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Characterization of a d_1-Optimal Motion for a Rod by Decomposing 3-Dimensional Configuration Space

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Recently, remarkable progress of the computer architecture has shortened the running time of programs and increased the available computer resource of the data storage and the memory space. This progress enables us to implement algorithms which did not usually work due to the large order in practical running time and resource for space. However, shortening the running time and increasing the available resource for space are not enough to solve NP-hard problems. Therefore we need to design approximation algorithms to decrease the order of the running time.

One of these NP-hard problems is the "robot motion planning" which is to find an optimal motion avoiding obstacles between given two points. This is an important problem to be solved in a view point of efficient robot motion. Motion planning for a point robot, which has only a location without area, has been well studied relatively. Hence in this paper we focus on motion planning for a rod, which is formally defined as follows:

- Motion planning for a rod:
  - Let $U \subset \mathbf{R}^2$ be a finite space which contains polygonal obstacles.
  - The space U includes an initial location S of the rod and a terminal location T of the rod.

  Find an optimal motion avoiding obstacles from S to T in the space U.

Motion planning for a rod in the 2-dimensional space proved NP-hard. Therefore, we must design an approximation algorithm to solve it by using not an optimal motion but an approximated one. For a rod which has only a length without width, an optimal motion of the rod is dependent on how it contacts obstacles and how often it contacts them. For a point robot which has only a coordinate of a location without area, an optimal motion of the point robot is independent of how it contacts obstacles. Hence, to fine an optimal motion planning for a rod, we need an approximation algorithm different from one for a point robot and need its characterization for a rod.

Here, we establish our terminology to specify a motion for a rod uniquely. Given a rod as a line segment AB, we assume that the rod has a fixed point F called *focus* in the middle of this line segment AB. We use a terminology: a *placement* is a tuple $Z(F(x,y),\theta) \in \mathbf{R}^2 \times \mathbf{R}^1$ where $F(x,y) \in \mathbf{R}^2$ is a coordinate of the *focus* point of the rod in the space U and $\theta \in \mathbf{R}^1$ is an angle of the rod measured counterclockwisely from the positive direction of the x-axis. This rod is allowed to translate and rotate about the *focus*. By using these terminologies, we characterize a motion for a rod.

In the previous methods for motion planning for a rod, motion was characterized using 2-dimensional cell decomposition based on the relative location between the rod and obstacles. This decomposition may miss existence of some states of a rod which must be considered, and it is not proved that any state of a rod in the space U is considered exhaustively. Lately, new characterization of a motion for a rod has been proposed by using a decomposition 3-dimensional configuration space. This configuration space is defined by identifying a placement $Z(F(x,y),\theta) \in \mathbf{R}^2 \times \mathbf{R}^1$ with $\mathcal{Z}(x,y,\theta) \in \mathbf{R}^3$. In this new characterization, a motion for a rod in the space U is identified with that in the 3-D configuration space. Then, using this identification, characterizing a motion for a rod in the 3-D configuration space means characterizing it in the space U. Therefore, by characterizing a motion for a rod in this configuration space, we can obtain a characterization for a rod in the space U. Furthermore, utilizing this new characterization, a new approximation algorithm for a rod is proposed. This new characterization is much more concise compared with the previous one. We aim to design more concise characterization. In this paper, we do not distinguish two endpoints of a rod nor its direction. By doing so, we reduce the complexity of the decomposition of the 3-dimensional configuration space and show that it is possible to characterize a motion for a rod by using this fewer decomposed 3-D configuration space. Finally, we prove that our characterization exhausts all the states of a rod and show its correctness.

Based on the characterization shown above, we are able to identify motion planning for a rod in the 2-dimensions with that for a point in the 3-dimensions. Thus, to solve the motion planning for a rod, we can utilize a number of useful approximation algorithms for NP-hard problems in 3-dimensions which have been well studied. Hence, by using our characterization, we are able to solve motion planning for a rod in the 2-dimensions which is originally NP-hard.