(p_5)=\cdots=\deg(p_{k-2})=2$. Using this notion, we obtain the condition for the immovable independent set on a caterpillar graph. Then we cannot slide any token in Π on G at all if and only if there exist a set of locked paths $P_1\cdots P_h$ for some h such that Π is a union of them. Intuitively, for any caterpillar graph G and its independent set Π, if Π contains a locked path P, we cannot slide any token through the vertices in P. Therefore, P splits G into two subgraphs, and we obtain two completely separated subproblems. Therefore, we obtain the following reduction. For any given caterpillar graph $G=(S\cup L,E)$ and two independent sets Π_b and Π_r on G, there is a linear time reduction from them to another caterpillar graph $G^*=(S^*\cup L^*,E^*)$ and two independent sets $\Pi_b{}^*$ and $\Pi_r{}^*$ such that (1) G, Π_b, and Π_r are a yes-instance of the Sliding Token Problem if and only if G^*, $\Pi_b{}^*$, and $\Pi_r{}^*$ are a yes-instance of the Sliding Token Problem, (2) Both of $\Pi_b{}^*$ and $\Pi_r{}^*$ contain no locked path.

Finally, the Sliding Token Problem for a caterpillar graph G and two independent sets Π_b and Π_r of G can be solved in $O(n)$ time and $O(n)$ space.

Further, we investigate for finding a shortest sequence of the Sliding Token Problem, which is called the shortest Sliding Token Problem. That is, our problem is formalized as follows:

Input: A graph $G=\{V, E\}$ and two independent sets Π_b, Π_r with $|\Pi_b|=|\Pi_r|$.

Output: A shortest reconfiguration sequence $\Pi_b=\Pi_1, \Pi_2,\cdots,\Pi_\ell=\Pi_r$, such that Π_i can be obtained from Π_{i-1} by sliding exactly one token on a vertex $u\in\Pi_{i-1}$ to its adjacent vertex v along $\{u, v\}\in E$ for each i, $2\leqq i\leqq \ell$.

When the Sliding Token Problem for a caterpillar graph is a yes-instance, a shortest reconfiguration sequence between them can be output in $O(n^2)$ time and $O(n)$ space.