An efficient solvable case of three-dimensional transportation problem

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There is a man, there is a thing.From ancient times, men wish to get things in faraway places.Today, with the development of media, we can came to get such things.Furthermore, movement of goods is expanding according to wide spread of computer networks.There are various kinds of transportation networks in the air, in the sea, and on the ground.Transportation costs may become enormous.

Reduction of costs and energy in transportation is a social demand, because it is related to earth environment and making comfortable society.

There is a research category called a transportation problem under such concern.It is widely known as one of classical problems in operations research (OR).Moreover, after the World War II mathematical programming methods offered mathematical frameworks for formulating problems in some concrete forms, and also give methods and algorithms to solve them efficiently. As a result, we can deal with various problems with different appearance in a systematic way.

Thus, OR has accomplished remarkable development, and has big influence on management science, social science, engineering, etc.OR is utilized in order to solve actual problems scientifically.To make a model of the problem, mathematical expressions are often used because they are easily handled by mathematics.One of remarkable points of OR is that it aims to solve the problem in a gentle and elegant way.Moreover, results are evaluated by computation time and simplicity of the method.

The transportation problem is a problem of finding how to move goods from producers to consumers with the cheapest expense.Today, transportation of goods takes immense expense.There exist goods such that 80% of the cost is spent for the transportation.Not only for consumers, but also for suppliers, reduction of transportation cost will yield new

profit and so it fulfills various needs in the society.For example, "the problem of a canned factory" and "the problem of a soap factory" are know as famous problems for a long time.

Let's start with describing the problem studied in this research.The problem is divided into some types by conditions described as follows.Hitchcock type transportation problem is defined as follows: There are several producers and consumers. Given the amount of production by each producer, the amount of consumption by each consumer, and the cost needed for transporting one unit of goods between each pair of producer and consumer, find how to transport goods form producers to consumers that minimizes the total cost.The name comes from F.L. Hitchcock since he first gave the formulation and a solution method in 1941. Although its formulation preceded that of general linear programming problem, it is treated as a special case of linear programming problems.

There are a lot of way how to find feasible solutions such as northwest corner rule and minimun cost rule.In this research, we study problems with more than three parameters.It is called the multi-dimension transportation problem.In present situation, many goods are moving simultaneously through transportation networks in the air, in the sea, and on the ground. Goods usually reach at the destination via several relay points.We study reduction of the cost in such a situation.

Monge property is important in solving the transportation problem.This idea has a long history. It was discovered in 1781 by a mathematician G. Monge and an engineer in France.A.J. Hoffman showed that the problem is solvable by a simple method in a reasonable time if the input of the problem satisfies some condition. Furthermore, he also showed that a problem that can be transformed into some form satisfying the Monge property is also solvable by a similar method.His method is applied to problems in various areas, such as applied molecular theory and computer geometry.

This research considers general type multi-dimension transportation problems.The transportation problem is extended to the multi-dimension transportation problem. Monge property for multi-dimensional cases is also proposed as an extension of the 2-dimensional case.Using this idea, if a cost function fulfills the multi-dimentional Monge property, the problem is solvable by a method with small computation time.We have found a condition for the existence of a solution of the problem with upper bound constraints.If the constraint fulfills some condition, then the problem has a feasible solution.Firthermore, we propose an algorithm that finds an optimum solution of the problem.

As the number of parameters of a multi-dimension transportation problem increases, the problem becomes more and more complicated.Therefor, even finding feasible solutions becomes hard.In solving such problems, we need to find whether there is a feasible solution or not.In such situations, conditions for the existence of solutions may work effectively in solving the problem.

As a future subject, we need to study the condition for the existence of solutions when the upper bounds become tight.For this problem, we describe in what condition the obtained result becomes invalid.This is a barrier we need to overcome in studying the problem.We may still have a chance to find a necessary and sufficient condition for the existence of feasible solutions by an independent way.