Title	アルゴリズム的コード化定理を満たすChaitnマシンの特徴付けの研究
Author(s)	天谷, 良章
Citation	
Issue Date	2001-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/1479
Rights	
Description	Supervisor:石原 哉, 情報科学研究科, 修士

Characterization of Chaitin Machine Satisfying the Algorithmic Coding Theorem

Yoshibumi Amaya

School of Information Science,
Japan Advanced Institute of Science and Technology

February 15, 2001

Keywords: Chaitin machine,prefix-code,programsize-complexity,Algorithmic Coding Theorem,minimal program.

1 Background

Algorithmic information theory (AIT) is the result of combining Shannon's information theory and Turing's computability theory. In algorithmic information theory, the primary concept is to measure the complexity of a individual object by the size in bits of the smallest program for computing it. This notion is also known as information-theoretic complexity or Kolmogorov complexity. This field is independently introduced by A.N.Kolmogorov, A.N.Kolmogorov, R.J.Solomonoff, G.J.Chaitin.

Later on, the prefix complexity (AIT_2) was introduced. It was first introduced by L.A.Levin and G.J.Chaitin in 1970s. The strategy is to measure the program size complexity by (Chaitin) machine having the prefix-free set as its domain. AIT_2 has nicer mathematical properties than the original AIT, and has therefore become something of standard in the field.

One of important results in AIT is the Algorithmic Coding Theorem. According to the Algorithmic Coding Theorem, the complexity of any universal machine is asymptotically optimal (i.e.optimal up to at most a constant) with respect to the entropy of the machine's algorithmic probabilities.

In order to make this result more general, we are interested in a class of machines, not necessarily universal, and any semi-distribution, an unknown constant. So, the aim of this thesis is, after adequately investigating and considering fundamental properties of the noiseless coding and the program-size complexity with Chaitin machine, to investigate a class of machines satisfying the Algorithmic Coding Theorem, not necessarily universal, any semi-distribution. Finally, we characterize all machines satisfying the Algorithmic

Coding Theorem, and we show a class of machines satisfying the Algorithmic Coding Theorem with constant $c = 0$.

2 Noiseless Coding

We give definitions of the prefix-free sets, prefix codes and Shannon's entropy, and show the mainly property of the prefix code. Properties are that the prefix-code is the uniqueness of decodability, that the set of the prefix code-strings (prefix-free set) satisfy Kraft's inequality and that the average lengths of the prefix code-strings is about equal to the Shannon' entropy.

3 Program-size Complexity

In order to measure the content of information, we employ a special model of deterministic Turing machine computation, namely Chaitin machines M having prefix-free domains. The complexity $H_M(x)$ of a string x is defined as the size in bits of the smallest program p for computing x by the Chaitin machine M ;

$$H_M(x) = \min\{|p| \mid M(p) = x\}.$$

AIT come from the Invariance Theorem as the following.
For a universal machine U, for every machine M, there exists a constant $c \geq 0$ for each finite object (string) x,

$$H_U(x) \leq H_M(x) + c.$$

It means that each object (string) x has an intrinsic complexity which is independent from the ways U, M of description. Hence, we can measure a program-size complexity by fixing the universal machine. There exist the other important properties in non-computability of program-size complexity, computability of approximations of the program-size complexity, and so on.

We give two important theorems, in AIT.
One is the Kraft-Chaitin Theorem:

Given a recursively enumerable (r.e.) set $S = \{(x_i, n_i) \in \Sigma^* \times N \mid i \geq 0\}$ such that $\sum_i 2^{-n_i} \leq 1$, we can effectively construct a machine M satisfying $M(u_i) = x_i$, $|u_i| = n_i$ for all i.

The other is, by using Kraft-Chaitin Theorem,
the Algorithmic Coding Theorem:

The complexity $H_U(x)$ for any universal machine U are asymptotically optimal (i.e. optimal up to at most a constant) with respect to the emtropy of the machine's algorithmic probabilities P_U,

$$H_U(x) \leq -\log P_U(x) + (1 + c).$$

$$H_U(x) = \min\{|u| \mid U(u) = x\},\ P_U(x) = \sum_{U(u)=x} 2^{-|u|}$$

We make this result more general.

4 Characterization

We investigate Chaitin machine satisfying the Algorithmic Coding Theorem conditions under a given semi-distribution P,

We give the basic result:

> When P is a semi-distribution and $S \subset \Sigma^* \times N$ and a constant $c \geq 0$ such that the following two conditions are satisfied for every $x \in \Sigma^*$:
>
> *(1)* $\sum_{(x,n)\in S} 2^{-n} \leq P(x)$,
>
> *(2) for all* $n \in N$, *if* $P(x) > 2^{-n}$, *then there exist some* $k \leq n + c$ *such that* $(x, k) \in S$ *and* k.
>
> Then, there exists a machine M (depending upon S) such that for all $x \in \Sigma^*$, $-\log P(x) \leq H_M(x) \leq (1 + c) - \log P(x)$.

By using this result, we investigate under variable conditions.

When P is a semi-distribution semi-computable from below, there exists a machine M satisfying the Algorithmic Coding Theorem with $c = 1$.When P is a computable semi-distribution, there exists a machine M satisfying the Algorithmic Coding Theorem with $c = 1$.

Finally, a class of the machines satisfying the Algorithmic Coding Theorem satisfy the following condition that there exists constant $c \geq 0$ for all natural n, if $P_M(x) > 2^{-n}$, then $H_M(x) \leq n + c$.

Furthermore, a class of the machines satisfying the Algorithmic Coding Theorem with $c = 0$ was under condition that for all different programs $u \neq u'$, $M(u) = M(u')$ implies $|u| \neq |u'|$.

References

[1] Cristian.S.Calude, *Information and Randomness :An Algorithmic Perspective*, Springer-Verlag, 1994.

[2] Cristian.S.Calude, Hajime Ishihara and Takeshi Yamaguchi, *Minimal programs are almost optimal*, CDMTCS Reserch Report Series, CDMTCS-116, 1999, Spring.

[3] Greg.J.Chaitin, *Information, Randomness and Imcompleteness, Paper on Algorithmic Information Theory*, World Scientific, Singapore, 1990(2nd ed.,).