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<th>Joint Optimization of Power Allocation and Relay Position for Lossy-Forwarding Relaying</th>
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<td>Author(s)</td>
<td>Qian, Shen; Juntti, Markku; Matsumoto, Tad</td>
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Description

The paper discusses the joint optimization of power allocation and relay position for lossy-forwarding relaying. It presents a framework that enhances the efficiency of the communication system by optimizing the power allocation and relay position. The authors propose a method that improves the performance of the system, particularly in scenarios where data loss is acceptable. The results demonstrated in the paper show significant improvements in system efficiency and reliability.
Joint Optimization of Power Allocation and Relay Position for Lossy-Forwarding Relaying

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Abstract—Joint optimization of power allocation (PA) and relay position (RP) is investigated for a lossy-forwarding relaying in order to minimize the outage probability. We investigate adaptive PA with fixed relay position, adaptive RP with fixed PA ratio, and joint optimization of the PA and RP under total transmit power constraint. A closed-form expression of the outage probability is derived at the high signal-to-noise ratio regime. It is shown that the closed-form expression is sufficiently accurate compared to numerical calculation results. Then, the optimum PA and the optimum RP can be formulated as a convex optimization problem. It is found that the system performance with the adaptive optimum PA outperforms that with equal power allocation. The outage performance with the adaptive optimum RP outperforms that with midpoint RP. However, the joint optimization of the PA and RP is superior to the semi-adaptive optimization algorithms.

I. INTRODUCTION

Cooperative communications, which share antennas with other users and generate a virtual antenna array for achieving diversity [1], [2], is one of the advantageous techniques towards the development of next generation wireless communication networks. Among the various types of cooperative communication techniques, decode-and-forward (DF) relay has drawn significant and practical attention due to its simplicity, and has been widely studied [3] [4].

A novel lossy-forwarding (LF) relaying technique based on the DF protocol which allows source-relay (S-R) link errors has been proposed in [5]. It has been shown that the system can improve its performance in terms of bit error rate (BER), outage or throughput. In [6], it has been found that analysing the LF relaying can capitalize the results available in the source coding with side information (SCwSI) in the network information theory. Furthermore, exact expression of outage probability can be deduced to a simple, yet accurate approximation by replacing the SCwSI theorem by the Slepian-Wolf (SW) theorem.

The outage probability bound of the LF relaying has been evaluated in [7]. The power allocation scheme for the LF relaying has been provided in [8], in which assumes the S-R link to be modelled as a static binary symmetric channel (BSC). In [9], a more practical model, which considers the error probability after decoding as also a random variable, has been considered as the framework of the optimal resource allocation problems.

It is well known that one-way relaying techniques need two transmission phases, which reduces system throughput. Furthermore, channel capacity relies on the receive signal-to-noise power ratio (SNR). The optimization of power allocation (PA) and relay position (RP) have been utilized to handle the coding gain (diversity order) and large-scale propagation effect (path-loss), respectively. The uniform PA among the nodes and the midpoint RP may not be optimal. Therefore, investigating the optimal RP and optimal PA for source (S) and relay (R) has very important practical benefit [10] [11]. The existing PA schemes focus on optimizing the transmit power for the LF relaying, while joint optimization of PA and RP has not been addressed.

In this paper, we investigate the optimal RP with fixed PA, optimal PA with fixed RP, and joint optimization of PA and RP in order to minimize the outage probability of the LF relaying over independent and identically distributed (i.i.d) fading channels.

The contributions of the paper are highlighted as follows: 1) We formulate the optimization over PA and RP under standard convex optimization problem. 2) We investigate the optimum
PA with certain RP and optimum RP with certain PA, under the total transmit power constraint. 3) We jointly optimize the PA and RP for minimizing the outage probability for the LF relaying. 4) We compare the different optimization schemes. It is found that by selecting the optimal PA ratio, the system can significantly reduce the outage probability compared to that with equal PA. Moreover, by identifying the optimal RP, a lower outage probability can be achieved compared to the case that R is located at the midpoint between S and D. It is also found that the optimum RP scheme outperforms the fixed PA&RP scheme. Furthermore, a lower outage probability can be achieved with the optimum PA scheme compared to that with the optimum RP scheme. The jointly optimized PA and RP further improves the outage performance.

II. SYSTEM MODEL

We consider a simple one-way relaying with three nodes as shown in Fig. 1. The source S communicates with the destination D with the help of one relay R. We assume a time-division channel allocation where the transmission has two time slots. During the first time slot, the original binary information sequences $b_1$ are broadcasted from S. R aims at recovering the transmitted information sequences $b_1$. However, because of the S-R link errors, $b_2$ obtained as the result of decoding at R may contain errors. Nevertheless, R re-interleaves $b_2$, re-encodes and forwards it to D during the second time slot.

The position of R is assumed to vary along the line between S and D to minimize the effect of path loss. With $d$ representing the length of the R-D link, and with the gain of the S-D link $G_1$ and the length of the S-D link being normalized to one, the geometric gains of the R-D and S-R links, $G_2$ and $G_3$, respectively, can be defined as [12]

$$G_2 = \left(\frac{1}{d}\right)^\alpha, \quad G_3 = \left(\frac{1}{1-d}\right)^\alpha,$$

where $\alpha$ is the path loss factor.

The S-R link is described by a BSC model [13] with the crossover probability $p_c$, as a parameter. $p_c$ represents the information bits flipping probability, during the S-R transmission. Hence, $b_2 = b_1 \oplus e$, where $\oplus$ denotes modulo-2 addition and $e$ is a binary variable representing errors, $Pr(e = 1) = p_c$. The $p_c$ value changes block-by-block.

In this paper, all the links are assumed to experience block Rayleigh fading. The probability density function (pdf) with instantaneous SNR $\gamma_i$ is given by

$$p(\gamma_i) = \frac{1}{\Gamma_i} \exp\left(-\frac{\gamma_i}{\Gamma_i}\right), \quad (i = 1, 2, 3),$$

where $\Gamma_i$ represents the average SNR of the S-D, R-D, and S-R links, respectively.

III. OUTAGE PROBABILITY DERIVATION

As shown in [6], the approximated outage expression derived from the SW theorem is accurate enough compared to the exact outage derived from the theorem for SCwSI. Therefore, for simplicity, the outage probability is calculated based on the SW theorem. The rate constrains are given by

$$\begin{align*}
R_1 &\geq H(b_1 | b_2), \\
R_2 &\geq H(b_2 | b_1), \\
R_1 + R_2 &\geq H(b_1, b_2) = 1 + H\left[\frac{C_0}{R_e}\right],
\end{align*}$$

where $R_1$ and $R_2$ are the source rate for $b_1$ and $b_2$, respectively. $R_{c,3}$ denotes the spectrum efficiency including the channel coding scheme and the modulation multiplicity and $C_3$ is the S-R channel capacity. $R_{c,3}^{-1}$ is the inverse binary rate-distortion function. $H(\cdot | \cdot)$ and $H(\cdot, \cdot)$ represent the binary conditional and the binary joint entropy function, respectively.

If $p_c = 0$, the inadmissible rate region is shown as area 1 in Fig. 2(a). If $0 < p_c \leq 0.5$, the inadmissible region is shown as areas 2 and 3 in Fig. 2(b). Therefore, the outage probability of the LF relaying can be expressed as $P_{out} = P_1 + P_2 + P_3$, where $P_1$, $P_2$, and $P_3$ denote the probabilities that $(R_1, R_2)$ falls into the inadmissible areas 1, 2, and 3, respectively.

![Slepian-Wolf rate regions of the proposed LF relaying](image)

Fig. 2. Slepian-Wolf rate regions of the proposed LF relaying.

It is found that the outage probability can be calculated by using a triple integral with respect to the joint pdf of the instantaneous SNRs $p(\gamma_1, \gamma_2, \gamma_3)$ [9], given the range defined in (3). We assume that the fading of each link is statistically independent. According to the source-channel separation theorem, the relationship between the instantaneous channel SNR $\gamma_i$ and its corresponding rate $R_i$ is given by $R_i \leq \frac{1}{\gamma_i} \log_2 \left(1 + \gamma_i\right), (i = 1, 2, 3)$, where the Gaussian codebook is assumed. Then the theoretical calculation of the outage probabilities $P_1$, $P_2$, and $P_3$ can be mathematically expressed as

$$P_1 = Pr\{0 < R_1 < H(b_1), 0 < R_2 < H(b_1, b_2), R_3 \geq 1\}$$

$$= \frac{1}{\Gamma_1} \exp\left(-\frac{1}{\Gamma_3}\right) \int_0^{\gamma_1} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \left[ 1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_2)} - 1}{\Gamma_2}\right) \right] d\gamma_1,$$

$$P_2 = Pr\{0 < R_1 < H(b_1 | b_2), R_2 > 0, 0 \leq R_3 < 1\}$$

$$= \frac{1}{\Gamma_3} \int_0^{\gamma_3} \exp\left(-\frac{\gamma_3}{\Gamma_3}\right) \left[ 1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_3)} - 1}{\Gamma_1}\right) \right] d\gamma_3,$$

$$P_3 = Pr\{0 < R_2 < H(b_2 | b_1), R_1 > 0, 0 \leq R_3 < 1\}$$

$$= \frac{1}{\Gamma_1} \int_0^{\gamma_1} \exp\left(-\frac{\gamma_1}{\Gamma_1}\right) \left[ 1 - \exp\left(-\frac{2^{1-\log_2(1+\gamma_3)} - 1}{\Gamma_2}\right) \right] d\gamma_1,$$
and
\[ P_3 = \Pr[H(b_1 | b_2) < R_1 < H(b_1)] = \frac{1}{\Gamma_1} \int_0^1 \frac{1}{\Gamma_3} \int_{\gamma_3=0}^{\gamma_3=2^{1-\log_2(1+\gamma_3)-1-\gamma_3}} \exp \left( -\frac{\gamma_1}{\Gamma_1} \right) \exp \left( -\frac{\gamma_3}{\Gamma_2} \right) \left[ 1 - \exp \left( -\frac{2^{2-\log_2(1+\gamma_3)-\log_2(1+\gamma_3)} - 1}{\Gamma_2} \right) \right] d\gamma_1 d\gamma_3. \] (6)

The values of \( P_1, P_2 \) and \( P_3 \) can be calculated by numerical method [14], with accurate enough numerical calculation error control.

IV. OPTIMAL PA AND RP TO MINIMIZE THE OUTAGE PROBABILITY

The goal of this section is to minimize the outage probability obtained at previous section, while the total transmit power \( E_T \) is fixed. By invoking the property of exponential function \( e^{-x} \approx 1 - x \) for small \( x \), at the high SNR regime, the outage probability \( P_{\text{out}} \) can be approximated by a closed-form expression, as,
\[ P_{\text{out}} \approx \frac{2 \ln 2 - 1}{\Gamma_1 \Gamma_2} + \frac{2 \ln 2 - 1}{\Gamma_1 \Gamma_2} + \frac{2 \ln 2 - 2 \ln 2 - 2}{\Gamma_1 \Gamma_2 \Gamma_3}. \] (7)

With the noise variance of each channel being normalized to the unity, the transmit power, which is equivalent to their corresponding average SNR, allocated to S and R are denoted as \( E_T k \) and \( E_T (1 - k) \), respectively. \( k \) (0 < \( k < 1 \)) is the power allocation ratio. Since \( \Gamma_1 = E_T k G_1, \Gamma_2 = E_T (1 - k) G_2, \) and \( \Gamma_3 = E_T k G_3, P_{\text{out}} \) can be written as
\[ P_{\text{out}} \approx A \left( k (1 - k) \right)^{-1} + A \left( \frac{k^2}{1 - d} \right)^{-1} + B \left( \frac{k^2}{1 - d} \right)^{-1}, \] (8)

where \( A = \frac{2 \ln 2 - 1}{E_T^2}, B = \frac{2 \ln 2 - 1}{E_T^2}. \)

The outage probability curves obtained by using the approximated expression (8) and by numerically calculating the (4)-(6) are presented in Figs. 3 and 4 with fixed \( k \) and fixed \( d \), respectively. A good match is observed between the curves of approximate and numerically calculated exact outage probability, with which indicates the usefulness of the approximation.

A. Optimal PA Ratio: RP Fixed

In this subsection, we minimize the outage probability by adjusting PA between S and R while keep the RP fixed. The optimization problem can be formulated as
\[ k^* = \arg \min_k P_{\text{out}}(k) \] subject to: \( k - 1 < 0, \quad -k < 0, \quad -E_T < 0. \) (9)

By taking second-order partial derivative of \( P_{\text{out}} \) with respect to \( k \), we can get
\[ \frac{\partial^2 P_{\text{out}}}{\partial k^2} = \frac{A d^\alpha (2 k^2 - 6 k + 2) - 6 A (1 - d)^\alpha}{k^3 (1 - k)^3} \frac{4}{k^4} + \frac{B d^\alpha (1 - d)^\alpha (12 k^2 - 16 k + 6)}{k^4 (1 - k)^3}. \] (10)

\[ \frac{\partial^2 P_{\text{out}}}{\partial k^2} \] is obviously positive in the range \( k \in (0, 1) \). This indicates that the objective function is convex with respect to \( k \) in (0, 1). Hence, taking the first-order derivative of \( P_{\text{out}} \) with respect to \( k \) and setting the derivative result equal to zero,
\[ \frac{\partial P_{\text{out}}}{\partial k} = \frac{4 A d^\alpha (2 k - 1) - 2 A (1 - d)^\alpha}{k (k - 1)^2} \frac{4}{k^3} + \frac{B d^\alpha (1 - d)^\alpha (3 k - 2)}{k^3 (k - 2)^2} = 0, \] (11)

the optimal PA ratio \( k^* \) can be obtained by solving (11) as:
\[ k^* = \frac{-4 A d' + 3 C'}{2 A' - 2 B'}, \] (12)
respectively. It is clearly seen that the smaller the value (a shorter R-D distance), the more transmit power should be allocated to S. We found that the optimal $k^*$ is almost the same with different total transmit power $E_T$.

**B. Optimal RP: PA Ratio Fixed**

In this subsection, we investigate the optimal RP, given a fixed PA ratio. The problem can be formulated as

$$d^* = \arg \min_d P_{\text{out}}(d)$$

subject to: $d - 1 < 0$, $-d < 0$, $-E_T < 0$. (13)

Taking the second-order derivative of $P_{\text{out}}$ with respect to $d$, we have

$$\frac{\partial^2 P_{\text{out}}}{\partial d^2} = A'\alpha(\alpha + 1) d^{(\alpha+2)} + 2A'\alpha d^{(\alpha+1)}$$

$$+ B''2\alpha(1-d)^{(\alpha+1)} + 2B''\alpha(1+(1-d)^{\alpha+2})$$

$$+ C''' \left( \frac{2\alpha}{d(1-d)} + \frac{2\alpha}{d^3(1-d)^2} + \frac{2\alpha}{d^3(1-d)^2} \right) (d(1-d))^{-(\alpha-2)}$$

where $A' = \frac{2\ln 2-2}{E_T^2 k(1-k)}$, $B'' = \frac{2\ln 2-1}{E_T^2 k^2}$, and $C''' = \frac{2\ln 2-1}{E_T^2 k^2(1-k)}$.

Again, it is obvious that $\frac{\partial^2 P_{\text{out}}}{\partial d^2} > 0$ in $d \in (0, 1)$, which proves the convexity of (14) with respect to $d$. Hence, taking the first-order derivative of $P_{\text{out}}$ with respect to $d$ and setting the derivative result equal to zero,

$$\frac{\partial P_{\text{out}}}{\partial d} = A'\alpha d^{(\alpha+1)} + B''\alpha(1-d)^{(\alpha+1)}$$

$$+ C''' \alpha(1-d)^{(\alpha+1)}(1-2d) = 0.$$ (15)

The optimal RP $d^*$ can be obtained by solving (15). It is excessively complex to derive the explicit expression for $d^*$. However, iterative root-finding algorithm is used to numerically calculate the solution to (15) efficiently with high enough accuracy.

Table II shows the optimal RP $d^*$ for outage probability with different PA ratio $k$, when $E_T$ is 16 dB and 20 dB, respectively. It is clearly seen that the smaller the $k$ value

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<th>$k$</th>
<th>optimal $d^*$ ($E_T = 16$ dB)</th>
<th>optimal $d^*$ ($E_T = 20$ dB)</th>
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<td>0.9</td>
<td>0.2949</td>
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<td>0.8</td>
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<td>0.5</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5401</td>
<td>0.5401</td>
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<td>0.3</td>
<td>0.5833</td>
<td>0.5833</td>
</tr>
<tr>
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<td>0.6342</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7051</td>
<td>0.7051</td>
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</table>

(less transmit power allocated to S), R should be located the point closer to S in order to achieve a lower outage probability. It is also found from Table II that, the optimal $d^*$ are almost the same with different total transmit power $E_T$. These observations indicate that the optimal $k^*$ and optimal $d^*$ are effective for any $E_T$.

**C. Joint Optimal RP and PA Ratio**

For joint optimizing the PA ratio and RP, the problem can be formulated as,

$$k^*, d^* = \arg \min_{k,d} P_{\text{out}}(k,d)$$

subject to: $d - 1 < 0$, $-d < 0$, $k - 1 < 0$, $-k < 0$, $-E_T < 0$. (16)

The Hessian matrix of $P_{\text{out}}$ is shown to be positive definite in Appendix. Hence the convexity of the approximated outage probability expression (8) is proved. By setting the first-order partial derivatives with respect to $k$ and $d$ to zero, respectively, the joint optimal PA ratio and RP can be obtained by jointly solving (11) and (15). Similarly, the iterative root-finding algorithm is used to numerically calculate $k^*$ and $d^*$.

**V. NUMERICAL RESULTS**

In this section, the outage performances of 1) fixed PA & RP 2) semi-adaptive algorithms: optimization of PA with fixed RP and optimization of RP with fixed PA, respectively; 3) and joint
optimization of PA and RP algorithms, are demonstrated. The path loss factor $\alpha = 3.52$ is used [15].

Fig. 5 presents the theoretical outage probabilities with equal and optimal PAs, when the total transmit power $E_T = 16$ dB and $E_T = 20$ dB, respectively. It can be found that the outage probability curves are symmetric to the midpoint between S and D with equal fixed PA ($k = 0.5$). On the other hand, with the optimal PA ratio $k$, the outage probability can be significantly reduced compared to that with equal PA if the relay is allocated close to the D (a smaller $d$). While if relay is near to the S (a larger $d$), outage performances are almost same. This indicates that optimal PA scheme enables the system to find/set proper relays for cooperation in a further place for achieving better outage performance.

Fig. 6 compares the outage performance of the LF relaying with midpoint and with optimal RPs. The $E_T$ is set to 16 dB and 20 dB as a parameter. It can be obviously seen from Fig. 6 that at the near equal PA region ($k$ is close to 0.5) , RP does not make impact on the outage performance. However, a lower outage probability can be yielded by choosing the optimal RP if S and R have unbalanced PA ratio ($k > 0.5$ or $k << 0.5$).

Fig. 7 presents the outage comparison between the 1) fixed PA&RP ($d = 0.3, k = 0.3$), 2) optimal PA ratio (with $d = 0.3$), 3) optimal RP (with $k = 0.3$), and 4) joint optimal RP and PA ratio. It is found that the optimal RP is significantly superior to fixed PA&RP algorithm. Furthermore, choosing the optimal PA ratio further reduces the outage probability. When the PA ratio and RP are jointly optimized, the outage performance can further be improved.

Fig. 8 plots the PA ratio ($k$) versus the optimal RP ($d^*$), as well as $d$ with $k^*$. The curves with different $E_T$ show almost the same tendency as shown in Tables I and II, therefore we just plot only for $E_T = 16$ dB. We can see that the smaller the $d$ value (R moves closer to D), the larger the optimal ratio $k^*$ (more power should be allocated to S). This is because that with the shorter R-D link, the outage performance is more dependent on the S-R link. In this case, allocating more power to the source node will improve the outage performance. On contrary, we can observe that the larger the ratio $k$ (more transmit power allocated to S), the smaller the $d^*$ value (R should be located close to D). This is because that with the less R transmit power, the R-D link has more impact on the outage performance. Fig. 8 also shows the joint optimal PA ratio and RP point with total power constraint. The joint optimal solution is the point where the contributions of the optimal PA ratio and optimal RP are balanced. It also can be seen that, with the certain RP $d$, joint optimal PA is not only the superior scheme in the sense that it can achieve a lower outage probability, but also in the sense that it can reduce the power consumption of S (a smaller $k$), which has great practical benefit.

VI. Conclusion

We have investigated the optimal schemes for a LF relaying to minimize outage probability. First, we optimized PA with fixed RP under the total transmit power constraint. Then, we derived the optimal RP with fixed PA. Additionally, we jointly optimized the PA and RP, and made comparisons between different optimization schemes. The optimization problem has been formulated by the convex optimization framework. The analytical results show that, the adaptive RP scheme outperforms the fixed RP&PA scheme in terms of outage performance. A lower outage probability can be achieved by adjusting the power allocated to S and R. It has been also shown that joint optimal PA and RP scheme further reduce the outage probability compared to fixed and semi-adaptive optimal schemes.

Appendix

The Hessian matrix of (8) can be calculated as

$$H(P_{\text{out}}) = \begin{bmatrix}
\frac{\partial^2 P_{\text{out}}}{\partial k^2} & \frac{\partial^2 P_{\text{out}}}{\partial k^3} & \frac{\partial^2 P_{\text{out}}}{\partial k^4} \\
\frac{\partial^2 P_{\text{out}}}{\partial k^3} & \frac{\partial^2 P_{\text{out}}}{\partial k^4} & \frac{\partial^2 P_{\text{out}}}{\partial k^5} \\
\frac{\partial^2 P_{\text{out}}}{\partial k^4} & \frac{\partial^2 P_{\text{out}}}{\partial k^5} & \frac{\partial^2 P_{\text{out}}}{\partial k^6}
\end{bmatrix}.$$ (17)

$$\frac{\partial^2 P_{\text{out}}}{\partial k^2} = \frac{Ad^n(6k^2 - 6k + 2)}{k^3(1-k)^3} + \frac{6A(1-d)^n}{k^4} + \frac{Bd^n(1-d)^n(12k^2 - 16k + 6)}{k^4(1-k)^3},$$ (18)
\[
\frac{\partial^2 P_{\text{out}}}{\partial d \partial d} = \frac{\partial^2 P_{\text{out}}}{\partial d \partial d} = \\
= \frac{A(2k-1)\alpha d^{(\alpha+1)}}{k(k-1)^2d^2} + \frac{2A\alpha(d-1)^{(\alpha+1)}}{k^3(d-1)^2} \\
+ \frac{B(3k-2)\alpha d((1-d)^{(\alpha+1)})(1-2d)}{k^3(k-2)^2d^2(d-1)^2}, \quad (19)
\]
where
\[
A = \frac{2\ln 2 - 1}{E_p^2}, \quad B = \frac{2\ln 2 - 1}{E_p^2}.
\]

\[
\frac{\partial^2 P_{\text{out}}}{\partial d^2} = \frac{A''\alpha(\alpha + 1)d^{(\alpha+2)}}{d^3} + \frac{2A''\alpha d^{(\alpha+1)}}{d^3} \\
+ \frac{B''(1-d)^{(\alpha+1)}}{(1-d)^3} + \frac{B''\alpha(\alpha + 1)(1-d)^{\alpha+2}}{(1-d)^4} \\
+ \frac{C''}{(d(1-d))^{\alpha+1}} + \frac{\alpha(1)}{d(1-d)^3} + \frac{\alpha(1)}{(d(1-d))^{\alpha+2}} \quad (20)
\]
where \(A'' = \frac{2\ln 2 - 1}{E_p^{2\alpha}(1-\alpha)}, \quad B'' = \frac{2\ln 2 - 1}{E_p^{2\alpha}(1-\alpha)}, \quad \text{and} \quad C'' = \frac{2\ln 2 - 1}{E_p^{2\alpha}(1-\alpha)}.\)

The determinant of \(\mathbf{H}(P_{\text{out}})\) is given by
\[
\mathbf{H}(P_{\text{out}}) = \begin{pmatrix}
A d^{3}(6k^2 - 2k + 2) + 6A(1-d)^{\alpha} \\
Bd^{3}(1-d)^{(\alpha+1)}(12k^2 - 16k + 6) + 2A''\alpha d^{(\alpha+2)} \\
2A''\alpha d^{(\alpha+2)} + B''\alpha(\alpha + 1)(1-d)^{\alpha+2} \\
C''(\alpha(1)) + \alpha(1) \left(\frac{1}{(d(1-d))^{\alpha+1}} + \frac{1}{d(1-d)^3} + \frac{1}{(d(1-d))^{\alpha+2}}\right)^2 \\
\end{pmatrix}
\]
(21)

It can be seen that for \(k \in (0, 1)\) and \(d \in (0, 1)\), (21) is non-negative. Therefore, the outage probability expression (8) can be proven to be convex.

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