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[Image of JAIST logo]
Achieving Accurate Geo-location Detection Using Joint RSS-DOA Factor Graph Technique

Muhammad Reza Kahar Aziz, Shofiyati Nur Karimah, Niimi Yoshio, Khoirul Anwar, and Tad Matsumoto

Abstract—This paper proposes a detection technique based on factor graph (FG) to estimate the position of radio wave emitter. To obtain accurate estimation, we combine received signal strength (RSS) and direction of arrival (DOA) schemes into a single factor graph, called joint RSS-DOA, where soft information as mean and variance of the estimated target position are exchanged between the two schemes. The performance of DOA in this paper is used to modify variance approximation of the target location. We introduce the weighting factors for RSS and DOA to avoid the soft information of DOA factor graph be ignored. With the proposed technique, the complexity is kept low, because only mean and variance are exchanged between the factor nodes. Ray-tracing data is used in outdoor application to create power delay profile (PDP) for the RSS-based factor graph and evaluate 5, 10, and 20 training points. The results confirmed that proposed joint RSS-DOA has best accuracy in detection compared to RSS-based or DOA-based only factor graph. To the best of our knowledge, we are the first showing successful results of FG based geo-location using Ray-tracing data.

I. INTRODUCTION

Wireless geo-location has been recognized as a key of technology with significant importance for recent and future location based service applications, e.g., location-sensitive billing, Emergency 911, smart transportation systems, vehicle navigation, fraud detection, people tracking, and public safety systems [1]–[3]. One of geo-location techniques exploits the advantage of factor graph, which global function factors into products of local functions, to reduce the complexity. Factor graph consist of two main nodes, i.e., factor nodes and variable nodes. Factor node contains function to treat soft information from some variable nodes. In this paper, factor node is represented by the black box, while variable node is by white circle, as shown in Fig. 1. Variable node multiplies all soft information if more than two factor nodes are connected. However, it only pass the soft information if there are only two factor nodes [4].

The input of factor graph for geo-location technique can be anything that can be converted to a geo-location coordinate \((x, y)\). It can be Received Signal Strength (RSS) [5], Direction of Arrival (DOA) [6], [7], Time of Arrival (TOA) [8], [9], and Time Difference Of Arrival (TDOA) [10]. However, TOA and TDOA need perfect synchronisation since an error of 1 \(\mu s\) leads to error around 300 meters [8]. TOA parameter is also difficult to estimate an unknown transmitter, since the time stamp is also unknown.

In this paper, to achieve high accuracy we consider a joint RSS-DOA schemes exploiting factor graph. The joint RSS-DOA technique does not require perfect synchronisation. Another advantage of the RSS-based technique is its low complexity, because RSS-based measurement does not need additional hardware and software infrastructures. In addition, the functionality for measuring RSS is already imposed in IEEE 802.11 [5]. However, we need additional hardware and software infrastructures. In addition, the functionality for measuring RSS is already imposed in IEEE 802.11 [5].

To increase the accuracy of DOA measurement, we propose iterative detection technique for joint use RSS and DOA with factor graph. We found that with the proposed technique, the accuracy of the position identification is significantly improved over RSS-only and/or DOA-only techniques without requiring prohibitive heavy hardware for each sensor. The computational complexity at the fusion center is also very low, because the messages to be exchanged between the factor and variable nodes are simply means and variances of the measurement data.

A. Related Works

Refs. [8], [9] propose a technique to estimate the target location identification using TOA-based factor graph where the sum-product algorithm is performed, while [6] uses both TOA and DOA\(^1\) for message passing between the factor and variable nodes. Each sensor performs multiple measurements on TOA and DOA and calculates the mean and variance empirically. The message to be exchanged are mean and variance of those parameters, which is assumed to be Gaussian distributed. In DOA factor graph, it estimates the distances from each sensors to the target by utilizing the tangent function.

In [8], it is stated that its original version of the algorithm presented in [9] does not take into account in TOA measurement error included in each sensor measurement data, \(^1\)In this paper, we use the term of DOA instead of Angle of Arrival (AOA) for better expression.
when converting the parameters into the distance. We noticed that in technique presented in [9] is also used in [6], where parameter TOA and DOA are used. Hence, this paper aims to reformulate the relationship between the measurement and the distance, as noted above, in the DOA-based technique. Ref. [7] aims to revise the mean and variance of the distance in \((x, y)\)-coordinate taking into account the relationship in the DOA-based location identification. However, the equations provided in Table I of [7] are still unclear (argument of the distance \(r_i\) is not an angle \(\theta\)). Therefore, we use variance of tangent function derived by using Taylor series approximation in DOA-based factor graph following [12].

Authors of [5] propose a geo-location technique using RSS-based factor graph. They transmit training sequence from several representative points in the region, referred to as monitoring spots, to obtain received signal strength in watt and then convert it to dB. The reference data obtained through the training is then used to produce RSS - information surface of Power Delay Profile (PDP) by interpolation, which local linearity is exploited in estimating the target location. With the STD values of the measurement data being \(2 \times 10^{-6} - 7 \times 10^{-6}\) watts. Ref. [5] shows that the location identification accuracy of 0.6 – 0.9 m, can be achieved in indoor scenario with size of 100 m².

Authors in [13] develop TOA-based factor graph for mobile station positioning in non-line-of-sight environment. They apply the maximum error value of distance mean 1,000 meters in uniform distribution to be convoluted by Gaussian distribution, which variances \((\sigma^2)\) between 20 and 70 dB, as non LOS (NLOS) measurement error part. The factor node in factor graph uses probability density function (pdf) which is derived by probability LOS and probability NLOS.

Furthermore, we have detailed some other wireless geo-location techniques for detecting unknown wave emitter in [14] utilizing several measurement data including Difference Received Signal Strength (DRSS), Time Difference of Arrival (TDOA), and join RSS and Voronoi.

**B. Contributions**

Our contributions are as follows: 1) We jointly combine RSS and DOA based factor graph with weighting factor to a single in factor graph to get the good accuracy for any channel conditions. 2) We use Ray-tracing collected by real experiment in Shinjuku area as shown in Fig. 2 for outdoor application to create PDP profile for the RSS-based factor graph and evaluate the performances using 5, 10, and 20 training points.

The joint RSS-DOA scenario, as shown in Fig. 1, combines two independent RSS and DOA factor graph, where the final variable nodes \((x, y)\) is the target position. We also propose weighting factors \(w_p\) and \(w_\theta\), for RSS and for DOA, respectively.

**II. Factor Graph Model of Joint RSS-DOA**

We explain briefly the factor graph algorithm from independent each DOA and RSS as our basic step to develop the technique.
DOA (With the modified function)

The modified function of DOA technique is basically based on our previous work in [12] and [14]. Target position \((x, y)\) has connection to the direction (angle) of arrival from target transmitted signal to sensors by using tangent and cotangent operations with the range between target position and sensor position \((X_i, Y_i)\).

DOA-based factor graph receive angle or direction of received signal from target in \(\theta_i\). As shown in Fig. 1, factor nodes \(D_1\) introduce the measurement error \((\sigma_{\theta_i})\) to get Gaussian distribution from number of angle measurement samples. The soft information of \(\theta_i\) that is in mean \((m_{\theta_i})\) and variance \((\sigma_{\theta_i}^2)\) of Gaussian distribution will be processed in factor nodes \(C_i\) to obtain soft information of range between target position and sensor position \((\Delta x_{\theta_i}, \Delta y_{\theta_i})\) in term of mean \((m_{\Delta x_{\theta_i}}, m_{\Delta y_{\theta_i}})\) and variance \((\sigma_{\Delta x_{\theta_i}}^2, \sigma_{\Delta y_{\theta_i}}^2)\) [6].

In [12] we derive the new variance formula in factor node \(C_i\) because we need to revise the variance formula in [6] according to [8]. The mean \((m_{f(\theta)})\) and variance \((\sigma_{f(\theta)}^2)\) of first order Taylor series for tangent function approximation is applied to derive new variance formula \((\sigma_{\Delta x_{\theta_i}}^2, \sigma_{\Delta y_{\theta_i}}^2)\) from DOA in main factor node \(C_i\), as expressed below

\[
m_{f(\theta)} = f(m_{\theta}), \quad m_{\tan(\theta_i)} = \tan(m_{\theta_i}), \quad m_{\cot(\theta_i)} = \cot(m_{\theta_i}),
\]

\[
\sigma_{f(\theta)}^2 = (f'(m_{\theta}))^2 \sigma_{\theta_i}^2, \quad \sigma_{\tan(\theta_i)}^2 = \sec^4(m_{\theta_i}) \sigma_{\theta_i}^2, \quad \sigma_{\cot(\theta_i)}^2 = \csc^4(m_{\theta_i}) \sigma_{\theta_i}^2.
\]

where \(f'(m_{\theta})\) is the first derivative. \(f(\theta)\) values are either \(\tan(\theta)\) or \(\cot(\theta)\). Then we get

\[
m_{\tan(\theta_i)} = \tan(m_{\theta_i}), \quad m_{\cot(\theta_i)} = \cot(m_{\theta_i}),
\]

\[
\sigma_{\tan(\theta_i)}^2 = \sec^4(m_{\theta_i}) \sigma_{\theta_i}^2, \quad \sigma_{\cot(\theta_i)}^2 = \csc^4(m_{\theta_i}) \sigma_{\theta_i}^2.
\]

The main factor node \(C_i\) for each sensor is contained a function as follow

\[
\Delta y_{\theta_i} = \Delta x_{\theta_i} \cdot \tan \theta_i,
\]

where \((\Delta y_{\theta_i}, \Delta x_{\theta_i})\) is the distance between sensor position \((X_i, Y_i)\) with target position \((x, y)\). We get the value of \(\Delta x_{\theta_i}\) by using the opposite manner with cotangent function. In this case, we obtain the soft information, \(m_{\Delta y_{\theta_i}} = \Delta x_{\theta_i}\), where \(m_{\Delta y_{\theta_i}}\) is the mean of \(\Delta x_{\theta_i}\) Gaussian distribution, then we apply the same manner to get \(m_{\Delta x_{\theta_i}}\). The variance for product of two independent variables is expressed as below

\[
\text{Var}(xy) = m_x^2 \sigma_y^2 + m_y^2 \sigma_x^2 + \sigma_x^2 \sigma_y^2.
\]

Then we apply equation (8) to equation (7) to obtain the variance \((\sigma_{\Delta x_{\theta_i}}^2, \sigma_{\Delta y_{\theta_i}}^2)\) as below

\[
\sigma_{\Delta y_{\theta_i}}^2 = m_{\Delta x_{\theta_i}}^2 \sec^4(m_{\theta_i}) \sigma_{\theta_i}^2 + \sigma_{\Delta x_{\theta_i}}^2 \tan^2(m_{\theta_i}) + \sigma_{\Delta x_{\theta_i}}^2 \sec^4(m_{\theta_i}) \sigma_{\theta_i}^2,
\]

\[
\sigma_{\Delta x_{\theta_i}}^2 = m_{\Delta y_{\theta_i}}^2 \csc^4(m_{\theta_i}) \sigma_{\theta_i}^2 + \sigma_{\Delta y_{\theta_i}}^2 \cot^2(m_{\theta_i}) + \sigma_{\Delta y_{\theta_i}}^2 \csc^4(m_{\theta_i}) \sigma_{\theta_i}^2.
\]

factor node \(A_i\) and \(B_i\) produce soft information for variable node \((x, y)\) in term mean \((m_{x_{\Delta x_i}}, m_{y_{\Delta y_i}})\) and variance \((\sigma_{x_{\Delta x_i}}^2, \sigma_{y_{\Delta y_i}}^2)\) simply by subtracting the position of sensors \((X_i, Y_i)\) with mean \((m_{\Delta x_{\theta_i}}, m_{\Delta y_{\theta_i}})\). Then the variance \((\sigma_{x_{\Delta x_i}}^2, \sigma_{y_{\Delta y_i}}^2)\) is passed through because variance does not change while a constant is added. The mean and variance equations in factor node \(A_i\) and \(B_i\) are reversible.

These soft information in mean \((m_{\Delta x_{\theta_i}}, m_{\Delta y_{\theta_i}}, m_{x_{\Delta x_i}}, m_{y_{\Delta y_i}})\) and variance \((\sigma_{\Delta x_{\theta_i}}^2, \sigma_{\Delta y_{\theta_i}}^2, \sigma_{x_{\Delta x_i}}^2, \sigma_{y_{\Delta y_i}}^2)\) are exchanged in factor graph via iteration in factor graph algorithm until it reaches the convergence point.

The soft information in mean within factor nodes \(A_i\), \(B_i\), \(C_i\) and variable nodes \(x, y, \Delta x_i, \Delta y_i, \theta_i\) are still kept as in [6], however we adopt the variance function of factor nodes \(A_i\) and \(B_i\) as in [8]. For detail derivation of equation (1) to equation (11) to obtain variance formula used in factor node \(C_i\) can be found in [12].

B. RSS

With RSS position of target \((x, y)\) can be obtained via the relationship between the received signal power and PDP obtained from training data. The soft information \(x\) and \(y\) are exchanged between variable and factor nodes via iterations. RSS is the cheapest solution because it only needs received signal power. This RSS-based factor graph technique is applied for indoor application in [5]. However, for outdoor detection,
RSS may need many training points. In this paper, we use Ray-tracing data in outdoor application with total area $10^6$ m$^2$ for RSS data from training point dan target.

Factor graph of RSS is simpler than DOA since we do not need factor nodes $A_i$ and $B_i$ of DOA as shown in Fig. 1. Therefore, factor nodes $C_i$ directly produce target position variable node $x$ and $y$. First, we need training data to make PDP, where the target should be in the area of PDP. With five training points and three sensors, the plane equation for $i$-th sensors is expressed as

$$\alpha_{x,i} \cdot x + \alpha_{y,i} \cdot y + a_{p,i} \cdot \tilde{p}_{i,j} = c, \quad j = 1, 2, 3, 4, 5. \quad (11)$$

From this equation, we get matrix

$$B \cdot A = C,$$  

(12)

where $B$ is matrix of $x_j, y_j$, and $\tilde{p}_{i,j}$, $A$ is vector of $\alpha_{x,i}, \alpha_{y,i}$, and $a_{p,i}$, and $C$ is vector of constant. Using least square (LS), we obtain

$$A = (B^T \cdot B)^{-1} \cdot B^T \cdot C. \quad (13)$$

Finally, the relationship between target position $(x, y)$ and the power measurement $p_{i,t}$ at $i$-th sensor is expressed in

$$a_{x,i} \cdot x + a_{y,i} \cdot y + a_{p,i} \cdot p_{i,t} = c. \quad (14)$$

Then we can re-write equation (14) as below

$$y = \frac{c}{a_{y,i}} + \frac{a_{p,i}}{a_{y,i}} \cdot x - \frac{a_{p,i}}{a_{y,i}} \cdot p_{i,t} = \alpha_{y,i} + \beta_{y,i} \cdot x + \gamma_{y,i} \cdot p_{i,t} \quad (15)$$

$$x = \frac{c}{a_{x,i}} + \frac{a_{p,i}}{a_{x,i}} \cdot x - \frac{a_{p,i}}{a_{x,i}} \cdot p_{i,t} = \alpha_{x,i} + \beta_{x,i} \cdot y + \gamma_{x,i} \cdot p_{i,t}. \quad (16)$$

Therefore, we get the mean $(m_{y_{p,i}}, m_{x_{p,i}})$ and variance $(\sigma_{y_{p,i}}^2, \sigma_{x_{p,i}}^2)$ as below

$$m_{y_{p,i}} = \alpha_{y,i} + \beta_{y,i} \cdot m_{x_{p,i}} + \gamma_{y,i} \cdot m_{p,i} \quad (17)$$

$$m_{x_{p,i}} = \alpha_{x,i} + \beta_{x,i} \cdot m_{y_{p,i}} + \gamma_{x,i} \cdot m_{p,i} \quad (18)$$

$$\sigma_{y_{p,i}}^2 = \sigma_{p,i}^2 + \gamma_{y,i} \cdot \sigma_{x_{p,i}}^2 + \gamma_{y,i}^2 \cdot \sigma_{p,i}^2 \quad (19)$$

$$\sigma_{x_{p,i}}^2 = \sigma_{p,i}^2 + \gamma_{x,i} \cdot \sigma_{y_{p,i}}^2 + \gamma_{x,i}^2 \cdot \sigma_{p,i}^2 \quad (20)$$

The soft information of variable nodes $x$ and $y$ are exchanged between sensors until they are convergence to get estimated target position $(x, y)$. The entire function of mean and variance of factor nodes and variable nodes can be found in [5] and [14].

C. Joint RSS-DOA

Here we get the value of variable nodes $(x, y)$ from factor nodes $C_{pi}$, of RSS-based factor graph, $A_{0i}$ and $B_{0i}$ of DOA-based factor graph, respectively. We product all soft information, assumed the Gaussian distribution is independent, iteratively coming from all factor nodes $j$ in all sensors except soft information towards one factor node $i$

$$\prod_{j=1, j\neq i}^{J} N(x, m_{j,i}, \sigma_{x_{j,i}}^2) \propto N(x, m_{\lambda_{x,i}}, \sigma_{x_{\lambda,i}}^2), \quad (21)$$

if the variables are statistically independent. We can rewrite equation (21) with introduce the weighting factor $w_p$ for RSS part and $w_{\theta}$ for DOA part as below

$$\frac{1}{\sigma_{x_{\lambda,i}}^2} = \sum_{j=1}^{J} \frac{w_{p,j}}{\sigma_{x_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}}{\sigma_{x_{\theta,j}}^2}, \quad (22)$$

$$\frac{1}{\sigma_{x_{\lambda,i}}^2} = \sum_{j=1}^{J} \frac{w_{p,j}}{\sigma_{x_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}}{\sigma_{x_{\theta,j}}^2}, \quad (23)$$

$$m_{x_{\lambda,i}} = \sigma_{x_{\lambda,i}}^2 \left( \sum_{j=1}^{J} \frac{w_{p,j}m_{x_{p,j}}}{\sigma_{x_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}m_{x_{\theta,j}}}{\sigma_{x_{\theta,j}}^2} \right), \quad (24)$$

$$m_{x_{\lambda,i}} = \sigma_{x_{\lambda,i}}^2 \left( \sum_{j=1}^{J} \frac{w_{p,j}m_{x_{p,j}}}{\sigma_{x_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}m_{x_{\theta,j}}}{\sigma_{x_{\theta,j}}^2} \right), \quad (25)$$

where $w_p$ and $w_{\theta}$ are weighting factors determined empirically in this paper. $\sigma_{\lambda}$ and $m_{\lambda}$ is joint RSS-DOA variance and mean, respectively. We need to add weighting factor because the variance from DOA factor graph is too large compare to variance from RSS. If the weighting factor is removed, then soft information from DOA factor graph is ignored.

When the iteration converges, $j = i$ is included in (21). Since the final value, $m_{\lambda_{x}}$ and $m_{\lambda_{y}}$ are taken as the value of estimated target position $(x, y)$ as below

$$\frac{1}{\sigma_{y_{\lambda}}^2} = \sum_{j=1}^{J} \frac{w_{p,j}}{\sigma_{y_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}}{\sigma_{y_{\theta,j}}^2}, \quad (26)$$

$$\frac{1}{\sigma_{y_{\lambda}}^2} = \sum_{j=1}^{J} \frac{w_{p,j}}{\sigma_{y_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}}{\sigma_{y_{\theta,j}}^2}, \quad (27)$$

$$x = m_{\lambda_{x}} = \sigma_{x_{\lambda}}^2 \left( \sum_{j=1}^{J} \frac{w_{p,j}m_{x_{p,j}}}{\sigma_{x_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}m_{x_{\theta,j}}}{\sigma_{x_{\theta,j}}^2} \right), \quad (28)$$

$$y = m_{\lambda_{y}} = \sigma_{y_{\lambda}}^2 \left( \sum_{j=1}^{J} \frac{w_{p,j}m_{y_{p,j}}}{\sigma_{y_{p,j}}^2} + \sum_{j=1, j\neq i}^{J} \frac{w_{\theta,j}m_{y_{\theta,j}}}{\sigma_{y_{\theta,j}}^2} \right). \quad (29)$$

D. Ray-tracing Data and Measurement Error

Ray-tracing data has transmitter as target and reference point. There are 25 paths signal received in one antenna in sensors. Total antenna in one sensor is 3 antennas. Each path contains phase ($\phi$), delay ($\tau$) in second, and power ($p$) in dB. Normalization by the highest power of signal is applied because the value of RSS is very small. The detail parameters used for Ray-tracing data collection is shown in Table I.

In this paper, RSS of target from Ray-tracing data is obtained in complex number from total 75 paths of Ray-tracing data. Then we combine all complex numbers as below

$$\tilde{p}_{w,i} = \sum_{i=1}^{n} P_i \cos \phi + jP_i \sin \phi \quad ; n = 75, \quad (30)$$

where $P_i$ is power amplitude of path in watt. This $\tilde{p}_{w,i}$ is corrupted to Gaussian noise as measurement error in number of sample to become as, then convert it to $p_i$ in logarithmic value.
TABLE I
RAY-TRACING PARAMETERS

<table>
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<th>Parameter</th>
<th>Value</th>
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<td>Number of sensor as receiver</td>
<td>8 sensors</td>
</tr>
<tr>
<td>Number of target as transmitter</td>
<td>12 monitors</td>
</tr>
<tr>
<td>Number of reference point as transmitter</td>
<td>1,616 monitors</td>
</tr>
<tr>
<td>Number of antenna per sensor</td>
<td>3 antennas</td>
</tr>
<tr>
<td>Number of path in one pair of transmitter and receiver</td>
<td>25 paths</td>
</tr>
<tr>
<td>Channel Phase</td>
<td>-180°-180°</td>
</tr>
<tr>
<td>DOA azimuth</td>
<td>0°-360°</td>
</tr>
<tr>
<td>DOA altitude</td>
<td>0°-360°</td>
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</tbody>
</table>

We consider two types of error in DOA measurement. If the condition is Line-of-Sight (LOS), the measurement results are corrupted by zero-mean Gaussian noise. However, in Non LOS condition the measurement is corrupted by non-zero mean Gaussian noise [15].

In case of multipath fading, DOA measurement error is further caused by many signals arrived from different directions. On the other hand, flat fading makes high variation in RSS that cost in accuracy in the detection. In [6] and [5], the authors assumed simple model using zero-mean Gaussian noise for DOA and RSS, respectively.

In this paper, DOA soft information \((\theta)\) is created by directly obtain angle from target to sensor \((\theta)\), then it is corrupted with Gaussian noise in number of sample.

III. SIMULATION RESULTS

We evaluate the performance of the proposed technique using computer simulation with normalized RSS Ray-tracing data collected from real experiment in Shinjuku area. The true target location is at \((230, -440)\) m with 5, 10, and 20 training points around the target. We use three sensors located at \((423.5, -517)\) m, \((141, -957)\) m, \((296, -157)\) m, where the initial target is assumed at \((0, 0)\) m. The simulation runs on 100,000 independent trials, which each trial has 100 samples. Total iteration in this simulation is 10 times.

For the weighting factors, we search empirically via computer simulation. We found that the best weighting factor for RSS and DOA is \(w_p = 0.9 \cdot 10^{-6}\) and \(w_\theta = 1 - w_p\), respectively. Fig. 3 shows trajectory of RSS, DOA, and joint RSS-DOA factor graph iteration to get geo-location estimate of target over the map of Shinjuku area.

Linearity equations in RSS-based factor graph are obtained from Ray-tracing data with 20 monitoring points as follow

\[
\begin{bmatrix}
1.7 \cdot 10^{-3} \\
3.62 \cdot 10^{-5} \\
1.4 \cdot 10^{-3}
\end{bmatrix} x + \begin{bmatrix}
-0.8 \cdot 10^{-3} \\
-1.2 \cdot 10^{-3} \\
-1.4 \cdot 10^{-3}
\end{bmatrix} y + \begin{bmatrix}
-0.0199 p_1 \\
-0.0089 p_2 \\
0.0019 p_3
\end{bmatrix} = 1,
\]

(31)

where \(p_1\), \(p_2\), and \(p_3\) is power profile in dB at 1st, 2nd, and 3rd sensor, respectively. RSS of target, \((230, -440)\) m, in each sensor is obtained from Ray-tracing after normalization as follow, \(p_1 = 0.1399\) W, \(p_2 = 9.0514 \cdot 10^{-8}\) W, and \(p_3 = 0.0782\) W. However, this RSS of target from Ray-tracing data is not equal to RSS from power profile of lineiar equation as follow, \(p_1 = 0.0570\) W, \(p_2 = 5.4151 \cdot 10^{-6}\) W, and \(p_3 = 1.7599 \cdot 10^3\) W. Fig. 4 shows MSE performance of RSS-based factor graph for 5, 10, and 20 training points. MSE performance of RSS-based factor graph increases with more training points.

There are two type of simulations. The first simulation employs STD measurement error in fixed power \(1 \cdot 10^{-5}\) watt\(^2\) and in angle from 1° to 45° as shown in Fig. 5. Furthermore, the second simulation employs STD measurement error in fixed angle 30° and in variable power from \(0.7 \cdot 10^{-5}\) to \(1.6 \cdot 10^{-5}\) watt.

It can be observed from Fig. 5 that the proposed joint RSS-DOA technique provides good accuracy MSE performance at DOA measurement error standard deviation above 10°. It is also shown in Fig. 5 that the variance of tangent function of DOA produces accurate detection. Fig. 6 shows that the proposed technique has the best accuracy MSE performance compared with the stand-alone RSS-based and DOA-based factor graph.

From the results discussed above, it is confirmed that the proposed technique contributes the following unique points: a)
Fig. 5. MSE performances of DOA FG, RSS FG, and Joint RSS-DOA FG with fixed STD power at $1 \cdot 10^{-5}$ watt.

Fig. 6. MSE performances of DOA FG, RSS FG, Joint RSS-DOA FG with fixed STD angle at 30°.

High accuracy, b) low computational complexity.

IV. CONCLUSION

We have proposed a joint RSS-DOA factor graph-based technique for geo-location estimation. We combined variance approximation of tangent function of the DOA detection scheme with the RSS detection scheme utilizing Ray-tracing data collection and then add weighting factors in both RSS part and DOA part. From computer simulation, we found that the best weighting factor for RSS and DOA is $w_p = 0.9 \cdot 10^{-6}$ and $w_{\theta} = 1 - w_p$, respectively. The simulation results show the proposed joint RSS-DOA based factor graph scheme provides higher accuracy in detection over the RSS-based and DOA-based only in term of mean square error (MSE) versus noise in degree and MSE versus noise in watt. It also keeps low computational complexity in indoor and outdoor applications due to the mere use of mean and variance.

V. ACKNOWLEDGMENT

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