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Author(s)	Xu, Dawei
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Description	Supervisor:上原　隆平，情報科学研究科，博士

Abstract

In this thesis, we study the folding and unfolding problems of polycubes. A polycube is a kind of polyhedron that composed of identical cubes. It has an important role in the field of computational geometry.

The first problem is about the common developments that can fold to two or more incongruent polycubes, boxes. These developments are simple polyominoes that consist of unit squares of four connected so that the boxes and their corresponding developments have the same surface area. In searching for common developments that fold to two or more boxes, we start from a relatively simple task: finding two or more boxes that their surface areas are equal, but sizes are different.

The smallest surface area is 22 which admits to fold to two boxes of size 1*1*5 and 1*2*3. All common developments of these two boxes were enumerated by Matsui in 2011. The next smallest integer n such that surface area n can fold to two different boxes is 30. Previous study tried to list the common developments of two different boxes of size 1*1*7 and 1*3*3 of surface area 30, however, it failed due to the limitation of computational power. We fill this gap and research further.

By a new algorithm on a parallel computer, we first enumerate all common developments of two boxes of size 1*1*7 and 1*3*3. Starting from the partial development of surface area 1, we repeat adding one new unit square to all possible places of the partial development to generate new developments. In each loop, the new developments will get checked whether they can cover the boxes or not. The algorithm loops n times, where n = 30.

Finally, we obtained all common developments of two boxes of size 1*1*7 and 1*3*3. There are 1080 common developments.

Surface area 30 is consistent to the surface area of a cube of size √5*√5*√5. Therefore, next, we try to find a polyomino of surface area 30 that fold to these three different boxes. It is a great improvement from previously known one that the smallest surface area that folds to three different boxes is greater than 500. To achieve that, we propose a new algorithm. It determines if a polyomino P that can fold to two boxes of size 1*1*7 and 1*3*3 can also fold to a box of size √5*√5*√5.

As a result, nine out of 1080 developments can fold to the cube of size √5*√5*√5. While eight developments have only one way of folding to the cube of size √5*√5*√5, the other development has two different ways of folding to it, for there is an angle of 26.6 degrees to the unit square's edge clockwise and counter-clockwise. That is, the last development can actually fold to 3 boxes in 4 different ways of folding:

the box of size 1*1*7, the box of size 1*3*3, and 2 ways of the box of size $\sqrt{5}*\sqrt{5}*\sqrt{5}$.

In these results, there are some common developments with a special property that they are central symmetric. We call them the centrosymmetric common developments.

The way to enumerate the centrosymmetric common developments is a bit different from the enumeration of normal common developments, which needs more computations. However, the size of data is relatively small, and it runs in loops of n/2 in total, where n is its surface area.

As a result, in 2263 common developments of two boxes of 1*1*5 and 1*2*3, only 45 centrosymmetric common developments exist. We confirm the exact numbers of centrosymmetric common developments from surface area 22 to surface area 54.

Especially, we show that there is no centrosymmetric common development of three different boxes of surface area 46. The surface area 46 is the smallest candidates of three boxes of integral size of 1*1*11, 1*2*7, and 1*3*5. It is still open that if there is a common development of three boxes of these sizes, and we give a partial answer to this open problem.

The second part of this thesis is research on the "rep-cube", which is a new notion derived from the classic notion of "rep-tile" with the idea of the development. A rep-tile of order k is a polygon that can be divided into k replicas congruent to each another and similar to the original one. It was proposed by Solomon W. Golomb in 1962.

In 2016, a new notion of rep-cube was proposed. A polyomino is a rep-cube of order k if it is a development of a cube, and it can be divided into k polyominoes such that each of them can fold to a cube. If each of these k polyominoes has the same size, we call the original polyomino a regular rep-cube of order k.

A recent study shows that there exist regular rep-cubes of order k for each k=2, 4, 5, 8, 9, 36, 50, 64, and also k=36gk'2 for any positive integer k' and an integer g in 2, 4, 5, 8, 9, 36, 50, 64. That is, there are infinitely many k that allow regular rep-cube of order k. These results lead us to the following natural question: how many rep-cubes of order k exist for some k?

As a consequence, we enumerate all regular rep-cubes of order 2 and 4 of surface area 12 and 24, respectively. For example, there are 33 rep-cubes of order 2 of surface area 12; that is, there are 33 dodecominoes that can fold to a cube of size $\sqrt{2}*\sqrt{2}*\sqrt{2}$ and each of them can be divided to two developments of the unit cube. Similarly, there are 7185 regular rep-cubes of order 4 of surface area 24.

Key words: Origami, Folding algorithm, Polyomino, Polycube, Rep-cube.