Joint Optimization of Power Allocation and Relay Position for Lossy-Forwarding Relaying

Shen Qian
School of Information Science
Japan Advanced Institute of
Science and Technology (JAIST)
1-1 Asahidai, Nomi, Ishikawa,
Japan 923-1292,
Centre for Wireless Communication
University of Oulu
FI-90014 Finland
Email: shen.qian@jaist.ac.jp

Markku Juntti
Centre for Wireless Communication
University of Oulu
FI-90014 Finland
Email: markku.juntti@oulu.fi

Tad Matsumoto
School of Information Science
Japan Advanced Institute of
Science and Technology (JAIST)
1-1 Asahidai, Nomi, Ishikawa,
Japan 923-1292,
Centre for Wireless Communication
University of Oulu
FI-90014 Finland
Email: tadashi.matsumoto@ee.oulu.fi

***Abstract*—Joint optimization of power allocation (PA) and relay position (RP) is investigated for a lossy-forwarding relaying in order to minimize the outage probability. We investigate adaptive PA with fixed relay position, adaptive RP with fixed PA ratio, and joint optimization of the PA and RP under total transmit power constraint. A closed-form expression of the outage probability is derived at the high signal-to-noise ratio regime. It is shown that the closed-form expression is sufficiently accurate compared to numerical calculation results. Then, the optimum PA and the optimum RP can be formulated as a convex optimization problem. It is found that the system performance with the adaptive optimum PA outperforms that with equal power allocation. The outage performance with the adaptive optimum RP outperforms that with midpoint RP. However, the joint optimization of the PA and RP is superior to the semi-adaptive optimization algorithms.**

I. Introduction

Cooperative communications, which share antennas with other users and generate a virtual antenna array for achieving diversity [1], [2], is one of the advantageous techniques towards the development of next generation wireless communication networks. Among the various types of cooperative communication techniques, decode-and-forward (DF) relay has drawn significant and practical attention due to its simplicity, and has been widely studied [3] [4].

A novel lossy-forwarding (LF) relaying technique based on the DF protocol which allows source-relay (S-R) link errors has been proposed in [5]. It has been shown that the system can improve its performance in terms of bit error rate (BER), outage or throughput. In [6], it has been found that analysing the LF relaying can capitalize the results available in the source coding with side information (SCwSI) in the network information theory. Furthermore, exact expression of outage probability can be deduced to a simple, yet accurate approximation by replacing the SCwSI theorem by the Slepian-Wolf (SW) theorem.

The outage probability bound of the LF relaying has been evaluated in [7]. The power allocation scheme for the LF relaying has been provided in [8], in which assumes the S-R link to be modelled as a static binary symmetric channel (BSC). In [9], a more practical model, which considers the error probability after decoding as also a random variable, has been considered as the framework of the optimal resource allocation problems.

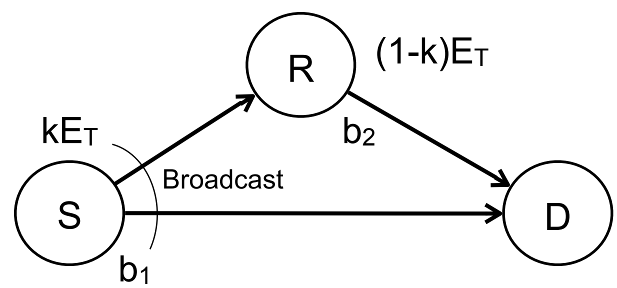


Fig. 1. Single relay lossy-forwarding relaying network.

It is well known that one-way relaying techniques need two transmission phases, which reduces system throughput. Furthermore, channel capacity relies on the receive signal-to-noise power ratio (SNR). The optimization of power allocation (PA) and relay position (RP) have been utilized to handle the coding gain (diversity order) and large-scale propagation effect (path-loss), respectively. The uniform PA among the nodes and the midpoint RP may not be optimal. Therefore, investigating the optimal RP and optimal PA for source (S) and relay (R) has very important practical benefit [10] [11]. The existing PA schemes focus on optimizing the transmit power for the LF relaying, while joint optimization of PA and RP has not been addressed.

In this paper, we investigate the optimal RP with fixed PA, optimal PA with fixed RP, and joint optimization of PA and RP in order to minimize the outage probability of the LF relaying over independent and identically distributed (i.i.d) fading channels.

The contributions of the paper are highlighted as follows: 1) We formulate the optimization over PA and RP under standard convex optimization problem. 2) We investigate the optimum

PA with certain RP and optimum RP with certain PA, under the total transmit power constraint. 3) We jointly optimize the PA and RP for minimizing the outage probability for the LF relaying. 4) We compare the different optimization schemes. It is found that by selecting the optimal PA ratio, the system can significantly reduce the outage probability compared to that with equal PA. Moreover, by identifying the optimal RP, a lower outage probability can be achieved compared to the case that R is located at the midpoint between S and D. It is also found that the optimum RP scheme outperforms the fixed PA&RP scheme. Furthermore, a lower outage probability can be achieved with the optimum PA scheme compared to that with the optimum RP scheme. The jointly optimized PA and RP further improves the outage performance.

II. System model

We consider a simple one-way relaying with three nodes as shown in Fig. 1. The source S communicates with the destination D with the help of one relay R. We assume a time-division channel allocation where the transmission has two time slots. During the first time slot, the original binary information sequences b_1 are broadcasted from S. R aims at recovering the transmitted information sequences b_1. However, because of the S-R link errors, b_2 obtained as the result of decoding at R may contain errors. Nevertheless, R re-interleaves b_2, re-encodes and forwards it to D during the second time slot.

The position of R is assumed to vary along the line between S and D to minimize the effect of path loss. With d representing the length of the R-D link, and with the gain of the S-D link G_1 and the length of the S-D link being normalized to one, the geometric gains of the R-D and S-R links, G_2 and G_3, respectively, can be defined as [12]

$$G_2 = \left(\frac{1}{d}\right)^{\alpha}, G_3 = \left(\frac{1}{1-d}\right)^{\alpha}, \tag{1}$$

where α is the path loss factor.

The S-R link is described by a BSC model [13] with the crossover probability p_e as a parameter. p_e represents the information bits flipping probability, during the S-R transmission. Hence, $b_2 = b_1 \oplus e$, where $\oplus$ denotes modulo-2 addition and e is a binary variable representing errors, $\Pr(e=1) = p_e$. The p_e value changes block-by-block.

In this paper, all the links are assumed to experience block Rayleigh fading. The probability density function (pdf) with instantaneous SNR γ_i is given by

$$p(\gamma_i) = \frac{1}{\Gamma_i}\exp(-\frac{\gamma_i}{\Gamma_i}), (i = 1, 2, 3), \tag{2}$$

where Γ_i represents the average SNR of the S-D, R-D, and S-R links, respectively.

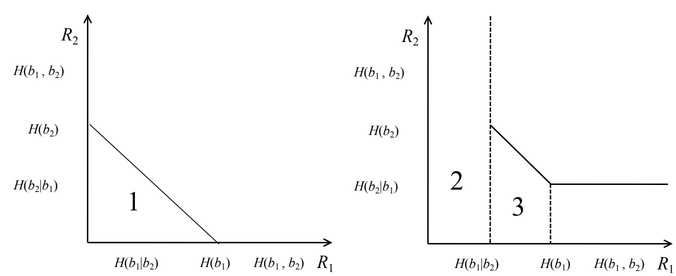
III. Outage probability derivation

As shown in [6], the approximated outage expression derived from the SW theorem is accurate enough compared to the exact outage derived from the theorem for SCwSI. Therefore, for simplicity, the outage probability is calculated based on the SW theorem. The rate constrains are given by

$$\begin{cases} R_1 \geqslant H(b_1 \mid b_2), \\ R_2 \geqslant H(b_2 \mid b_1), \\ R_1 + R_2 \geqslant H(b_1, b_2) = 1 + H\left[R_b^{-1}\left(\frac{C_3}{R_{c3}}\right)\right], \end{cases} \tag{3}$$

where R_1 and R_2 are the source rate for b_1 and b_2, respectively. R_{c3} denotes the spectrum efficiency including the channel coding scheme and the modulation multiplicity and C_3 is the S-R channel capacity. R_b^{-1} is the inverse binary rate-distortion function. $H(\cdot|\cdot)$ and $H(\cdot,\cdot)$ represent the binary conditional and the binary joint entropy function, respectively.

If $p_e = 0$, the inadmissible rate region is shown as area 1 in Fig. 2(a). If $0 < p_e \leq 0.5$, the inadmissible region is shown as areas 2 and 3 in Fig. 2(b). Therefore, the outage probability of the LF relaying can be expressed as $P_{out} = P_1 + P_2 + P_3$, where P_1, P_2, and P_3 denote the probabilities that (R_1, R_2) falls into the inadmissible areas 1, 2, and 3, respectively.



(a) $p_e = 0$ in the S-R link. (b) $0 < p_e \leqslant 0.5$ in the S-R link.

Fig. 2. Slepian-Wolf rate regions of the proposed LF relaying.

It is found that the outage probability can be calculated by using a triple integral with respect to the joint pdf of the instantaneous SNRs $p(\gamma_1, \gamma_2, \gamma_3)$ [9], given the range defined in (3). We assume that the fading of each link is statistically independent. According to the source-channel separation theorem, the relationship between the instantaneous channel SNR γ_i and its corresponding rate R_i is given by $R_i \leq \frac{1}{R_{ci}}\log_2(1+\gamma_i), (i = 1, 2, 3)$, where the Gaussian codebook is assumed. Then the theoretical calculation of the outage probabilities P_1, P_2, and P_3 can be mathematically expressed as

$$\begin{aligned} P_1 =& \Pr\left[0 < R_1 < H(b_1), 0 < R_2 < H(b_1, b_2), R_3 \geq 1\right] \\ =& \frac{1}{\Gamma_1}\exp\left(-\frac{1}{\Gamma_3}\right)\int_1^{\gamma_1=0}\exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \\ &\cdot\left[1-\exp\left(-\frac{2^{1-\log_2(1+\gamma_1))}-1}{\Gamma_2}\right)\right]d\gamma_1, \end{aligned} \tag{4}$$

$$\begin{aligned} P_2 =& \Pr\left[0 < R_1 < H(b_1 \mid b_2), R_2 > 0, 0 \leq R_3 < 1\right] \\ =& \frac{1}{\Gamma_3}\int_{\gamma_3=0}^{1}\exp\left(-\frac{\gamma_3}{\Gamma_3}\right) \\ &\cdot\left[1-\exp\left(-\frac{2^{1-\log_2(1+\gamma_3)}-1}{\Gamma_1}\right)\right]d\gamma_3, \end{aligned} \tag{5}$$

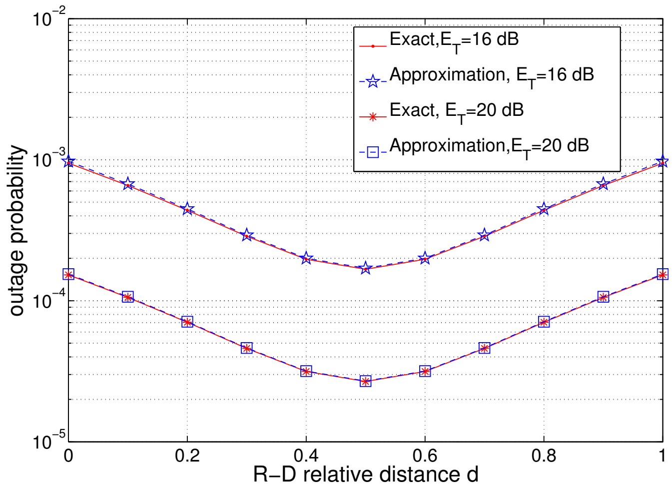


Fig. 3. Comparison between the exact and approximate outage probabilities for $k = 0.5$.

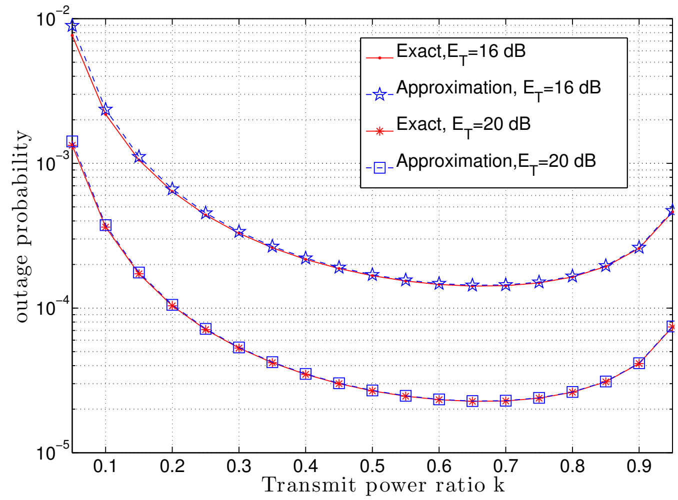


Fig. 4. Comparison between the exact and approximate outage probabilities for $d = 0.5$.

and

$$\begin{aligned} P_3 =& \Pr\left[H(b_1 \mid b_2) < R_1 < H(b_1), \right. \\ & \left. 0 < R_2 < H(b_1, b_2) - R_1, 0 \leq R_3 < 1\right] \\ =& \frac{1}{\Gamma_1}\frac{1}{\Gamma_3}\int_{\gamma_1 = 2^{1-\log_2(1+\gamma_3)}-1}^{1}\int_{\gamma_3=0}^{1} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right)\exp\left(-\frac{\gamma_3}{\Gamma_3}\right) \\ & \cdot\left[1 - \exp\left(-\frac{2^{2-\log_2(1+\gamma_3)-\log_2(1+\gamma_1)} - 1}{\Gamma_2}\right)\right] d\gamma_1 d\gamma_3. \end{aligned} \tag{6}$$

The values of P_1, P_2 and P_3 can be calculated by numerical method [14], with accurate enough numerical calculation error control.

IV. Optimal PA and RP to Minimize the Outage Probability

The goal of this section is to minimize the outage probability obtained at previous section, while the total transmit power E_T is fixed. By invoking the property of exponential function $e^{-x} \approx 1 - x$ for small x, at the high SNR regime, the outage probability P_{out} can be approximated by a closed-form expression, as,

$$P_{out} \approx \frac{2\ln 2 - 1}{\Gamma_1\Gamma_2} + \frac{2\ln 2 - 1}{\Gamma_1\Gamma_3} + \frac{2\ln^2 2 - 2\ln 2 - 2}{\Gamma_1\Gamma_2\Gamma_3}. \tag{7}$$

With the noise variance of each channel being normalized to the unity, the transmit power, which is equivalent to their corresponding average SNR, allocated to S and R are denoted as $E_T k$ and $E_T(1-k)$, respectively. k ($0 < k < 1$) is the power allocation ratio. Since $\Gamma_1 = E_T k G_1$, $\Gamma_2 = E_T(1-k)G_2$, and $\Gamma_3 = E_T k G_3$, P_{out} can be written as

$$\begin{aligned} P_{out} \approx& A\left(k\left(1-k\right)\left(\frac{1}{d}\right)^{\alpha}\right)^{-1} + A\left(k^2\left(\frac{1}{1-d}\right)^{\alpha}\right)^{-1} \\ & + B\left(k^2\left(1-k\right)\left(\frac{1}{1-d}\right)^{\alpha}\right)^{-1}, \end{aligned} \tag{8}$$

where $A = \frac{2\ln 2 - 1}{E_T^2}$, $B = \frac{2\ln 2 - 1}{E_T^3}$.

The outage probability curves obtained by using the approximated expression (8) and by numerically calculating the (4)-(6) are presented in Figs. 3 and 4 with fixed k and fixed d, respectively. A good match is observed between the curves of approximate and numerically calculated exact outage probability, with which indicates the usefulness of the approximation.

A. Optimal PA Ratio: RP Fixed

In this subsection, we minimize the outage probability by adjusting PA between S and R while keep the RP fixed. The optimization problem can be formulated as

$$\begin{aligned} & k^* = \arg\min_{k} \quad P_{out}(k) \\ & \text{subject to:} \quad k - 1 < 0, \quad -k < 0, \quad -E_{\mathrm{T}} < 0. \end{aligned} \tag{9}$$

By taking second-order partial derivative of P_{out} with respect to k, we can get

$$\begin{aligned} \frac{\partial^2 P_{out}}{\partial k^2} =& \frac{Ad^{\alpha}(6k^2 - 6k + 2)}{k^3(1-k)^3} + \frac{6A(1-d)^{\alpha}}{k^4} \\ & + \frac{Bd^{\alpha}(1-d)^{\alpha}(12k^2 - 16k + 6)}{k^4(1-k)^3}. \end{aligned} \tag{10}$$

$\frac{\partial^2 P_{out}}{\partial k^2}$ is obviously positive in the range $k \in (0,1)$. This indicates that the objective function is convex with respect to k in (0, 1). Hence, taking the first-order derivative of P_{out} with respect to k and setting the derivative result equal to zero,

$$\begin{aligned} \frac{\partial P_{out}}{\partial k} =& \frac{Ad^{\alpha}(2k-1)}{k(k-1)^2} - \frac{2A(1-d)^{\alpha}}{k^3} \\ & + \frac{Bd^{\alpha}(1-d)^{\alpha}(3k-2)}{k^3(k-2)^2} = 0, \end{aligned} \tag{11}$$

the optimal PA ratio k^* can be obtained by solving (11) as:

$$k^* = \frac{-4B' + 3C' \pm \sqrt{9C'^2 + 8A'B' + 8A'C' + 8B'C'}}{2(A' - 2B')}, \tag{12}$$

TABLE I. OPTIMAL PA RATIO k^*

d	optimal k^* (E_T = 16 dB)	optimal k^* (E_T = 20 dB)
0.9	0.5002	0.5002
0.8	0.5037	0.5037
0.7	0.5221	0.5222
0.6	0.5752	0.5752
0.5	0.6667	0.6667
0.4	0.7743	0.7743
0.3	0.8719	0.8719
0.2	0.9436	0.9436
0.1	0.9855	0.9855

where $A' = \frac{d^\alpha(2\ln 2-1)}{E_T^2}$, $B' = \frac{(1-d)^\alpha(2\ln 2-1)}{E_T^2}$, and $C' = \frac{d(1-d)^\alpha(2\ln 2-1)}{E_T^3}$.

The optimal PA ratio k^* with RP d are shown in Table I. Obviously, the smaller the d value (a shorter R-D distance), the more transmit power should be allocated to S. We found that the optimal k^* is almost the same with different total transmit power E_T.

B. Optimal RP: PA Ratio Fixed

In this subsection, we investigate the optimal RP, given a fixed PA ratio. The problem can be formulated as

$$\begin{aligned} & d^* = \arg\min_{d} \quad P_{out}(d) \\ & \text{subject to:} \quad d-1<0, \quad -d<0, \quad -E_{\text{T}}<0. \end{aligned} \tag{13}$$

Taking the second-order derivative of P_{out} with respect to d, we have

$$\begin{aligned} \frac{\partial^2 P_{out}}{\partial d^2} &= \frac{A''\alpha(\alpha+1)d^{(\alpha+2)}}{d^4} + \frac{2A''\alpha d^{(\alpha+1)}}{d^3} \\ &+ \frac{B''2\alpha(1-d)^{(\alpha+1)}}{(1-d)^3} + \frac{B''\alpha(\alpha+1)(1-d)^{\alpha+2}}{(1-d)^4} \\ &+ \frac{C''\left(\frac{2\alpha}{d(1-d)^3}+\frac{2\alpha}{d^2(1-d)^2}+\frac{2\alpha}{d^3(1-d)}\right)(d(1-d))^{(-\alpha-2)}}{\frac{1}{(d(1-d))^{(\alpha+1)}}+\alpha(\alpha+1)\left(\frac{1}{d(d-1)^2}+\frac{1}{d^2(d-1)}\right)^2}, \end{aligned} \tag{14}$$

where $A'' = \frac{2\ln 2-1}{E_T^2 k(1-k)}$, $B'' = \frac{2\ln 2-1}{E_T^2 k^2}$, and $C'' = \frac{2\ln 2-1}{E_T^3 k^2(1-k)}$. Again, it is obvious that $\frac{\partial^2 P_{out}}{\partial d^2} > 0$ in $d \in (0,1)$, with which proves the convexity of (14) with respect to d. Hence, taking the first-order derivative of P_{out} with respect to d and setting the derivative result equal to zero,

$$\begin{aligned} \frac{\partial P_{out}}{\partial d} &= \frac{A''\alpha d^{(\alpha+1)}}{d^2} + \frac{B''\alpha(1-d)^{(\alpha+1)}}{(d-1)^2} \\ &+ \frac{C''\alpha d(1-d)^{(\alpha+1)}(1-2d)}{d^2(d-1)^2} = 0. \end{aligned} \tag{15}$$

The optimal RP d^* can be obtained by solving (15). It is excessively complex to derive the explicit expression for d^*. However, iterative root-finding algorithm is used to numerically calculate the solution to (15) efficiently with high enough accuracy.

Table II shows the optimal RP d^* for outage probability with different PA ratio k, when E_T is 16 dB and 20 dB, respectively. It is clearly seen that the smaller the k value (less transmit power allocated to S), R should be located the point closer to S in order to achieve a lower outage probability. It is also found from Table II that, the optimal d^* are almost the same with different total transmit power E_T. These observations indicate that the optimal k^* and optimal d^* are effective for any E_T.

TABLE II. OPTIMAL RP d^*

k	optimal d^* (E_T = 16 dB)	optimal d^* (E_T = 20 dB)
0.9	0.2949	0.2949
0.8	0.3659	0.3658
0.7	0.4167	0.4167
0.6	0.4599	0.4599
0.5	0.5000	0.5000
0.4	0.5401	0.5401
0.3	0.5833	0.5833
0.2	0.6341	0.6342
0.1	0.7051	0.7051

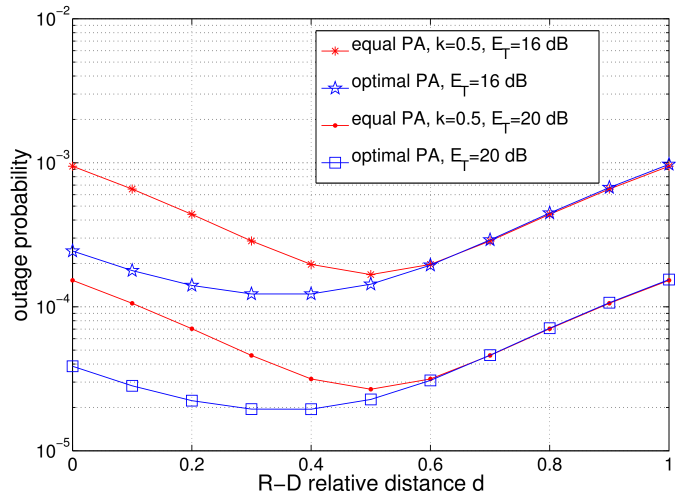


Fig. 5. Outage probabilities with and without optimal power allocation, total transmit power fixed.

C. Joint Optimal RP and PA Ratio

For joint optimizing the PA ratio and RP, the problem can be formulated as,

$$\begin{aligned} & k^*, d^* = \arg\min_{k,d} \quad P_{out}(k,d) \\ & \text{subject to:} \quad d-1<0, -d<0, k-1<0, \\ & \qquad\qquad\quad -k<0, -E_{\text{T}}<0. \end{aligned} \tag{16}$$

The Hessian matrix of P_{out} is shown to be positive definite in Appendix. Hence the convexity of the approximated outage probability expression (8) is proved. By setting the first-order partial derivatives with respect to k and d to zero, respectively, the joint optimal PA ratio and RP can be obtained by jointly solving (11) and (15). Similarly, the iterative root-finding algorithm is used to numerically calculate k^* and d^*.

V. NUMERICAL RESULTS

In this section, the outage performances of 1) fixed PA&RP 2) semi-adaptive algorithms: optimization of PA with fixed RP and optimization of RP with fixed PA, respectively; 3) and joint

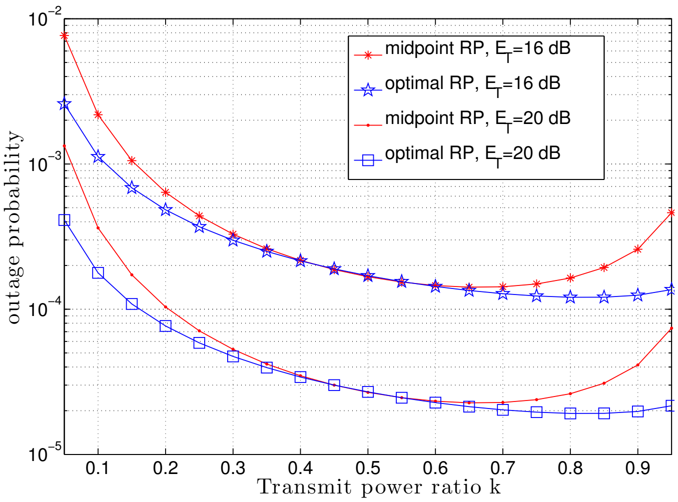


Fig. 6. Comparison between the outage probabilities with midpoint relay position and optimal relay position.

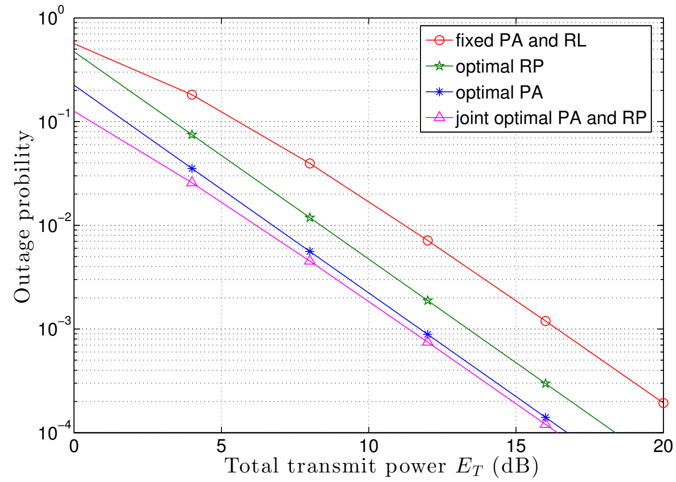


Fig. 7. Outage probability comparison between the optimal PA, optimal RP, joint optimal, and fixed schemes.

optimization of PA and RP algorithms, are demonstrated. The path loss factor $\alpha = 3.52$ is used [15].

Fig. 5 presents the theoretical outage probabilities with equal and optimal PAs, when the total transmit power $E_T = 16$ dB and $E_T = 20$ dB, respectively. It can be found that the outage probability curves are symmetric to the midpoint between S and D with equal fixed PA ($k = 0.5$). On the other hand, with the optimal PA ratio k, the outage probability can be significantly reduced compared to that with equal PA if the relay is allocated close to the D (a smaller d). While if relay is near to the S (a larger d), outage performances are almost same. This indicates that optimal PA scheme enables the system to find/set proper relays for cooperation in a further place for achieving better outage performance.

Fig. 6 compares the outage performance of the LF relaying with midpoint and with optimal RPs. The E_T is set to 16 dB and 20 dB as a parameter. It can be obviously seen from Fig. 6 that at the near equal PA region (k is close to 0.5) , RP does not made impact on the outage performance. However, a lower outage probability can be yielded by choosing the optimal RP if S and R have unbalanced PA ratio ($k \gg 0.5$ or $k \ll 0.5$).

Fig. 7 presents the outage comparison between the 1) fixed PA&RP ($d = 0.3$, $k = 0.3$), 2) optimal PA ratio (with $d = 0.3$), 3) optimal RP (with $k = 0.3$), and 4) joint optimal RP and PA ratio. It is found that the optimal RP is significantly superior to fixed PA&RP algorithm. Furthermore, choosing the optimal PA ratio further reduces the outage probability. When the PA ratio and RP are jointly optimized, the outage performance can further be improved.

Fig. 8 plots the PA ratio (k) versus the optimal RP (d^*), as well as d with k^*. The curves with different E_T show almost the same tendency as shown in Tables I and II, therefore we just plot only for $E_T = 16$ dB. We can see that the smaller the d value (R moves closer to D), the larger the optimal ratio k^* (more power should be allocated to S). This is because that with the shorter R-D link, the outage performance is more dependent on the S-R link. In this case, allocating more power to the source node will improve the outage performance. On contrary, we can observe that the larger the ratio k (more transmit power allocated to S), the smaller the d^* value (R should be located close to D). This is because that with the less R transmit power, the R-D link has more impact on the outage performance. Fig. 8 also shows the joint optimal PA ratio and RP point with total power constraint. The joint optimal solution is the point where the contributions of the optimal PA ratio and optimal RP are balanced. It also can be seen that, with the certain RP d, joint optimal PA is not only the superior scheme in the sense that it can achieve a lower outage probability, but also in the sense that it can reduce the power consumption of S (a smaller k), which has great practical benefit.

VI. Conclusion

We have investigated the optimal schemes for a LF relaying to minimize outage probability. First, we optimized PA with fixed RP under the total transmit power constraint. Then, we derived the optimal RP with fixed PA. Additionally, we jointly optimized the PA and RP, and made comparisons between different optimization schemes. The optimization problem has been formulated by the convex optimization framework. The analytical results show that, the adaptive RP scheme outperforms the fixed RP&PA scheme in terms of outage performance. A lower outage probability can be achieved by adjusting the power allocated to S and R. It has been also shown that joint optimal PA and RP scheme further reduce the outage probability compared to fixed and semi-adaptive optimal schemes.

Appendix

The Hessian matrix of (8) can be calculated as

$$\mathbf{H}(P_{out}) = \begin{bmatrix} \frac{\partial^2 P_{out}}{\partial k^2} & \frac{\partial^2 P_{out}}{\partial k \partial d} \\ \frac{\partial^2 P_{out}}{\partial d \partial k} & \frac{\partial^2 P_{out}}{\partial d^2} \end{bmatrix}. \tag{17}$$

$$\frac{\partial^2 P_{out}}{\partial k^2} = \frac{A d^\alpha (6k^2 - 6k + 2)}{k^3 (1-k)^3} + \frac{6A(1-d)^\alpha}{k^4} + \frac{B d^\alpha (1-d)^\alpha (12k^2 - 16k + 6)}{k^4 (1-k)^3}, \tag{18}$$

$$\begin{aligned}\frac{\partial^2 P_{out}}{\partial k \partial d} = \frac{\partial^2 P_{out}}{\partial d \partial k} &= \\ &= \frac{A(2k-1)\alpha d^{(\alpha+1)}}{k(k-1)^2 d^2} + \frac{2A\alpha(d-1)^{(\alpha+1)}}{k^3(d-1)^2} \\ &+ \frac{B(3k-2)\alpha d(1-d)^{(\alpha+1)}(1-2d)}{k^3(k-2)^2 d^2 (d-1)^2}, \end{aligned} \quad (19)$$

where $A = \frac{2\ln 2 - 1}{E_T^2}$, $B = \frac{2\ln 2-1}{E_T^3}$.

$$\begin{aligned}\frac{\partial^2 P_{out}}{\partial d^2} &= \frac{A''\alpha(\alpha+1)d^{(\alpha+2)}}{d^4} + \frac{2A''\alpha d^{(\alpha+1)}}{d^3} \\ &+ \frac{B''2\alpha(1-d)^{(\alpha+1)}}{(1-d)^3} + \frac{B''\alpha(\alpha+1)(1-d)^{\alpha+2}}{(1-d)^4} \\ &+ \frac{C''\left(\frac{2\alpha}{d(1-d)^3} + \frac{2\alpha}{d^2(1-d)^2} + \frac{2\alpha}{d^3(1-d)}\right)(d(1-d))^{(-\alpha-2)}}{\frac{1}{(d(1-d))^{(\alpha+1)}} + \alpha(\alpha+1)\left(\frac{1}{d(d-1)^2} + \frac{1}{d^2(d-1)}\right)^2}, \end{aligned} \quad (20)$$

where $A'' = \frac{2\ln 2-1}{E_T^2 k(1-k)}$, $B'' = \frac{2\ln 2-1}{E_T^2 k^2}$, and $C'' = \frac{2\ln 2-1}{E_T^3 k^2(1-k)}$.

The determinant of $\mathbf{H}(P_{out})$ is given by

$$\begin{aligned}|\mathbf{H}(P_{out})| &= \left(\frac{Ad^{\alpha}(6k^2-6k+2)}{k^3(1-k)^3} + \frac{6A(1-d)^{\alpha}}{k^4} \right. \\ &\left. + \frac{Bd^{\alpha}(1-d)^{\alpha}(12k^2-16k+6)}{k^4(1-k)^3}\right)\left(\frac{A''\alpha(\alpha+1)d^{(\alpha+2)}}{d^4}\right. \\ &+ \frac{2A''\alpha d^{(\alpha+1)}}{d^3}\frac{B''2\alpha(1-d)^{(\alpha+1)}}{(1-d)^3} + \frac{B''\alpha(\alpha+1)(1-d)^{\alpha+2}}{(1-d)^4} \\ &\left. + \frac{C''\left(\frac{2\alpha}{d(1-d)^3} + \frac{2\alpha}{d^2(1-d)^2} + \frac{2\alpha}{d^3(1-d)}\right)(d(1-d))^{(-\alpha-2)}}{\frac{1}{(d(1-d))^{(\alpha+1)}} + \alpha(\alpha+1)\left(\frac{1}{d(d-1)^2} + \frac{1}{d^2(d-1)}\right)^2}\right) \\ &- \left(\frac{2A(2k-1)\alpha d^{(\alpha+1)}}{k(k-1)^2 d^2} + \frac{4A\alpha(d-1)^{(\alpha+1)}}{k^3(d-1)^2}\right. \\ &\left. + \frac{2B(3k-2)\alpha d(1-d)^{(\alpha+1)}(1-2d)}{k^3(k-2)^2 d^2(d-1)^2}\right). \end{aligned} \quad (21)$$

It can be seen that for $k \in (0,1)$ and $d \in (0,1)$, (21) is non-negative. Therefore, the outage probability expression (8) can be proven to be convex.

Acknowledgment

This research was supported in part by European Unions FP7 project, ICT-619555 RESCUE (Links-on-the-fly Technology for Robust, Efficient and Smart Communication in Unpredictable Environments), in part by Academy of Finland NETCOBRA project, in part by the Japan Society for the Promotion of Science (JSPS) Kiban (B) No. 15H04007, in part by JAIST Doctoral Research Fellow program, and also in part by NEC C&C Grants for Non-Japanese Researchers.

References

[1] J. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec 2004.

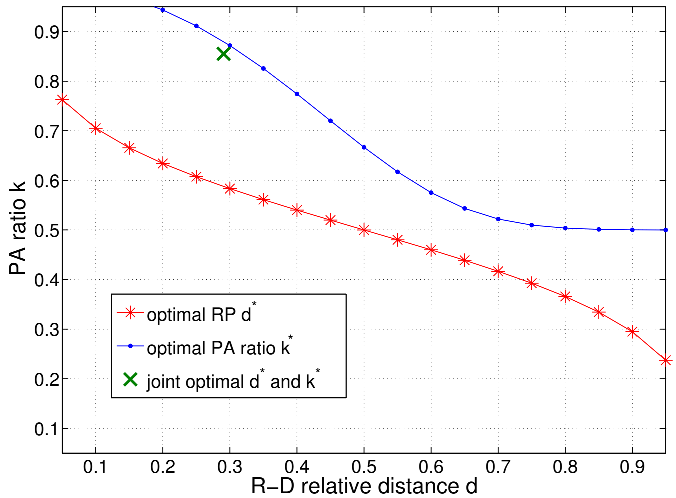


Fig. 8. Optimal PA ratio k^* with RP d; optimal RP d^* with PA k; joint optimal RP d^* and PA k^*.

[2] P. Liu, Z. Tao, Z. Lin, E. Erkip, and S. Panwar, "Cooperative wireless communications: a cross-layer approach," *IEEE Wireless Commun. Mag.*, vol. 13, no. 4, pp. 84–92, Aug 2006.

[3] S.-Q. Huang, H.-H. Chen, and M.-Y. Lee, "On performance bounds of mixed amplify-and-forward and decode-and-forward cooperative relay systems," in *Proc. Int. Conf. on Commun. and Networking in China*, Aug 2011, pp. 521–527.

[4] S. Ikki and M. Ahmed, "Performance analysis of decode-and-forward incremental relaying cooperative-diversity networks over rayleigh fading channels," in *Proc. IEEE Veh. Technol. Conf.*, April 2009, pp. 1–6.

[5] K. Anwar and T. Matsumoto, "Accumulator-assisted distributed turbo codes for relay systems exploiting source-relay correlation," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1114–1117, 2012.

[6] X. Zhou, M. Cheng, X. He, and T. Matsumoto, "Exact and approximated outage probability analyses for decode-and-forward relaying system allowing intra-link errors," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 7062–7071, Dec 2014.

[7] M. Cheng, K. Anwar, and T. Matsumoto, "Outage probability of a relay strategy allowing intra-link errors utilizing slepian-wolf theorem," *EURASIP J. Advances Signal Processing*, vol. 2013, no. 1, p. 34, 2013.

[8] ——, "Outage based power allocation: Slepian-wolf relaying viewpoint," in *Proc. IEEE Global Telecommun. Conf. (Workshop)*, Dec 2013, pp. 807–811.

[9] S. Qian, M. Cheng, and T. Matsumoto, "Outage based power allocation for a lossy-forwarding relaying system," in *Proc. IEEE Int. Conf. Commun. (Workshop)*, June 2015, pp. 2139–2144.

[10] M. Wang and Z. Zhong, "Optimal power allocation and relay location for decode-and-forward dual-hop systems over weibull fading channels," in *Proc. Int. Wireless Commun. & Mobile Computing Conf.*, Aug 2012, pp. 240–244.

[11] S. Ikki, M. Uysal, and M. Ahmed, "Joint optimization of power allocation and relay location for decode-and-forward dual-hop systems over nakagami-m fading channels," in *Proc. IEEE Global Telecommun. Conf.*, Nov 2009, pp. 1–6.

[12] D. Liang, S. X. Ng, and L. Hanzo, "Relay-induced error propagation reduction for decode-and-forward cooperative communications," in *Proc. IEEE Global Telecommun. Conf.*, Dec 2010, pp. 1–5.

[13] J. Garcia-Frias and Y. Zhao, "Near-Shannon/Slepian-Wolf performance for unknown correlated sources over AWGN channels," *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 555–559, Apr. 2005.

[14] L. Shampine, "Matlab program for quadrature in 2D," *Applied Math. and Comp.*, vol. 202, no. 1, pp. 266 – 274, 2008.

[15] R. Youssef and A. Graell i Amat, "Distributed Serially Concatenated Codes for Multi-Source Cooperative Relay Networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 253 –263, Jan. 2011.