Power Allocation in an Asymmetric Wireless Sensor Network

Weiwei Jiang, Xin He and Tadashi Matsumoto

***Abstract*—This letter investigates the power allocation problem for an asymmetric wireless senor network, where multiple sensors observe a common binary source and transmit their corrupted observations to a data fusion node. We propose a power allocation scheme by maximizing the weighted channel capacity subject to the sum power constraint and show that this problem is convex. The simulation results verify that the proposed power allocation scheme outperforms the uniform power allocation method. Furthermore, a scheduling scheme for binary data gathering is proposed by determining the sensors that dominate the bit error rate performance.**

***Index Terms*—Power Allocation, Binary Sensor Network, Rate-distortion**

I. Introduction

A TYPICAL wireless sensor network (WSN) is a network composed of a group of sensor nodes to monitor physical phenomena. The WSN is recognized as a promising technique to build information and communication systems where many sensing devices are involved such as in the Internet of things. Usually, in sensor networks, each sensor node is equipped with a battery, and the power is usually scarce owing to the limit battery size. Hence, the energy efficiency in sensor networks is an extremely important issue for constructing WSNs.

In [1], [2], an optimization framework for joint source coding, routing and resource allocation was presented in sensor networks. The distortion and power are weighted by two vectors in the optimization problem to achieve the optimal balancing between them. The optimization problem can then be solved efficiently in the dual domain. Optimal power allocation for Gaussian sensor network with distortion constraints was considered in [3], where both time division multiple access (TDMA) and non-orthogonal multiple access (NOMA) schemes are assumed in transmission phases of sensors. In [4], the power allocation scheme was considered by minimizing the outage probability with the aim of its application to wireless camera networks.

In this work, we focus on a specific scenario of WSN, where multiple sensor nodes observe a common source (object) and produce erroneous observations. They first convert their erroneous observations into binary sequence, which are then encoded in a distributed manner and transmitted to a data fusion node over independent additive white Gaussian noise (AWGN) channels. Such WSN is referred to as binary data gathering WSN, for which a series of encoding and decoding schemes were proposed [5]–[8]. However, in those known schemes, only uniform power allocation is assumed. In [9]–[12], various power allocation schemes are investigated for asymmetric Gaussian source WSNs though, such studies on binary source WSNs have not been well addressed yet. Therefore, we address the power allocation for the asymmetric binary data gathering WSN, where each sensor has different observation accuracy.

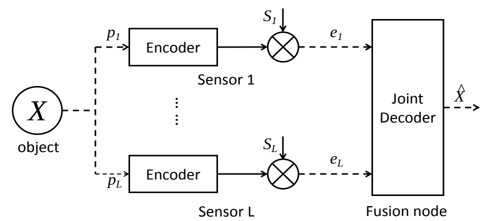

Fig. 1. System model: an asymmetric wireless sensor network.

Major Contributions. We propose a power allocation scheme for the network shown in Fig. 1 by maximizing the *mutual information* between the source information and its estimation subject to the sum power constraint. Our strategy is different from that proposed in [10], where rate-distortion function is used in the optimization. However, the mutual information is yet unknown according to the state-of-the-art studies. To solve this intractable problem in practice, we adopt mathematical tools which are used for solving problems in information theory, and reformulate the problem in the framework of convex optimization by maximizing the summation of weighted channel capacity of each link. Furthermore, a scheduling scheme is proposed by analyzing the sensor subset that dominates the bit error rate (BER) performance.

The rest of this letter is organized as follows. The system model is described in Section II. Section III discusses the proposed power allocation scheme. The numerical results are shown in Section IV. We conclude this letter in Section V.

II. System Model

Main notations are summarized in Table I. We consider an asymmetric wireless sensor network as showed in Fig. 1. For each sensor $i \in \{1, 2, ..., L\}$, the observation value X_i can be considered as a corrupted version of the true value X with a bit error rate (BER) p_i. Observation X_i is encoded and transmitted via an orthogonal AWGN channel to the data fusion node with a transmission power $S_i = \alpha_i S_T$, with S_T and α_i being

X. He is with the School of Mathematics and Computer Science, Anhui Normal University, China (e-mail: xin.he@ahnu.edu.cn).

W. Jiang was with the School of Information Science, JAIST, Japan (e-mail: weiweijiangcn@gmail.com).

T. Matsumoto is with the School of Information Science, JAIST, Japan.

TABLE I
MAIN NOTATIONS.

Notation	Meaning	Notation	Meaning
X, X_i, $\hat{\cdot}$	source, i-th observation, estimates	p_i, $P1 \sim P5$	observation error probability, vector of p_i
S_i, S_T	transmission power of i-th sensor, total power	α_i	power ratio of i-th sensor (optimizing variable)
p_e, $p_e^{(i)}$	system BER floor and that achieved by i sensors	e_i	BER of the link between i-th sensor and fusion node
F_k	all subsets of choosing k sensors from L sensors	E, E^c	subset of sensors containing errors and its complementary set
$\mathbf{w}$, w_i	vector of weights and its i-th element	$\mathcal{W}_i$	subset of first i elements of $\mathbf{w}$
$\widetilde{\mathcal{W}_i}$	unequal-weight partitions of $\mathcal{W}_i$	$\overline{\mathcal{W}_i}$	equal-weight partitions of $\mathcal{W}_i$
δ	dominating sensor number	ϵ	pre-set tolerance in BER

the total transmission power of the network and the power allocation ratio for each sensor, respectively. Without loss of generality, we assume the noise power and the geometric gain for each channel are normalized to unity.

The data fusion node performs joint decoding of the signals received from the sensors, which is considered as a binary CEO[1]. For a symmetric sensor network or a majority vote CEO, the theoretical bit error probability (BEP) p_e representing the error floor of system BER is given by [13]

$$p_e = \begin{cases} \sum\limits_{k>\frac{L}{2}} \text{Pb}(K=k) & L \text{ is odd} \\ \sum\limits_{k>\frac{L}{2}} \text{Pb}(K=k) + \frac{1}{2}\text{Pb}(K=\frac{L}{2}) & L \text{ is even,} \end{cases} \tag{1}$$

and $\text{Pb}(\cdot)$ is the Poisson Binomial function

$$\text{Pb}(K=k) = \sum_{E\in F_k} \prod_{i\in E} (p_i) \prod_{j\in E^c} (1-p_j), \tag{2}$$

where F_k is the set of all subset of k sensors selected from all L sensors, E is the set of sensors that contain errors and $card(E) = k$, while E^c is the complementary set of E as $E^c = \{1, 2, ..., L\} \setminus E$. The reason is that the BEP is determined by comparing the number of 1's and 0's in a group of L Bernoulli trails, which is equivalent to Poisson binomial distribution.

III. Power Allocation Schemes

Similarly to the water-filling problem, larger powers should be allocated to those sensors with less observation errors. In this section, we propose a power allocation scheme and a scheduling scheme for the system to optimize the BER performance. The convex optimization model for the power allocation is constructed and solved from the Karush-Kuhn-Tucker (KKT) conditions. A lemma for BEP of asymmetric sensor networks is derived, from which the scheduling scheme is developed.

A. Convex Optimization

We aim to minimize the BER given the total transmission power S_T, which is equivalent to maximizing the mutual information between the source X and its estimate $\hat{X}$. The reason of using mutual information is that it indicates the mutual dependence between X and $\hat{X}$. Hence, the problem of power allocation is modeled as

$$\max_{\alpha_1,\alpha_2,\dots,\alpha_L} I(X;\hat{X}), \tag{3}$$

subject to

$$\begin{cases} 1 - H_b(e_i) \leqslant C(S_i), \quad e_i \geqslant 0, \quad S_i = \alpha_i S_T \\ \sum\limits_{i=1}^{L} \alpha_i = 1, \quad \alpha_i \geqslant 0 \end{cases}, \tag{4}$$

where the first constraint is obtained from the Shannon's source-channel separation theorem [14] with e_i representing the BER between X_i and its estimates $\hat{X}_i$. The function $H_b(\cdot)$ calculates the binary entropy, and $C(\cdot)$ is the capacity function for each channel, which are defined as

$$H_b(e_i) = -e_i \log(e_i) - (1-e_i)\log(1-e_i) \tag{5}$$

and

$$C(S_i) = \log_2(1+S_i) \tag{6}$$

respectively. Using the chain rule of mutual information and the Markov property of $X \rightarrow \hat{X}_i \rightarrow \hat{X}$, we have

$$\begin{aligned} I(X;\hat{X}) &= H(X) - H(X|\hat{X}) \geqslant H(X) - \sum_{i=1}^{L} H(X|\hat{X}_i) \\ &= H(X) - \sum_{i=1}^{L} H_b(p_i * e_i), \end{aligned} \tag{7}$$

with $*$ denoting binary convolution, i.e., $a * b = a(1-b) + b(1-a)$. Since $H(X)$ is a constant, the objective function can be approximated by

$$\max \sum_{i=1}^{L} -H_b(p_i * e_i), \tag{8}$$

while the negative of binary entropy function is convex that may not be maximized, we further use the first order term $c_1 + c_2 p$ of the Taylor series of function $H_b(p)$ at a small enough p, where c_1 and c_2 are constants, and the definition of operator $*$ to simplify the objective function, as

$$\begin{aligned} &\min\, c_2 \sum_{i=1}^{L} p_i * e_i + Lc_1 = \min\, c_2 \sum_{i=1}^{L} (1-2p_i)e_i + p_i + Lc_1 \\ &\leftrightarrow \min \sum_{i=1}^{L} (1-2p_i)e_i \text{ (since } p_i,\ c_1 \text{ and } c_2 \text{ are fixed).} \end{aligned} \tag{9}$$

Assuming capacity achieving coding, we further derive e_i as

$$e_i = \begin{cases} 0 & S_i \geqslant 1 \\ H_b^{-1}[1 - C(S_i)] & S_i < 1 \end{cases}. \tag{10}$$

[1]The CEO tries to reconstruct the source, which he cannot directly observe, as accurate as possible from multiple copies of corrupted observations transmitted by the deployed agents.

Since the capacity function $C(\cdot)$ and the inverse binary entropy function $H_b^{-1}(\cdot)$ are monotonically increasing functions, the object function can be finally reduced to as follows.

$$\min\sum_{i=1}^{L}(1-2p_i)e_i = \min\sum_{i=1}^{L}(1-2p_i)H_b^{-1}[1-C(S_i)] \\ \sim \max\sum_{i=1}^{L}(1-2p_i)C(S_i). \quad (11)$$

It is clear that the objective function is concave. We are now able to adopt KKT conditions to derive the power allocation values α_i. The KKT conditions are

$$\nabla f(\alpha_i) = -(1-2p_i)\frac{S_T}{1+\alpha_i S_T}\ln 2 + \lambda_1 = 0, \quad (12)$$

$$\mu_1\alpha_i = 0, \quad (13)$$

where λ_1 and μ_1 are Lagrange multipliers. After solving these equations, we get the optimal value

$$\alpha_i^* = \frac{1-2p_i}{\sum_i(1-2p_i)}(1+\sum_i\frac{1}{S_T}) - \frac{1}{S_T}. \quad (14)$$

Alternatively, we may use computational tools such as *cvx* to calculate numerical results.

B. BEP and Sensor Scheduling

The BEP for an asymmetric sensor network is dominated by a subset of the sensors, while for symmetric scenarios each link impacts the BEP performance. Hence, determining such subset and only allocating power to the dominating sensors can have a considerable improvement in the sense of BER given limited sum power.

In an asymmetric sensor network, weighted vote scheme greatly advantages over the majority vote that is adopted in symmetric scenarios. The decoding scheme for our system performs a so-called f_c function for each sensor before the CEO makes a final decision [13]. In principle, it may be considered as a weighted vote scheme, for which the hard decision of $\hat{X}$ follows

$$\hat{X} = \mathbf{w}^T\mathbf{X}, \quad (15)$$

$$\mathbf{X} = [\hat{X}_1\ \hat{X}_2\ ...\ \hat{X}_L]^{\mathrm{T}}, \quad (16)$$

where $(\cdot)^T$ denotes the matrix transpose. The weight vector $\mathbf{w}$ for all sensors is given by

$$\mathbf{w} = [w_1, w_2, ..., w_L]^T, \quad (17)$$

$$w_i = \frac{\mathscr{L}_i}{\sum_{i=1}^{L}\mathscr{L}_i}, \quad (18)$$

where $\mathscr{L}_i$ is the log-likelihood ratio (LLR) of the observation error with each sensor, given by

$$\mathscr{L}_i = \log\frac{1-p_i}{p_i}. \quad (19)$$

To derive the subset of dominating sensors, we first assume the observation BER vector $P = [p_1, \cdots, p_L]$ is sorted in ascending order. Consequently, the weight vector $\mathbf{w}$ is also sorted but in descending order. Furthermore, let $\mathcal{W}_i$ denote the subset of the first i elements of $\mathbf{w}$, and $(\cdot)^c$ the complementary set. Then we have the following lemma.

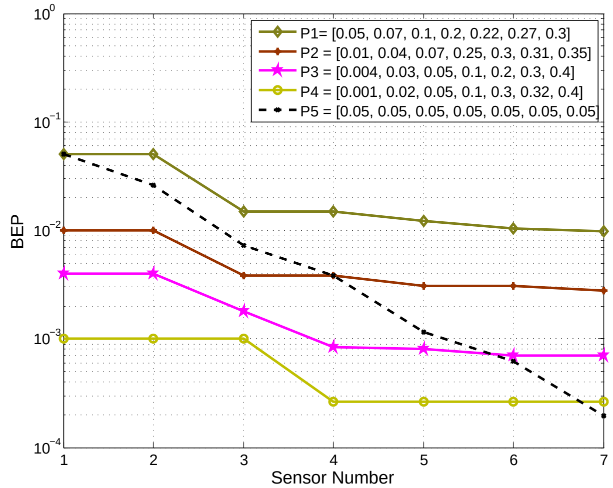

Fig. 2. BEP vs. number of sorted sensors.

Lemma 3.1: $\forall\ \widetilde{\mathcal{W}}_i, \overline{\mathcal{W}}_i \subseteq \mathcal{W}_i$, $\sum_{\widetilde{w}_i\in\widetilde{\mathcal{W}}_i}\widetilde{w}_i > \sum_{\widetilde{w}_i^c\in\widetilde{\mathcal{W}}_i^c}\widetilde{w}_i^c$, and $\sum_{\overline{w}_i\in\overline{\mathcal{W}}_i}\overline{w}_i = \sum_{\overline{w}_i^c\in\overline{\mathcal{W}}_i^c}\overline{w}_i^c$, the BEP $p_e^{(i)}$ for the sensor subset is given by

$$p_e^{(i)} = \sum_{\widetilde{\mathcal{W}}_i}\prod_{x\in\arg\widetilde{\mathcal{W}}_i}p_x\prod_{y\in\arg\widetilde{\mathcal{W}}_i^c}(1-p_y) \\ + \frac{1}{2}\sum_{\overline{\mathcal{W}}_i}\prod_{x\in\arg\overline{\mathcal{W}}_i}p_x\prod_{y\in\arg\overline{\mathcal{W}}_i^c}(1-p_y). \quad (20)$$

The BEP equation defines a *weighted Poisson Binomial* compared to (1), indicating that the decision is no longer made by the majority number but the majority sum weight, and the decision may be reversed in some cases when adding more sensors. Recall, however, that the weight vector $\mathbf{w}$ is in descending order, such reversal may occur less likely when increasing the number of sensors. This is because the sensors having large weights dominate the decision-making policy while the new comers are not strong enough to make any significant impact on the decision.

Hence, in asymmetric cases, there should exist a dominating sensor number δ, and after reaching δ, remaining sensors do not affect the BEP significantly. As a formal expression, the dominating sensor number δ is expressed as

$$\delta = \arg\min_i p_e^{(i)} - p_e^{(L)} \leqslant \epsilon, \quad (21)$$

where ϵ is predefined regarding to the Quality of Service (QoS) requirement of the sensor network. The sensor subset $\mathcal{D} = \{1,\ 2,\ ...\ \delta\}$ is then regarded as the set of dominating sensors.

Algorithm 1. We propose a scheduling algorithm in a straightforward way from the discussion above. We simply activate the dominating sensors with equal power allocation while leaving the rest sensors hibernating. When the observation error vector P changes, the dominating sensors can be identified adaptively and the sensor network can be rescheduled according to the new dominating sensors.

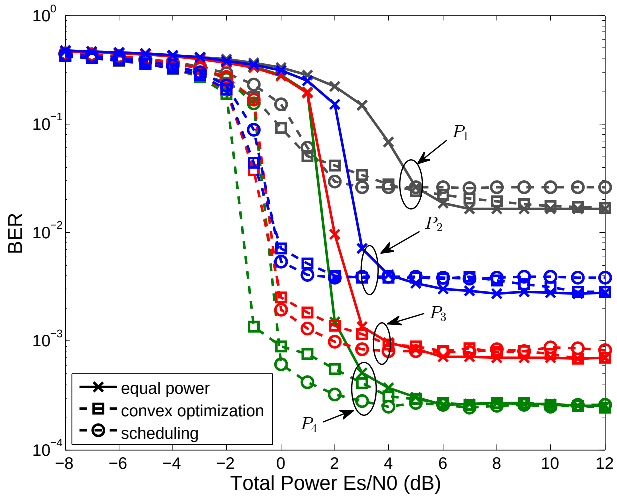

Fig. 3. BER performance with proposed power allocation and scheduling schemes. Convex optimization is the result of calculating (14), while the scheduling is the result of performing Algorithm 1.

IV. Numerical Results

To demonstrate our schemes, we exemplify five observation error vectors as labeled in the legend of Fig. 2, which depicts the relationship between the sensor subset and the corresponding BEP values. We predefine these five error vectors, $P1 \sim P5$, to identify dissimilar BEP values and the dominating sensor number. The numerical BEP results indicate that the dominating sensor number for $P1$, $P2$, $P3$ and $P4$ are 3, 3, 4, and 4 respectively. BEP improvement by adding more sensors, after reaching the dominating numbers, is negligible in scenarios $P1 \sim P4$. In contrast, in a symmetric scenario represented by $P5$, adding sensors reduces BEP. In the simulations of which results shown in Fig. 3, we adopt the joint CEO decoding scheme which is the same as authors proposed in [8]. The information length is 10000, and a memory-one Recursive Systematic Convolutional (RSC) encoder with generator polynomial $G = [03,\ 02]_8$ for each sensor, the doping ratio of Accumulator (ACC) is 1, and the modulation is Binary Phase Shift Keying (BPSK). The proposed technique can be also applied to higher order modulation. Compared to the equal power schemes, both of the convex optimization scheme and scheduling schemes have 2∼3.5 dB total power gains in the cliff region. The error floors of BER are also consistent with the BEP Lemma 3.1 and the numerical results in Fig. 2. The scheduling scheme may have minor sacrifice for the BEP in some scenarios as expected.

In fact, the total power is mostly allocated to the dominating sensors in low SNR region as the result of the convex optimization scheme, which is essentially consistent to the scheduling scheme and the intuition. However, the scheduling scheme only consider the power allocation among the dominating sensors, which may cause some performance loss in cliff region. It is also noticeable that the convex optimization scheme may have a small performance loss after reaching the BEP of dominating sensors. This is due to our approximation to the objective function in equations (7) and (9), which may cause acceptable performance loss in high SNR region.

V. Conclusion

We have proposed a power allocation scheme and a sensor scheduling scheme for asymmetric sensor networks. The power allocation scheme is based on convex optimization, of which objective is to maximize the mutual information between the source and its estimate. The scheduling scheme is built from the BEP lemma we have derived for a weighted vote CEO scheme. We have demonstrated that both schemes achieve 2∼3.5 dB total power gains in the cliff region, and they depend on the observation error vectors. All numerical results and the theoretical analysis have been shown to be consistent.

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