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A Study on the Number of Convex Configuration of Silhouette Puzzles

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Given a set of polygons and a target shape, a silhouette puzzle is a problem to decide whether the polygons can be arranged in the plane without any overlap so that their union exactly covers the target shape, where the polygons can be rotated and reflected. Tangram is one of the most famous silhouette puzzles. Wang and Hsiung [The American Mathematical Monthly 49:9 (1942) 596–599] showed that the number of convex target shapes that Tangram can form is 13. There is another silhouette puzzle called Sei Shonagon Chie no Ita, which is known in Japan since the 18th century. Recently, Fox-Epstein, Katsumata, and Uehara [IEICE Trans. E99-A:6 (2016) 1084–1089] have shown that the number of convex target shapes that Sei Shonagon Chie no Ita can form is 16. These two puzzles have a common property: each piece can be decomposed into some unit isosceles right triangles and the total number of such triangles is 16. The numbers of convex shapes that can be formed by the two puzzles are different. This difference can be seen as a difference of expressiveness. To investigate the expression power of silhouette puzzles, in this research, we focus on the number of convex shapes that a puzzle can form.

As mentioned above, there are a couple of previous studies on the number of convex shapes that can be formed by silhouette puzzles. Fox-Epstein et al. focused on the class of silhouette puzzles that can be decomposed into 16 unit isosceles right triangles. They studied the maximum number of convex shapes that can be formed by silhouette puzzles in the class with a fixed number of pieces. They determined such numbers exactly for the cases where the number of pieces is 6 or more. Shibuya [Unpublished (2016)] worked on this problem as his minor research at JAIST and showed lower bounds for the cases where the number of pieces is between 2 and 5. One of the aims of this research is to determine the maximum number of possible convex shapes for these open cases. Horiyama, Uehara, and Hosoya [FUN 2016, LIPIcs 20:1–20:14] studied a silhouette puzzle involving the golden ratio. They presented a general algorithm for searching all convex shapes with a certain property that a silhouette puzzle can form. In this research, we implemented their algorithm and applied it to the open cases. By separately proving that the cases where the algorithm cannot be applied form no more convex shapes than the known lower bound, we completely solved the open cases.

There is a silhouette puzzle called Lucky Puzzle with pieces decomposable into unit isosceles right triangles, where the total number of such triangles is 40. It is known that Lucky Puzzle can form 21 convex shapes, but it has not been proved that there are no other possible convex shapes. The second purpose in this research is to determine the number of convex shapes that Lucky Puzzle can form. Horiyama et al. applied their algorithm to Lucky Puzzle. They ran the algorithm for two weeks, but the algorithm did not finish the search. To make it faster, we modified the congruency-check part of the algorithm. We first applied the modified algorithm to Tangram and Sei Shonagon Chie no Ita and tried to find a way of ordering the pieces to be processed. We found that the ordering significantly affects the computation time and that some natural orderings like the area-increasing ordering are much better than others. Using these orderings, we applied the modified algorithm to Lucky Puzzle, but the computation did not end after two weeks. Thus we switched to a theoretical approach of manual proof to solve the problem for Lucky Puzzle.

We showed that the known lower bound is tight. We first showed that the number of convex shapes that can be formed by Lucky Puzzle is at most 47. Then, by considering the shapes of two large pieces of Lucky Puzzle, it is easy to show that 14 of them cannot be formed. For the remaining 12, we showed impossibility by a case-by-case analysis.