Title	アロー論理における決定可能性に関する研究
Author(s)	森岡，竜司
Citation	
Issue Date	2001-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/1459
Rights	
Description	Supervisor:小野 寛晰，情報科学研究科，修士

On the Decidability of Arrow Logics

Ryuji Morioka

School of Information Science,
Japan Advanced Institute of Science and Technology

February 15, 2001

Keywords: modal logic,arrow logic,decidability.

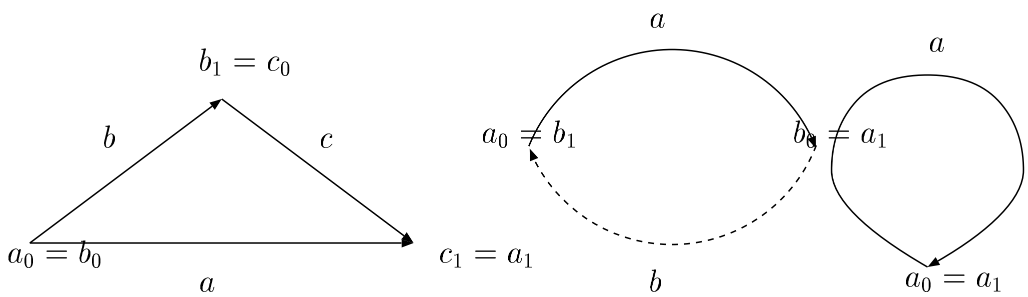
1 Introduction

Arrow logic was first introduced by J.van Benthem in 1990's in [4]. An arrow logic is a logic obtained by adding some modal operater to classical logic and is regarded as a modal logic in which state transitoins can be represented. Kripke-type semantics is introduced for these logics in which formulas are interpreted as "arrows". An arrow is a binary relation which is an abstract form of a state transition. Three basic relations are defined on arrows, i.e.the composition of two arrows, the inverse of a given arrow, and the identity arrow. By using these notions, we can represent dynamic situations of state transitions which can be described neither by classical logic nor standard modal logics.
The purpose of the present paper is to study the decidability of arrow logics. The filtration method doesn't work always when we have the associativity. In fact, the presence of associativity sometimes cause the undecidability. On the other hand, V. Gyuris succeeded to show that arrow logics with the associativity become decidable if we have no "modal distribution". By examining his proof, we can get the decidability of some more arrow logics.

2 Arrow Logic

Arrow frames are used for semantics of arrow logics. An arrow frame is $\mathbf{F} = (\mathbf{W}, \mathbf{C}, \mathbf{R}, \mathbf{I})$, which consists of ;
$\mathbf{W}$: a non empty set, whose elements are called arrows,
$\mathbf{C_{xyz}}$: which is read as "x is a composition of y and z".
$\mathbf{R_{xy}}$: which is read as "x is a reversal of y".
$\mathbf{I_x}$: which is read as "x is an identity arrow".

The figure in the next page shows their intuitive meaning.



These frames are used for semantics of modal language, i.e the language extending the language of the classical logic by adding one binary modality $\varphi \circ \psi$(composition), one unary modality $\otimes\varphi$(reverse) and a modal constant $\iota\delta$. Their interpretation is given as follows:

$x \models \varphi \circ \psi \Leftrightarrow \exists y, z\mathbf{C}_{xyz} y \models \varphi z \models \psi$

$x \models \otimes\varphi \Leftrightarrow \exists y\mathbf{R}_{xy} y \models \varphi$

$x \models \iota\delta \Leftrightarrow \mathbf{I}_x$

J.van Benthem [2] extended with the more operator like Kleene star *, the resulting logic is called Dynamic arrow logic.

The interpretation of Kleene star is as follows :

$x \models \varphi^* \Leftrightarrow x \models \varphi$ or $x \models x_1 \circ \cdots \circ x_n$ and $x_i \models \varphi for 1 \leq i \leq n.$

In this paper we treat only three modal operator such as $\circ, \otimes, \iota\delta$

3 Decidability

The aim of this paper is study of the sufficient conditions for a given arrow logic to be decidable. There exist two standard method of proving the decidability of a given logic Λ. The first one is to formalize Λ in a Gentzen-type sequent calculus and then to prove the cut elimination theorem. The second is to prove the finite model property of Λ. Then, the decidability of Λ follows from its finite model property, as long as it is finite axiomatizable.

In arrow frames, the method to prove the finite model property can not be used as long as the formula $(\varphi \circ \psi) \circ \chi \leftrightarrow \varphi(\psi \circ \chi)$ is valid. Therefore it is interesting to see whether the decidability hold or not, when the logic has the associativity of $\circ$. Gyuris [3] proved the decidability of arrow logic with the associativity but without the modal distrbutive law $\varphi \circ (\psi \vee \chi) \leftrightarrow (\varphi \circ \psi) \vee (\varphi \circ \chi)$.

So the system introduced by Gyuris is not a normal modal logic.

In this paper, we consider some extensions of the above system and show that the decidability still holds.

4 Conclusion

In this paper, the decidability of some extensions of the system by Gyuris are shown by using the technique of term rewriting system.
To show the decidability of the arrow logics with more rules of inference is left for future work.

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