

Title	Memory Constrained Algorithms for Geometric Problems
Author(s)	小長谷, 松雄
Citation	
Issue Date	2016-06
Type	Thesis or Dissertation
Text version	ETD
URL	<a href="http://hdl.handle.net/10119/13719">http://hdl.handle.net/10119/13719</a>
Rights	
Description	Supervisor:上原 隆平, 情報科学研究科, 博士

# Abstract

Due to recent advancement of technologies of CPU and memory grow in recent years, the possibility of lack of memory space decreases while executing a program. However, the constraint of using limited memory spaces can be required in process on small devices such as digital cameras and cellular phones, because of their volume restriction.

In our study on memory constrained algorithms, we define our computation model as follows: The memory space in which every stored item is allowed to be read, overwritten is called as the work-space. We use a standard random access machine, so that invoking an item in a memory space takes in constant time. We assume that input data is stored in a read-only array. Thus, no reordering and overwriting to the array are possible. In this paper, we proposed memory constrained algorithms for geometric problems as follows.

First, we consider computing a farthest-point Voronoi diagram using work-space of size  $O(\log n)$  bits. Given set  $S$  of  $n$  points in a plane, a farthest-point Voronoi diagram is the partition of the plane into regions, such that each region is the set of points of the region that has a common point as the farthest point in  $S$  from them. Using only work-space of size  $O(\log n)$  bits is the most strict constraint in our computation model, since at least  $\log n$  bits are necessary for the work-space to distinguish  $n$  data from each other. We call algorithms which is designed under such memory constraint as constant work-space algorithms. The algorithm for the Voronoi diagram can have quite simple implementation and also runs in reasonable running time. Moreover, we also consider the problem of finding the smallest enclosing circle for given points. which is applications of farthest-point Voronoi diagram which is the problem is one of fundamental geometric problems. We present an algorithm for finding the smallest enclosing circle using constant work-space.

Next, we turn to the Depth-First Search (DFS), which is a basic algorithm in many areas, using  $O(n)$  bits work-space. The  $O(n)$  bits space is sublinear in the input size, since  $O(n \log n)$  bits are required to represent  $n$  inputs. Typically, the sublinear work-space for an input size was the model when memory constrained algorithms were invented. In this paper, we provide algorithms performing DFS on a directed or undirected graph with  $n$  vertices and  $m$  edges using only  $O(n)$  bits. If the work-space of size  $O(n \log n)$  bits is available, DFS can be implemented easily by using stack data structure. The advantage of the stack is that can find the next vertex to be visited in constant time. To find such a vertex without  $O(n \log n)$  bits stack, all vertices are maintained by four colors instead of  $\log n$  bits.

Third, we provide an adjustable work-space algorithm for the segment intersection problems. Roughly speaking, the model of an adjustable work-space algorithm can perform with work-space of an arbitrary size between  $O(1)$  and  $O(n)$  words. Given  $n$  line segments in a plane, we invent adjustable work-space algorithms for detecting a segment intersection and for reporting all pairs of intersecting line segments. Specially, algorithm of detecting a segment intersection is practical, which means that no complicated data structure is needed in the algorithm. We also obtain practical algorithms of reporting segment intersecting pairs for input of  $c$  slopes line segments in which the input has at most  $c$  different slopes. In general, however, the number of line slopes can be  $n$ . In this case, we have to use a sophisticated data structure.

Finally, we give polynomial-time algorithms for subgraph isomorphism problems for small graph classes of perfect graphs. This work comes from on the way that we try to design memory constrained algorithms for geometric graphs. Although this work is out of the framework of memory constrained algorithms, we include into this paper as a future topics.

**Key words:** Computational geometry, Algorithms, Memory constrained algorithms, Space-Time tradeoffs