Extensions and Applications of Antichain Algorithms

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This thesis is the contribution to the study of automata-theoretic theorem proving. Automata-theoretic theorem proving checks the satisfiability / validity of a first-order formula under a fixed interpretation. Elements in the universe are encoded into words and every predicate has a corresponding automaton which accepts the support of the formula.

There are several existing tools for automata-theoretic theorem proving. MONA [6] translates formulas in weak monadic second-order logic of k successor (WSkS) to finite tree automata. Recently FORT [15] is implemented for the first-order theory of term rewriting. It is based on tree automata and ground tree transducers (GTT) for left-liner right-ground term rewriting systems. They determine not only whether a formula is valid, but also generate counter-examples from the automata if the formula is not valid.

Difficulty of the automata construction comes from the state explosion problem. WS1S requires tower of computation task corresponding to the height of quantifiers in the input formula. The determinization of automata causes state explosion, which is necessary to translate the complementation.

Antichain Algorithms The commonly used optimization to tackle the state explosion is the on-the-fly state space generation [10]. Antichain algorithm, an additional technique originally developed in the model checking, combines the on-the-fly determinization and minimization [19]. Abdulla, et al. [1] combined antichains and a simulation technique and further reduced the state space of the universality/inclusion checking. An interesting empirical observation of FORT is that the flattening of a formula into a prenex normal form triggers further state explosion, which motivated our work. This paper investigates a generalized antichain algorithms without flattening. We focus on monadic first-order logic, which has neither set variables (as MONA) nor transitive closure (as FORT), as the most simple case study.

One of our aim was to directly handle a nested formula with an antichain algorithm (i.e., to avoid flattening the input formula). The aim was achieved by introducing the composition terms that represent the automata construction and by finding ordering inductively. So that the generalized antichain algorithm is inductively defined for the structure of composition terms. As an optimization, we further introduced

1. the conversion rules of composition terms which preserve the accepted language, and
2. the distributive laws of emptiness checking into the composition terms.

We performed experiments on randomly generated 3000 Presburger formulas. We could not observe that the generalized antichain algorithm improved the performance. Due to the overhead of calculating orderings, it did not work for small problems. In the most cases, conversion of the composition term performed better than the generalized antichain algorithm. Still, there were some cases where the generalized antichain algorithm outperformed other algorithms, when the problem was sufficiently large.

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