JAIST Repository
https://dspace.jaist.ac.jp/

Title	Design of Polar Lattices Based on Subcode Rate Selection
Author(s)	Garcia Alvarez, Erick Christian
Citation	
Issue Date	2017-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/14176
Rights	
Description	Supervisor:KURKOSKI, Brian Michael, 情報科学研究科, 修士

Japan Advanced Institute of Science and Technology

Abstract

This thesis proposes a design of lattices based on subcodes. In literature it is known how to construct polar lattices, however construction of optimal lattices are still an open problem. This work aims to form lattices from polar codes and study their performance.

To obtain efficient polar lattices in terms of low error rate, we selected various polar subcodes to compare which produces lower bit-error rate.

In this work, lattices are formed using construction A and construction D. Both constructions require a binary linear code $\mathcal{C}$. A lattice constructed with a single code is called *single-level lattice*. A lattice constructed with two nested codes is called *two-level lattice*. Multilevel refers to the number of nested codes chosen to form a lattice.

Construction A produces lattices which, in general, are effective in lower dimensions. Lattices Λ are obtained by using a single binary code $\mathcal{C}$. Each codeword of the binary code is assumed to be a lattice point in the real space $\mathbb{R}^N$.

Construction D is one of the various lattice constructions which produces lattices from nested binary codes. In this work, lattices Λ are formed by selecting nested polar codes $\mathcal{C}_i$.

We selected polar codes as the binary code $\mathcal{C}$ denoted by $Polar(N, K, \mathcal{F})$. Polar codes are specified by the channel transition probabilities $W(y|x)$, N is the block code length (or the lattice dimension), K is the number of information bits and the index vector $\mathcal{F}$ that has $N - K$ elements, which in literature is commonly called frozen bits.

We are interested in the generator matrix $\mathbf{G}_N$ of the polar code, because sub polar codes can be obtained by using $\mathbf{G}_N$. The full rank generator matrix $\mathbf{G}_N$ is defined by: $\mathbf{G}_N = \mathbf{R}_N(\mathbf{F} \otimes \mathbf{I}_{\frac{N}{2}}) \cdot (\mathbf{I}_2 \otimes \mathbf{G}_{\frac{N}{2}})$, where $\mathbf{R}_N$ is the reverse shuffle permutation matrix, $\mathbf{F} = \left[\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right]$ and $\otimes$ is the Kronecker product. A given polar subcode contains some of the basis row vectors as the generator matrix $\mathbf{G}_N$.

Polar codes were chosen because polar codes have a structured construction providing easier identification of subcodes required to build up polar lattice by construction D.

Polar lattices Λ_P are specified by $\Lambda_P(N, K_i)$, where N is the dimension of the lattice and K is the dimension of the nested binary code i. In a few lines, the polar lattice

construction is explained as follows. From the generator matrix $\mathbf{G}_N$, subcodes with rate $(\frac{K_i}{N})$ are chosen, following the polar code construction, such as the selection of frozen bits by maximizing the symmetric capacity, which is explain in detail in section 3. To form polar lattices we use the polar subcodes and apply: (1) construction A requires a single polar code. Details are explained in section 4.4.1. (2) construction D requires nested polar codes. There is a restriction on the minimum distance $d_i \geq \frac{4^{(i)}}{j}$ for the nested polar codes, where j is either 1 or 2, and in this work we used 2. i is the number of nested polar codes.

To evaluate the best bit-error rate, symbol-error rate and word-error rate performance for each lattice, Monte Carlo simulation were performed for:

1. Single-level lattice construction. Polar lattices are formed by a single code with construction A and construction D. We investigated the best BER, SER and WER performance for a give polar lattice under lower volume to noise ratio (VNR). VNR is the metric for lattices which shows how dense a lattice is. We evaluate from 0dB to 5dB for the VNR.

2. Multilevel lattice construction. Polar lattices are formed by nested codes using construction D. The objective of this simulation is to identify with how many polar lattices levels the best BER, SER and WER under lower VNR (from 0dB to 5dB VNR).

For such simulations, the additive white Gaussian noise (AWGN) channel is considered. And at the receiver side, the decoding is done using a multilevel decoding and successive cancellation decoding (SCD).

Simulation results show that single-level polar lattices outperform the two-level and three-level polar lattices. Simulations also show that polar lattices with lower minimum distance has lower error rate performance, this positive result occurs because the best performance occurs when a higher amount of lattice points are packed on the same volume.

In this work we also compared the performance of polar lattices with another code. We chose BCH codes as another binary code to form lattices. BCH codes are powerful random error-correcting cyclic codes.

Simulation where performed with single-level BCH lattices by construction A. Results show that at a lower VNR (1dB), BCH lattice performed worse than polar lattices in terms of SER. However, on a little higher VNR (4dB), the best BCH lattice seems improve the performance, but still lower than some polar lattices.