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SHEN QIAN
STATISTICAL PERFORMANCE CHARACTERIZATION OF LOSSYFORWARD BASED COOPERATIVE WIRELESS NETWORKS OVER FADING CHANNELS

Academic dissertation to be presented with the assent of the Doctoral Training Committee of Technology and Natural Science of the University of Oulu for public defence in Collaboration room 7, Asahidai, on 5 October 2017, at 2 p.m.

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and
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#### Abstract

Cooperative wireless communications are investigated from the perspective of exploiting statistical nature of channel property variation. The target of this research is to provide analytical assessments and theoretical bounds of lossy-forward (LF) relaying based cooperative communications in various network topologies and propagation conditions, where the channel variation information is efficiently utilized.

The performance of three-node LF relaying over independent block Nakagami-m fading channels is investigated. Based on the source coding with a helper theorem, the exact outage probability expression with arbitrary values of the shape factor $m$ is derived. Furthermore, the decay of the outage curve, referred to as equivalent diversity order, and coding gain of LF relaying are identified based on a yet accurate high signal-to-noise ratio (SNR) outage probability approximation. It is found that the decay of the outage curve is dominated by the less reliable channel of either the source-relay or the relay-destination link. It is also found that in terms of the outage probability, LF relaying is superior to conventional decode-and-forward (DF) relaying where relay keeps silent if error is detected after decoding. This is because the relay in LF always forwards the decoder output to the destination via re-interleaving and re-encoding of the information sequence. Therefore, the whole system can be regarded as a distributed turbo code. Moreover, with LF relaying, not only the outage probability can be reduced, but also the search area for a relay (helper) can be increased compared to conventional DF relaying while keeping the same or even lower outage probability, resulting in significant coverage expansion of the system.

The outage probabilities of LF, decode-and-forward (DF) and adaptive decode-andforward (ADF) relaying are analyzed in block Rayleigh fading channels, with the aim of identifying the impact of the spatial and temporal correlations of the fading variations. It is proven that the coding gain with LF is larger than with DF but smaller than with ADF, where the ADF scheme utilizes error-free feedback from the relay to the source. It is found that compared to the independent fading case, in the correlated fading, to achieve the lowest outage probability, the relay should be located closer to the destination, or more transmit power should be allocated to the relay, both for reducing the gain loss caused by the fading correlation.


A comparative study on the outage probabilities of LF relaying with the two distributions, Rician and Nakagami-m, is conducted. Kullback-Leibler divergence (KLD) and Jensen-Shannon divergence (JSD) are used to identify the difference between the distributions. It is found that even with a specific parameter setting yielding the same line-of-sight (LOS) ratio, Rician is not equivalent to Nakagami-m model for representing the shape of the entire portion of the distribution.

Furthermore, we derive an upper bound of the outage probability for a two-way LF relaying system over Rician fading channels with a random $K$-factor. The $K$-factor is assumed to follow empirical distributions, normal or logistic distributions, which are derived from measurement data. Compared to the two-way DF transmission, the two-way LF transmission is found to achieve lower outage probability regardless of either logistic or normal distribution is used to represent the variation of $K$-factor. Because with LF, the relay always broadcast the decoder output regardless of whether error is detected after decoding in the information part or not.

The work is extended to a multi-source multi-relay transmission system, where all the links experience the $\kappa-\mu$ fading variations. It is found that, regardless of whether the LOS component exists in the channel or not, the outage performance of the system with orthogonal transmission with joint-decoding scheme is superior to that with maximum ratio transmission scheme.

Keywords: Outage probability, relay channels, lossy-forward (LF), Nakagami-m fading, Rician fading, $\kappa-\mu$ fading, Kullback-Leibler divergence (KLD), Jensen-Shannon divergence (JSD), line-of-sight (LOS) component, diversity order and coding gain

To my son Chisheng and my daughter Chicheng

## Preface

This research has been conducted under the framework of joint supervision of doctoral dissertation and for awarding double doctoral degree between University of Oulu (UOulu), Finland and Japan Advanced Institute Science and Technology (JAIST), Japan. At JAIST, the research was carried out at Information Theory and Signal Processing Laboratory, School of Information Science. At UOulu, the work was conducted at Centre for Wireless Communications (CWC).

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## List of abbreviations

| AF | Amplify-and-forward |
| :--- | :--- |
| AWGN | Additive white Gaussian noise |
| BER | Bit error rate |
| BPSK | Binary phase shift keying |
| BSC | Binary symmetric channel |
| cdf | Cumulative distribution function |
| CEO | Chief executive officer |
| CSI | Channel state information |
| DF | Decode-and-forward |
| EXIT | Extrinsic information transfer |
| FP7 | 7th Framework Program |
| FER | Frame error rate |
| HI | Horizontal iteration |
| ICT | Information and communication technology |
| i.i.d. | Independently and identically distributed |
| IoT | Internet of things |
| JSD | Jensen-Shannon divergence |
| KKT | Karush-Kuhn-Tucker |
| KLD | Kullback-Leibler divergence |
| LF | Lossy-forward |
| LLR | Log-likelihood ratio |
| LOS | Line-of-sight |
| MARC | Multiple access relay channel |
| MAC | Multiple access channel |
| MIMO | Multiple-input multiple-output |
| MUD | Multi-user detection |
| NLOS | Non-line-of-sight |
| P2P | Point-to-point |
| pdf | Probability density function |
| QPSK | Quadrature phase-shift keying |


| RESCUE | Links-on-the-fly Technology for Robust, Efficient and |
| :--- | :--- |
|  | Smart Communication in Unpredictable Environments |
| SINR | Signal-to-interference-plus-noise ratio |
| SNR | Signal-to-noise ratio |
| VI | Vertical iteration |
| VR | Virtual reality |
| XOR | Exclusive-OR |
| 5G | 5th generation |

## List of Symbols

| $C$ | channel capacity |
| :--- | :--- |
| $\mathscr{D}$ | distortions |
| $E_{S}$ | transmit energy per symbol |
| $E^{n}$ | dimensionality of channel input |
| $G$ | geometric gain |
| $G_{c}$ | coding gain |
| $G_{d}$ | diversity gain |
| $h$ | complex channel gain |
| $K$ | Rician factor |
| $m$ | Nakagami-m shape factor |
| $n$ | zero-mean AWGN |
| $N_{0}$ | noise variation |
| $p_{f}$ | bit flipping probability |
| $P_{T}$ | total transmit power |
| $R$ | source coding rate |
| $R^{\mathrm{c}}$ | channel coding rate |
| $R(\mathscr{D})$ | rate-distortion function |
| $\alpha$ | path loss exponent |
| $\gamma$ | instantaneous SNR |
| $\bar{\gamma}$ | average SNR |
| $\rho$ | correlation coefficient |
| $\rho_{s}$ | spatial correlation coefficient |
| $\rho_{t}$ | temporal correlation coefficient |
|  |  |
| $\mathrm{ACC}_{\mathrm{S}}$ | accumulator for the source |
| ACC |  |
| $\mathrm{C}_{\mathrm{S}}$ | decoder of the accumulator for the source |
| $\mathrm{D}_{\mathrm{S}}$ | encoder for the source |
| $\mathrm{M}_{\mathrm{S}}$ | decode for the source |
| $\mathrm{M}_{\mathrm{S}}^{-1}$ | modulator for the source |
| $\Pi_{\mathrm{S}}$ | demodulator for the source |
| interleaver for the source |  |


| $\Pi_{S}^{-1}$ | de-interleaver for the source |
| :---: | :---: |
| $b_{\text {S }}$ | binary information sequence of the source |
| $\mathrm{ACC}_{\mathrm{R}}$ | accumulator for the relay |
| $\mathrm{ACC}_{\mathrm{R}}^{-1}$ | decoder of the accumulator for the relay |
| $\mathrm{C}_{\mathrm{R}}$ | encoder for the relay |
| $\mathrm{D}_{\mathrm{R}}$ | decode for the relay |
| $\mathrm{M}_{\mathrm{R}}$ | modulator for the relay |
| $\mathrm{M}_{\mathrm{R}}^{-1}$ | demodulator for the relay |
| $\Pi_{\mathrm{R}}$ | interleaver for the relay |
| $\Pi_{R}^{-1}$ | de-interleaver for the relay |
| $b_{\text {R }}$ | binary information sequence of the relay |
| $\Pi_{0}$ | interleaver between the source and the relay |
| $\Pi_{0}^{-1}$ | de-interleaver between the source and the relay |
| $a * b$ | binary convolution, i.e., $a * b=a(1-b)+b(1-a)$ |
| $C(\cdot)$ | Shannon channel capacity |
| $C_{c c}(\cdot)$ | constellation constrained channel capacity |
| $C_{c c}^{-1}(\cdot)$ | inverse function of constellation constrained channel capacity |
| $d(\cdot, \cdot)$ | distortion measure function |
| $E[\cdot]$ | expectation of random variable |
| $\exp (\cdot)$ | natural exponential function |
| $H(\cdot, \cdot)$ | joint entropy function |
| $H(\cdot \mid \cdot)$ | conditional entropy function |
| $H(\cdot)$ | binary entropy function |
| $H^{-1}(\cdot)$ | inverse of binary entropy function |
| $I(\cdot ; \cdot)$ | mutual information function |
| $I_{0}(\cdot)$ | zero-th order modified Bessel's function of the first kind |
| min | minimization |
| $p(\cdot)$ | probability density function |
| $p(\cdot, \cdot)$ | joint probability density function |
| $p(\cdot \mid \cdot)$ | conditional probability density function |
| $Q_{1}(\cdot, \cdot)$ | Marcum Q function |
| $\Gamma(\cdot)$ | Gamma function |
| $\gamma(\cdot, \cdot)$ | lower incomplete gamma function |
| $\ln (\cdot)$ | natural logarithm |


| $\log (\cdot)$ | logarithm function with base 2 |
| :--- | :--- |
| $\lg (\cdot)$ | logarithm function with base 10 |
| $\operatorname{Pr}(\cdot)$ | probability |
| $\hat{\theta}$ | estimation |
| $\oplus$ | modulo-2 addition |

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## 1 Introduction

Next generation wireless systems (5G) is considered to be the foundation of information and communication technology (ICT)-based society including virtual reality (VR), autonomous vehicles, and internet of things (IoT). By exploiting underutilized frequency spectrum available in millimeter wave, 5 G is designed to handle thousands of times of more data traffic and can provide more than ten times the transmission speed compared to the existing networks. Millimeter wave cannot well travel in the presence of obstacles, e.g., buildings and shadowing objects, and tend to be affected by weather, e.g., absorbed by rain [1]. In cooperative communications, relays overhear the wireless signal and enhance the transmission between the nodes. Consequently, the wireless signal can be forwarded over the obstacles with the aid of relays, which can well solve the aforementioned problems. Cooperative communication in wireless networks is also of great importance as it has great potential for achieving diversity gain, enhancing network throughput, and extending communication coverage. This dissertation focuses on statistical performance characterization of the novel lossy-forward (LF)-based cooperative wireless networks and theoretical analysis of system performance by exploiting the statistical nature of channel variations.

In this chapter, we briefly introduce the basic concept of cooperative wireless communications. The wireless channel models are then described. The background and motivation of the research on exploiting channel variation in LF-based cooperative wireless communications are described, which is followed by the in-depth literature survey on the analyses and their results related to this research. The outline of this dissertation and the contributions of this research are also provided.

### 1.1 Cooperative Communications

Cooperative communication has been recognized as one of the most important techniques not only in designing the next generation wireless communication networks but also enhancing services and coverage of the existing systems, especially when it is used for the coverage expansion or diversity rather than static link design. It was initially introduced by Van der Meulen [2] in 1970s and the information theoretic analyses for the relay networks were intensively conducted by Cover and Gamal in [3]. Before the cooperative communications emerged, the transmission quality only relies on the
condition of the direct link between source and destination. However, with cooperative communications, nearby nodes (i.e., base stations, mobile devices, stationary devices) who overhear the source information can serve as relays for helping in transmission. By forming virtual antenna array [4] to cooperate with each other to mitigate the signal attenuation due to the variation of the propagation conditions, without having to impose strict constraints in deployment or excessively high hardware complexity compared with fixed multiple-input multiple-output (MIMO) techniques, cooperative communications are able to enhance power and spectrum efficiencies and improve communication reliability [5].

### 1.1.1 Relay Protocols

The relay protocol specifies the processing performed by the relay upon its received signals. Based on the operations at the relay, relay protocols can be mainly classified into following categories [6-9].

## Amplify-and-Forward

In amplify-and-forward (AF) relaying (also called observe-and-forward) [7], the received signal sent via the source-relay link is simply scaled and amplified by a relay, and forwarded to the destination. The destination make decision by properly combining signals sent from the source and relay, and thereby spatial diversity can be achieved by the AF protocol. The major problem of the AF protocol is that the noise at the relay is also amplified and forwarded, which causes performance degradation.

## Decode-and-Forward

Among the various relaying protocols, decode-and-forward (DF) relaying has drawn significant attentions, and been widely studied. In DF relaying, if the transmitted information sequence is successfully recovered at the relay, the recovered information sequence is re-encoded and forwarded to the destination. The relay keeps silent if error is detected after decoding at the relay to avoid error propagation. Several analyses on the diversity and multiplexing gains as well as their tradeoff of DF relaying have been conducted for half duplex and full duplex systems [10-12]. Various practical

Table 3. Forwarding Strategies

| Type | Approach |
| :---: | :--- |
| Static Relaying | A certain implementation for the cooperation is utilized at the <br> relay. |
| Adaptive Relaying |  |
| (Selection) | Transmitting terminals adapt transmission format, e.g., coop- <br> erative or non-cooperative, based upon the measured SNR. |
| Adaptive Relaying | Exploiting limited feedback from destination to decide <br> whether relay forwards what it received from the source <br> (Incremental) |

implementations of DF relaying in single-antenna scenarios have been analyzed in [13, 14], and in multiple-antenna scenarios in [15, 16].

## Compress-and-Forward

In compress-and-forward relaying, the relay quantizes and compresses the received information by utilizing the source-relay correlation, then forwards the compressed version to the destination. The destination estimates the received information sequence by utilizing the information sequence transmitted from source as a helper [17, 18].

## Compute-and-Forward

In compute-and-forward relaying, the relay decodes linear functions of the received signals by using the linear combinations provided by the channel, and forwards the decoded results to the destination. At the destination, the linear combinations are solved for recovering the original information [9].

## Derivative Strategies

Various forwarding strategies have been proposed to achieve the benefits of cooperative communications. Laneman et al. outlined a variety of strategies [8, 19] which are categorized in Table 3.

### 1.2 Wireless Channel Models

In wireless communications, radio signal propagation is subject to reflection, diffraction, and scattering due to the obstacles in the environment. These mechanisms affect the radio propagation in different ways, i.e., path-loss, shadowing, and fading [20]. Path-loss is related to transmission distance and free space loss; shadowing (sometimes called large scale fading) indicates the received signal variation around the path-loss attenuation and its statistics are typically well characterized by the log-normal distribution; the (small scale) fading or short-term signal variation is typically caused by multiple signals received via different propagation paths. Due to the existence of a great variety of fading environments, channel fading can be statistically described by different models [21].

Radio signals arrive at the destinations via multiple propagation paths where there are obstacles. Therefore, each component signal received has different energy ${ }^{1}$. The received signal is a superposition of the component signals, which results in amplitude variation and phase rotation. The Rayleigh fading model is widely used to characterize the variations where the amplitude follows Rayleigh distribution over $[0, \infty)$ and the phase follows uniform distribution over $[0,2 \pi)$. The speed of the variation depends on the velocity of the mobile node. Each component signal suffers from different Doppler shift, hence the received composite signal composed of the received signal components suffers from Doppler spread.

The Rayleigh fading ${ }^{2}$ is commonly used to describe random variations due to multipath effect having non-line-of-sight (NLOS) components, such as densely populated urban areas having a lot of buildings and other objects, resulting in no direct path between the transmitter and the receiver in the wireless channel.

However, in some scenarios, channels are composed of both line-of-sight (LOS) and NLOS components in wireless communications systems, e.g., terrestrial mobile and satellite mobile communications systems. A mathematical model of the channel having LOS and NLOS components is Rician fading, where the amplitude is characterized by a Rician distribution. The Rician fading model is widely used to characterize the channels that exploit the performance gain due to the LOS conditions. Rician factor $K$ denotes the ratio of the LOS component power-to-NLOS components average power. $K$

[^0]factor represents severity of fading variation. With $K=0$, Rician fading reduces to Rayleigh fading. With $K=\infty$, channel is equivalent to static additive white Gaussian noise (AWGN) channel (Receiver hardware always introduces AWGN to the received signal).

Although the Rician distribution is widely used to represent the statistical behaviors of the channels, it is still not accurate enough as the first-order statistics of the channel variations are compared with the measurement data gathered in the fields. The Nakagami$m$ distribution is derived empirically based on the measurement data [22]. It is known that the Nakagami-m fading is able to better represent the distribution of the channel behaviours compared to the Rician distribution [23]. Nakagami-m model can model a wider range of channel conditions including severe, moderate, light and no fading. It is well known that both Rayleigh and Rician distributions are connected to the Nakagami-m fading model by adjusting the shape factor $m$ representing the fading severity in Nakagami-m model [20, 24].

Yacoub et al. proposed two more generalized fading models, $\kappa-\mu$ and $\eta-\mu$, which are fully characterized in terms of measurable physical parameters and known to be better matched to measurement data than the other models [25]. The $\kappa-\mu$ distribution includes the Rician, the Nakagami-m, and the Rayleigh distributions as special cases, whereas, the $\eta-\mu$ distribution includes the Nakagami-m, the Rayleigh, and the one-sided Gaussian distributions as special cases. In particular, the $\kappa-\mu$ distribution better fits LOS scenarios. The $\eta-\mu$ distribution is better suited for NLOS applications.

### 1.3 Background and Research Motivations

### 1.3.1 Research Background

As stated before, cooperative communications have been recognized as the promising technologies for next generation wireless communication systems. A key to create efficient and flexible technologies for the further generation wireless communication networks is to know how the latest results of network information theory can be best utilized in wireless communication network design. For example, in the DF relaying systems, the information sequences sent from the same source are correlated, which, in the theoretical side, invokes the idea of utilizing the source coding with a helper theorem. In the practical system design side, the correlation knowledge can be utilized in the distributed turbo coding with the idea of log likelihood ratio (LLR) updating
corresponding to the errors occurring in the network [26, 27]. The theoretical and practical considerations have created the relaying technique called lossy-forward (LF) [27]. It has been proven that LF relaying can reduce outage probability, compared to conventional DF relaying [28].


Fig. 1. Coding and decoding structure of the LF relaying system.

In conventional DF relaying [3, 19], the recovered information sequence is discarded at the relay if error is detected after decoding. It has been believed that if the relay re-encodes the information sequence containing error and forwards it to the destination, error propagation will occur, resulting in even worse performance [29]. However, even though error may be detected at the relay, the information sequences transmitted from the source and relay are still correlated. Therefore, the source coding with a helper theorem [30] can be used in this scenario by utilizing the information sequence transmitted from the relay as a helper. In LF relaying, the relay does not aim to perfectly correct the errors occurring in the source-relay link. The decoded information sequence at the relay is re-interleaved, re-encoded, and transmitted to the destination, even though error may be detected in the information sequence after decoding. The probabilities of the errors occurring in the source-relay link can be estimated at the destination [31]. After converting the received signals from the source and the relay to the LLR sequences of the systematic bits, iterative processing for the systematic LLR exchange between the two decoders, one for decoding the signal received via the source-destination link and the other via the relay-destination link, is invoked. The systematic LLR is modified to best utilize the estimate of the error probability of the source-relay link by a LLR
modification function [26]. The LLR modification function makes compensation of the systematic LLR values, according to the knowledge of the source-relay link error probability, resulting in decoding performance improvement [27].

The coding/decoding structure of a three-node LF relaying system is shown in Fig. 1. At the source, the binary information sequence $b_{S}$ is first encoded by $\mathrm{C}_{S}$. Then, the encoded information sequences are interleaved by a random interleaver $\Pi_{S}$ and doped-accumulated by a $\mathrm{ACC}_{S}$ (The purpose using $\mathrm{ACC}_{\mathrm{S}}$ is to control the shape of the extrinsic information transfer (EXIT) curve of the demodulator and keep the convergence tunnel open. [40]). Then, the outputs of $\mathrm{ACC}_{\mathrm{S}}$ are mapped onto symbols by a modulator $\mathrm{M}_{\mathrm{S}}$, and broadcasted to both the relay and the destination in the first time slot. The received signal at the relay is first fed to a demodulator $\mathrm{M}_{\mathrm{S}}^{-1}$, followed by the decoder $\mathrm{ACC}_{\mathrm{S}}^{-1}$ of $\mathrm{ACC}_{\mathrm{S}}$. The extrinsic LLR output from $\mathrm{ACC}_{\mathrm{S}}^{-1}$ is fed to the de-interleaver $\Pi_{S}^{-1}$, followed by the decoder $\mathrm{D}_{\mathrm{S}}$ of $\mathrm{C}_{\mathrm{S}}$. Instead of performing the fully iterative decoding/detection between $\mathrm{M}_{\mathrm{S}}^{-1}$ and $\mathrm{D}_{\mathrm{S}}^{-1}$, only simple non-iterative decoding is performed. Hard decision is then performed on the output of $\mathrm{D}_{\mathrm{S}}^{-1}$ to obtain the information sequence $b_{\mathrm{R}}$ at the relay. With this technique, we can significantly reduce the complexity of the relay compared to conducting iteratively decoding.

Due to the weak decoding at the relay, $b_{\mathrm{R}}$ may contain errors. However, $b_{\mathrm{R}}$ is again random interleaved by $\Pi_{0}$, channel re-encoded by $C_{R}$, re-interleaved by a random interleaver $\Pi_{R}$ and fed to $A C C_{R}$. The purpose of the use of $\Pi_{0}$ is that the interleaved sequence $\Pi_{0}\left(b_{\mathrm{R}}\right)$ is made statistically independent of $b_{\mathrm{S}}$. Finally, the information sequence is re-mapped by $\mathrm{M}_{\mathrm{R}}$, and transmitted to the destination in the second time slot.

At the destination, detection process for the signal received in the first and second time slots, respectively, is first performed independently as shown in Fig. 1. At this stage, fully iterative docoding/detection is adopted between $\mathrm{M}_{\mathrm{S}}^{-1}$ and $\mathrm{D}_{\mathrm{S}}^{-1}$, which is referred to as horizontal iteration (HI), as well as between $\mathrm{M}_{\mathrm{R}}^{-1}$ and $\mathrm{D}_{\mathrm{R}}^{-1}$ [27]. After each round of HI, the extrinsic systematic LLRs obtained from the two decoders $D_{S}$ and $D_{R}$ are further exchanged with each other, which is referred to as vertical iteration (VI). The extrinsic systematic LLR is updated by the function LLR updating function $f_{c}$ [31]. By utilizing the function $f_{c}$, the extrinsic systematic LLRs, forwarded by the relay, help the decoder eliminate the errors in the original information sequence $b_{\mathrm{S}}$ by exploiting the correlation knowledge between the source and the relay. Finally, the detection of $b_{\mathrm{S}}$ can be completed by making hard decisions of the a posteriori LLRs of the systematic bits outputs from $\mathrm{D}_{\mathrm{S}}$.

The LF technique can be viewed as a distributed joint source-channel coding system with a helper [32-35]. It is shown that with the help of the accumulator, LF can achieve turbo-cliff-like bit error rate (BER) performance over AWGN channels [27, 36].

The LF concept is originated from the idea of Slepian-Wolf cooperation presented in [37]. The coding algorithms are proposed for fading relay channel in [38, 39]. The key concept of the coding technique for LF is introduced in [27], where it assumes that the relay does not need to necessarily recover the information sent from the source perfectly. In [40], a three-node LF relaying over Rayleigh fading channels is studied by identifying the relationship between the DF protocol and Slepian-Wolf coding [41]. However, a drawback of $[27,40]$ is that the admissible rate region is determined by the Slepian-Wolf theorem which does not perfectly match the problem setup, since only the information of the source needs to be recovered at the destination. Zhou et al. [28] eliminate the aforementioned drawback by utilizing the source coding with a helper theorem in the network information theory. Based on [28], the technique is further extended to multiple access relay channel (MARC) [42], where a pair of correlated sources are transmitted to a destination with the aid of one relay. Furthermore, He et al.[43] apply the LF technique to the multi-source multi-relay system. A two-relay LF transmission system is proposed and a power allocation scheme for minimizing the outage probability of the two-relay LF system is presented in [44]. The technique is applied to wireless sensor networks, where a simple, yet efficient, power allocation scheme for an arbitrary number of sensors is derived in [45]. The major contributions under the LF relaying framework are summarized in Table 6.

### 1.3.2 Motivations

The primary goal of this dissertation is to identify the statistical properties performance of the LF-based cooperative wireless networks, and to provide theoretical analysis of system performance by exploiting the statistical characteristics of channel variations. In wireless communications, channels experience variations due to fading. The direct source-destination link and the via-relay link propagations are often found to be in different conditions. Hence, it is quite reasonable to assume that the statistical properties of the fading variations are also different, link by link. A most probably scenario is that since the direct link from source to destination suffers from severe fading, the destination needs the help of a relay via source-relay and relay-destination links which suffer from moderate fading. Hence, we consider the cases that the source-destination
link suffers from block Rayleigh fading (or equivalent Rayleigh fading), while the other links (the source-relay and relay-destination links) experience mild fading, e.g., having LOS component. It is, hence, very interesting to identify the impact of the LOS component on system performances.

In practical cooperative networks such as vehicle-to-infrastructure communications, it is often the case that the fading conditions experienced by different links are correlated due to insufficient separation in the space or time domains between the nodes or transmissions [46-48]. Therefore, examining the performance of diversity techniques in correlated fading conditions has been a long lasting problem with great importance [49]. Motivated by this, system performance in correlated fading is also investigated.

Diversity gain shows how fast the outage probability or frame error rate (FER) can decrease by increasing the average $\operatorname{SNR} \bar{\gamma}$ [50], by which one can have the insights regarding the factors determining the system performance in fading channels. How many independently varying signal components exist in the propagation medium, and how many of them can be extracted, for example, by using multiple antennas, different and/or time slots determines the diversity order that corresponds to the decay of the outage probability or FER curves as a function of $\bar{\gamma}$. Coding gain appears in the form of the parallel shift of the outage probability/FER curves versus $\bar{\gamma}$. At high SNR, the outage probability $P_{\text {out }}$ can be asymptoticly expressed in terms of diversity order and coding gain, as,

$$
\begin{equation*}
P_{\text {out }}=\left(G_{c} \cdot \bar{\gamma}\right)^{-G_{d}}, \tag{1}
\end{equation*}
$$

where $G_{d}$ and $G_{c}$ indicates the diversity order and coding gain, respectively. It should be noted here that in Rayleigh fading environment having NLOS components, the diversity order has only integer values which corresponds to the decay of the outage probability curve. However, in the presence of the LOS component, the decay can take negative real values, depending on the ratio of LOS component to NLOS components. Therefore, we refer the decay of the outage probability curve as to equivalent diversity order. This dissertation derives the equivalent diversity order and coding gain for LF relaying over block Nakagami-m fading with arbitrary values of the shape factor $m$. Also, the account is took of the channel correlations when investigating the diversity order and coding gain for LF, DF, and adaptive decode-and-forward (ADF). Since the capacity-achieving channel codes and infinite frame length are assumed in the theoretical analyses, the obtained theoretical results can be used for evaluating the asymptotic performances of practical systems.

Table 4. Fading channel models.

| Channel <br> Type | Applicable <br> Scenarios | Extreme Cases | Remarks |
| :--- | :--- | :--- | :--- |
| Rayleigh | NLOS conditions | - | - |
| Rician | NLOS and LOS <br> conditions | Rayleigh fading with <br> Rcian factor $K=0 ;$ <br> nonfading (AWGN) <br> with $K \rightarrow \infty$ | - |
| Nakagami- | NLOS and LOS <br> conditions | Rayleigh fading with <br> Shape factor $m=1 ;$ <br> nonfading (AWGN) <br> with $m \rightarrow \infty$ | empirically derived <br> from measurement <br> data |
| $\kappa-\mu$ | NLOS and LOS <br> conditions | Nakagami-m fading <br> with $\kappa=0 ;$ Rician <br> fading with $\mu=0$ | general fading distri- <br> butions; better fit ex- <br> perimental data |

For characterizing the statistical performance under different channel conditions, we consider several fading models, as listed in Table 4. The Rayleigh fading is most widely used propagation model, applicable to the case where there is no signal component along a LOS. The Rician model describes a fading condition where there are both LOS and NLOS paths between the transmitter and the receiver [51]. The Nakagami-m fading is an empirically derived model using field measurement data, and hence is better matched to real propagation scenario compared to the Rician distribution. Therefore, we also consider Nakagami-m model as representing the propagation scenario including LOS and NLOS components. Moreover, a recently proposed general $\kappa-\mu$ model, which better fits experimental data than Rician or Nakagami-m models, is also used to represent the variation of the fading signal in the presence of LOS component.

The Rician and Nakagami-m models are utilized to represent the fading variation having both NLOS and LOS components. They are connected by the Rician factor $K$ and Nakagami-m shape factor $m$ both representing the fading severity. Therefore, the impact difference of the Rician and Nakagami-m fading on outage performance is evaluated. The Kullback-Leibler divergence (KLD) and Jensen-Shannon divergence (JSD) are used
to identify the difference between the Rician and Nakagami-m distributions. To further generalize the propagation model with a LOS component, this dissertation applies time-varying $K$ factor scenario, where the $K$ value in the Rician model is assumed to follow empirical distributions, normal or logistic distributions; they were derived from field measurement data [52], and hence recognized as being practical.

Theoretical analysis and performance evaluation of the generic cooperative network model is complicated. Furthermore, if dynamic network topology variation is taken into account, it imposes a lot of challenges. Instead, we decompose the general network into several network models having simple structures that are widely used in cooperative communication research, as summarized in Table 5.

Table 5. System model in each Chapter.

| Chapter | Network Topology | Fading Model |
| :---: | :--- | :--- |
| 2 | Three-Node One-Way Relaying | Nakagami-m Fading, and Cor- <br> related Rayleigh Fading |
| 3 | Three-Node Two-Way Relaying | Rician Fading with Random <br> $K$-factor |
| 4 | Two-Source Two-Relay Transmission | $\kappa$ - $\mu$ Fading |

Although only simple system structures are considered, the information theoretic analysis for these basic network models provides insights into understanding of more generic networks.

### 1.4 Outline of Dissertation

In Chapter 2, we investigate the performance of a three-node one-way relay system. First of all, the exact outage probability of LF relaying over Nakagami-m fading channels with arbitrary values of the shape factor $m$ is derived. With a yet accurate approximated expressions, the equivalent diversity order and coding gain of LF relaying are identified. Compared with conventional DF relaying, LF relaying can achieve even lower outage probability. We then analyze the impact of the spatial and temporal correlations of the fading variations on the system performance of LF, DF and ADF relaying. The diversity orders with $\mathrm{LF}, \mathrm{DF}$, and ADF are derived in the presence of
the spatial and temporal fading correlations. Obviously, the larger the correlation, the higher the outage probability. Chapter 2 provides formulas that represents this relationship in a mathematical way. It is found that the optimal relay locations yielding the smallest outage probability move towards to the destination, compared with the case in independent fading. Finally, the differences between the outage performances with LF relaying in Rician and Nakagami-m fading, are also investigated. The KullbackLeibler divergence (KLD) and Jensen-Shannon divergence (JSD), which represent the difference quantitatively between two probability distributions, are used to verify the performance difference on the outage curves with Rician and Nakagami-m distributions. The analytical results show that, even with the parameter settings yielding the same LOS-to-NLOS power ratio, Rician can not exactly replace the Nakagami-m model to represent the entire shape of distributions.

Chapter 3 propose a two-way LF relaying system utilizing non-orthogonal source-to-relay links transmission, and hence it can achieve significant spectral efficiency gain compared to one-way relaying. Another focus point of Chapter 3 is the impact analysis of time-varying LOS signal energy. We assume the channels suffer from Rician fading with random $K$ factor. The Rician $K$-factor is assumed to follow empirical distributions derived from measurement data, i.e., logistic and normal distributions. From the analytical results, we found that compared to two-way DF relaying, two-way LF relaying achieves lower outage probability regardless of either logistic or normal distribution is used to represent the variation of $K$-factor. This is because with LF relaying, the relay always broadcast the decoder output regardless of whether or not error is detected after decoding in the information part, and hence source correlation can be well exploited at the decoder of the destination.

Chapter 4 extends the major results of the previous chapters, and focuses on the problem of transmitting two sources to one destination over a two-relay transmission system. All the links experience $\kappa-\mu$ fading variation, which is more consistent to measurement data than Rician or Nakagami-m model. Two transmission, non-orthogonal maximum ratio transmission and orthogonal transmission with joint-decoding at the destination, are considered as the relay-destination transmission schemes. The theoretical analysis indicates that the outage performance of the two-source two-relay system with orthogonal transmission with joint-decoding scheme outperforms that with maximum ratio transmission scheme.

Chapter 5 summarizes the work and presents insight discussion for the future work.

### 1.5 Summary of Outcomes

This dissertation is written as a monograph based on two journal papers [53, 54], one letter [55], and two conference papers [56, 57]. The first journal paper [53] has already been published and the other [54] has been accepted. The author has the main responsibility for performing the analysis, generating the numerical results, and writing all the papers [53-57]. The other authors have provided comments and constructive criticisms.

Besides the aforementioned publications, the authors published four other conference papers [58-61] and co-authored several papers in the relevant topic [62-66] during the doctoral study. Furthermore, the author has been involved in creating Technical reports/Deliverables [67-71] under the 7th Framework Program (FP7) Links-on-thefly Technology for Robust, Efficient and Smart Communication in Unpredictable Environments (RESCUE) project.

Table 6. Summary of major contributions on LF relaying.

| Year | Authors | Contributions |
| :---: | :--- | :--- |
| 2005 | Hu and Li <br> $[37]$ | Proposed Slepian-Wolf cooperation, which exploits distributed <br> source coding technologies in wireless cooperative communica- <br> tions. |
| 2007 | Woldegebreal <br> and Karl <br> $[72]$ | Considered a network-coding-based MARC in the presence of <br> non-ideal source-relay links, and analyzed the outage performance <br> and coverage. |
| 2007 | Sneessens <br> et al. $[39]$ | Derived a decoding algorithm which enables the use of turbo- <br> coded DF relaying by taking into account the probability of error <br> between the source and the relay. |
| 2012 | Anwar and <br> Matsumoto <br> $[27]$ | Proposed an iterative decoding technique, accumulator-assisted <br> distributed turbo code, where the correlation knowledge between <br> the source and the relay is estimated and exploited. |
| 2013 | Cheng et al. <br> $[40]$ | Proposed a scheme for exploiting the source-relay correlation in <br> joint-decoding process at the destination, based on the Slepian- <br> Wolf theorem. |
| 2014 | Zhou et al. <br> $[28]$ | Derived the exact outage probability by utilizing the lossy source- <br> channel separation and source coding with a helper theorems. |
| 2015 | Wolf et al. <br> $[44]$ | Proposed an optimal power allocation strategy for a two-relay sys- <br> tem based on convex optimization to minimize outage probability. |
| 2015 | Lu et al. <br> $[42]$ | Derived the outage probability for orthogonal MARC for corre- <br> [4ated source transmission where erroneous source information <br> estimated at the relay is forwarded. |

## 2 On the Performance of One-Way <br> Lossy-Forward Relay Wireless Networks

In this chapter, we investigate the performance of three-node LF relaying over independent block Nakagami-m fading channels. Based on the source coding with a helper theorem, the outage probability expression with arbitrary values of the shape factor $m$ is derived. Then, we identify the impact of the spatial and temporal correlations of the fading variations on the outage performances of LF, DF and ADF relaying in correlated Rayleigh fading. Finally, the impact difference of the Rician and Nakagami-m fading on outage performance of the one-way LF relaying system is evaluated.

### 2.1 Lossy-Forward Transmission over independent Nakagami-m Fading Channels

### 2.1.1 System Model

We consider a simple three-node relaying system as shown in Fig. 2. The source S communicates with the destination D with the help of a single relay R . The location of R varies in a line parallel to the line connecting $S$ and $D$ between $x=0$ (nearest to $S$ ) and $x=1$ (nearest to D ). Unless otherwise specified, the distant between R and the line connecting S and D is set at $\frac{1}{2}$ of S-D link length. We assume time-division transmission, where the overall transmission is divided into two time slots. In the first time slot, the original uniformly distributed binary information sequence $b_{S}$ is encoded and broadcast from S . The relay R aims to recover the information sequence, and always re-interleaves the information sequence, re-encodes it and forwards the encoder output to D in the second time slot, even though the decoding result may contain error in the original information sequence.

## LF Relaying

In conventional DF relaying, R keeps silent if error is detected after decoding in the information sequence sent through the S-R link. With LF relaying, after receiving the signal from $\mathrm{S}, \mathrm{R}$ attempts to recover $b_{\mathrm{S}}$. Although the decoding result of $b_{\mathrm{S}}$ at R ,


Fig. 2. The schematic of one relay aided communication system, where all the links (the S-R, R-D and S-D links) in Nakagami-m fading.
denoted by $b_{\mathrm{R}}$, may be found to contain error, R re-interleaves the information sequence $b_{\mathrm{R}}$, re-encodes the re-interleaved sequence, and forwards it to D .

The S-R link is virtually modeled by a binary symmetric channel (BSC) model with a crossover probability $p_{f}$. More specifically, $p_{f}$ represents the bit flipping probability between the information sequence obtained after decoding at R and the original information sequence sent from S. Hence, $b_{\mathrm{R}}=b_{\mathrm{S}} \oplus e$, where $\oplus$ denotes the modulo- 2 addition and $e$ is a realization of a binary random variable $E$ with $\operatorname{Pr}(E=1)=p_{f} . p_{f}$ stays the same within each block while changes block-by-block with its value being determined by the instantaneous SNR.

At D , after receiving the signals from S and R , joint decoding is performed to retrieve the original information $b_{\mathrm{S}}$. Iterative decoding is utilized between two decoders for decoding the information sent from $S$ and $R$. In the decoding process, the $\mathrm{S}-\mathrm{R}$ link error probabilities $p_{f}$ can be estimated at D . The estimated $p_{f}$ value is used as the correlation knowledge between $b_{\mathrm{S}}$ and $b_{\mathrm{R}}$ [27]. The LLRs of the systematic bits are exchanged between the two decoders via the interleaver/de-interleaver. Therefore, the system, as a whole, can be viewed as a distributed turbo code.

## Channel Model

The signals received at D and R in the first time slot, $y_{\mathrm{D}, 1}$ and $y_{\mathrm{R}, 1}$, respectively, and the signal received at D in the second time slot, $y_{\mathrm{D}, 2}$, are expressed as ${ }^{3}$

$$
\begin{align*}
& y_{\mathrm{D}, 1}=\sqrt{G_{\mathrm{SD}}} h_{\mathrm{SD}} x_{1}+n_{\mathrm{D}, 1},  \tag{2}\\
& y_{\mathrm{R}, 1}=\sqrt{G_{\mathrm{SR}}} h_{\mathrm{SR}} x_{1}+n_{\mathrm{R}, 1},  \tag{3}\\
& y_{\mathrm{D}, 2}=\sqrt{G_{\mathrm{RD}}} h_{\mathrm{RD}} x_{2}+n_{\mathrm{D}, 2}, \tag{4}
\end{align*}
$$

respectively, where $G_{i j}(i \in(\mathrm{~S}, \mathrm{R}), j \in(\mathrm{R}, \mathrm{D}), i \neq j)$ are the geometric gains related to the transmit distance of each link. The modulated symbols transmitted from S and R are denoted by $x_{1}$ and $x_{2}$, respectively. $h_{i j}$ denotes the complex channel gain and $n_{j, 1}$ and $n_{j, 2}$ are zero-mean AWGN with variance of $N_{0} / 2$ per dimension. It is assumed that $E\left[\left|h_{i j}\right|^{2}\right]=1$ and $h_{i j}$ stays constant over one block duration due to the block fading assumption. We assume that the channel state information (CSI) is only available at the receiver side.

The transmit energy of each symbol is denoted as $E\left\{\left|x_{1}\right|^{2}\right\}=E\left\{\left|x_{2}\right|^{2}\right\}=E_{s}$. Therefore, the average and instantaneous SNR of each link is expressed as $\bar{\gamma}_{i j}=G_{i j} \frac{E_{s}}{N_{0}}$ and $\gamma_{i j}=\left|h_{i j}\right|^{2} \bar{\gamma}_{i j}(i \in(\mathrm{~S}, \mathrm{R}), j \in(\mathrm{R}, \mathrm{D}), i \neq j)$, respectively. For the sake of simplicity, the variations due to shadowing and the fading frequency selectivity are not taken into account.

We assume that all the links (i.e., the S-R, S-D, and R-D links) suffer from independent block Nakagami-m fading, with the probability density function (pdf) of $\gamma_{i j}$ given by

$$
\begin{equation*}
p\left(\gamma_{i j}\right)=\frac{m_{i j}^{m_{i j}}\left(\gamma_{i j}\right)^{m_{i j}-1}}{\left(\bar{\gamma}_{i j}\right)^{m_{i j}} \Gamma\left(m_{i j}\right)} \exp \left(-\frac{m_{i j} \gamma_{i j}}{\bar{\gamma}_{i j}}\right), m_{i j}>0.5, \tag{5}
\end{equation*}
$$

where $\Gamma(\cdot)$ is the Gamma function. The shape factor $m_{i j}$ represents the severity of the fading variation of each link.

We consider a scenario that the S-D link suffers from severe fading and the destination needs the help of a relay via S-R and R-D links, which suffer from mild fading, i.e., not as severe as the S-D link. Hence, we set the shape factor of the S-D link $m_{\mathrm{SD}}=1$, corresponding to Rayleigh fading, while for other links (the S-R and R-D links), we set their $m$ values as parameters $\left(m_{\mathrm{SR}}>1, m_{\mathrm{RD}}>1\right)$.

[^1]

Fig. 3. Block diagram for the coding/decoding of $b_{\mathrm{S}}$ and $b_{\mathbf{R}}$ from the view of source coding with a helper. $b_{\mathbf{R}}$ is the bit-flipped version of $b_{\mathbf{S}}$ and serves as a helper for the decoding of $b_{\mathrm{S}}$ at the decoder.

### 2.1.2 Outage Probability Analysis

In this section, the definition of admissible rate region and the derivation for outage probability of LF relaying are provided. The outage probability is derived based on both Gaussian codebook capacity and constellation constrained capacity assumptions.

## Admissible Rate Region Based on Source Coding with a Helper Theorem

In LF relaying, D aims to recover $b_{\mathrm{S}}$ only and the $b_{\mathrm{R}}$ transmitted from R does not need to be decoded successfully. Therefore, the analysis falls into the problem category of source coding with a helper. In other words, in the LF relay system, $b_{\mathrm{R}}$ servers as a helper for the successful recovery of $b_{\mathrm{S}}$, as shown in Fig. 3. Assume $b_{\mathrm{S}}$ and $b_{\mathrm{R}}$ are described with rates $R_{\mathrm{S}}$ and $R_{\mathrm{R}}$, respectively. According to the source coding with a helper theorem [30, Section10.4]

Theorem 1. Lossless Source Coding with a Helper $b_{\mathrm{S}}$ can be recovered with an arbitrarily small probability of error if the rate pair $\left(R_{\mathrm{S}}, R_{\mathrm{R}}\right)$ satisfies

$$
\left\{\begin{array}{l}
R_{\mathrm{S}} \geq H\left(b_{\mathrm{S}} \mid \hat{b}_{\mathrm{R}}\right),  \tag{6}\\
R_{\mathrm{R}} \geq I\left(b_{\mathrm{R}} ; \hat{b}_{\mathrm{R}}\right),
\end{array}\right.
$$

where $\hat{b}_{\mathrm{R}}$ is the estimate of $b_{\mathrm{R}}$ obtained at the decoder of the destination. $H(\cdot \mid \cdot)$ and $I(\cdot ; \cdot)$ denote the conditional entropy and the mutual information between their arguments, respectively. Eq. (6) indicates that $I\left(b_{\mathrm{R}} ; \hat{b}_{\mathrm{R}}\right)$ bits per symbol can be used to describe $b_{\mathrm{R}}$. Then, $b_{\mathrm{S}}$ can be described at the rate of $H\left(b_{\mathrm{S}} \mid \hat{b}_{\mathrm{R}}\right)$ bits per symbol in the presence of a helper $\hat{b}_{\mathrm{R}}$.


Fig. 4. Rate region for $\mathbf{S}$ and $\mathbf{R}$ when $p_{f}=0$; the red solid line with bars separates the admissible and inadmissible regions.

With the block fading assumption, we also use a BSC model to represent the R-D link, as $\hat{b}_{\mathrm{R}}=b_{\mathrm{R}} \oplus e^{\prime}$ with $\operatorname{Pr}\left(E^{\prime}=1\right)=p_{f}^{\prime}$, where $e^{\prime}$ is a realization of a binary random variable $E^{\prime}$. Since the source is assumed to be binary, uniform, and independently and identically distributed (i.i.d), (6) can be expressed as [28]

$$
\left\{\begin{array}{l}
R_{\mathrm{S}} \quad \geq H\left(p_{f} * p_{f}^{\prime}\right)  \tag{7}\\
R_{\mathrm{R}} \quad \geq H\left(\hat{b}_{\mathrm{R}}\right)-H\left(\hat{b}_{\mathrm{R}} \mid b_{\mathrm{R}}\right)=1-H\left(p_{f}^{\prime}\right)
\end{array}\right.
$$

where $p_{f} * p_{f}^{\prime}=\left(1-p_{f}\right) p_{f}^{\prime}+\left(1-p_{f}^{\prime}\right) p_{f}$ and $H(\cdot)$ denotes the binary entropy function. $p_{f}=0$ indicates perfect decoding at R , and hence $H\left(b_{\mathrm{S}} \mid b_{\mathrm{R}}\right)=H\left(b_{\mathrm{R}} \mid b_{\mathrm{S}}\right)=0$. In this case, the inadmissible rate region becomes the triangle area A as shown in Fig. 4. When $0<p_{f} \leq 0.5$, the inadmissible region, which can be divided into two areas, B and C, is shown in Fig. 5.

If the relay sends $b_{\mathrm{R}}$ to the destination without error, $\hat{b}_{\mathrm{R}}=b_{\mathrm{R}}$ and $p_{f}^{\prime}=0$, then, the condition is $R_{\mathrm{R}} \geq H\left(b_{\mathrm{R}}\right)$ and $R_{\mathrm{S}} \geq H\left(b_{\mathrm{S}} \mid b_{\mathrm{R}}\right)$. In the case $p_{f}^{\prime}=0.5$, which indicates that $\hat{b}_{\mathrm{R}}$ is totally wrong and does not contain any useful information about $b_{\mathrm{R}}$. Therefore, the condition becomes as $R_{\mathrm{S}} \geq 1$ and $R_{\mathrm{R}}=0$. In any case of $0<p_{f}^{\prime}<0.5$, the condition is $R_{\mathrm{S}} \geq H\left(p_{f} * p_{f}^{\prime}\right)$ and $R_{\mathrm{R}} \geq 1-H\left(p_{f}^{\prime}\right)$.


Fig. 5. Rate region for $\mathbf{S}$ and $\mathbf{R}$ when $p_{f} \neq 0$; the red solid line with bars separates the admissible and inadmissible regions.

## Outage Event of LF relaying

If the rate pair ( $R_{\mathrm{S}}, R_{\mathrm{R}}$ ) falls into the inadmissible regions in Fig. 4 or Fig. 5, the outage event occurs and D cannot guarantee the reconstruction of $b_{\mathrm{S}}$ with an arbitrarily small error probability. The outage probability of LF relaying can be expressed as

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}}=P_{\mathrm{A}}+P_{\mathrm{B}}+P_{\mathrm{C}}, \tag{8}
\end{equation*}
$$

where $P_{\mathrm{A}}, P_{\mathrm{B}}$, and $P_{\mathrm{C}}$ denotes the probability that $\left(R_{\mathrm{S}}, R_{\mathrm{R}}\right)$ falls into the inadmissible areas $\mathrm{A}, \mathrm{B}$, and C , respectively. Taking into account the impact of $p_{f}$ and $p_{f}^{\prime}, P_{\mathrm{A}}, P_{\mathrm{B}}$, and $P_{\mathrm{C}}$ can further be expressed as

$$
\begin{align*}
& P_{\mathrm{A}}=\operatorname{Pr}\left[p_{f}=0,0 \leq R_{\mathrm{S}} \leq 1,0 \leq R_{\mathrm{R}} \leq H\left(p_{f} * p_{f}^{\prime}\right)\right],  \tag{9}\\
& P_{\mathrm{B}}=\operatorname{Pr}\left[0<p_{f} \leq 0.5,0 \leq R_{\mathrm{S}}<H\left(p_{f}\right), R_{\mathrm{R}} \geq 0\right],  \tag{10}\\
& P_{\mathrm{C}}=\operatorname{Pr}\left[0<p_{f} \leq 0.5, H\left(p_{f}\right) \leq R_{\mathrm{S}} \leq 1,\right. \\
& \left.\quad 0 \leq R_{\mathrm{R}}<H\left(p_{f} * p_{f}^{\prime}\right)\right] . \tag{11}
\end{align*}
$$

## (A) Outage Derivation with Gaussian Codebook Capacity

For calculating the outage probability, first we establish the relationship between $\gamma_{\mathrm{SD}}$ and $R_{\mathrm{S}}$, and the relationship between $\gamma_{\mathrm{RD}}$ and $R_{\mathrm{R}}$. According to the Shannon's lossless source channel separation theorem [73], if the total information transmission rates over the S-D and R-D links satisfy

$$
\left\{\begin{array}{l}
H\left(b_{\mathrm{S}}\right) \cdot R_{\mathrm{SD}}^{c} \leq R_{\mathrm{S}} \cdot R_{\mathrm{SD}}^{c} \leq C_{\mathrm{SD}}\left(\gamma_{\mathrm{SD}}\right)  \tag{12}\\
H\left(b_{\mathrm{R}}\right) \cdot R_{\mathrm{RD}}^{c} \leq R_{\mathrm{R}} \cdot R_{\mathrm{RD}}^{c} \leq C_{\mathrm{RD}}\left(\gamma_{\mathrm{RD}}\right)
\end{array}\right.
$$

the error probability can be arbitrarily small at the destination. $C_{i \mathrm{D}}$ and $R_{i \mathrm{D}}^{c}$, respectively, denote the channel capacity of the $i$-D link and the normalized spectrum efficiency of the corresponding transmission chain. The normalized spectrum efficiency includes the channel coding and the modulation multiplicity (constellation size). With the assumption that Gaussian codebook is used, the channel capacity $C_{i \mathrm{D}}$ of each link can be expressed as

$$
\begin{equation*}
C_{i \mathrm{D}}\left(\gamma_{i \mathrm{D}}\right)=\frac{E^{n}}{2} \log \left(1+\frac{2 \gamma_{i \mathrm{D}}}{E^{n}}\right),(i \in \mathrm{~S}, \mathrm{R}), \tag{13}
\end{equation*}
$$

where $E^{n}$ denotes the dimensionality of the channel input, e.g., $E^{n}=1$ for binary phase shift keying (BPSK) and $E^{n}=2$ for quaternary phase shift keying (QPSK). Hence, the relationship between the instantaneous channel SNR $\gamma_{i \mathrm{D}}$ and its corresponding source coding rate $R_{i}$ is given by

$$
\begin{equation*}
R_{i} \leq \Theta\left(\gamma_{i \mathrm{D}}\right)=\frac{C_{i \mathrm{D}}\left(\gamma_{i \mathrm{D}}\right)}{R_{i \mathrm{D}}^{c}}=\frac{E^{n}}{2 R_{i \mathrm{D}}^{c}} \log \left(1+\frac{2 \gamma_{i \mathrm{D}}}{E^{n}}\right),(i \in \mathrm{~S}, \mathrm{R}) \tag{14}
\end{equation*}
$$

with its inverse inequality

$$
\begin{equation*}
\gamma_{i \mathrm{D}} \geq \Theta^{-1}\left(R_{i}\right)=\frac{E^{n}}{2}\left(2^{\frac{2 R_{i} i i_{i \mathrm{D}}^{c}}{E^{n}}}-1\right) \tag{15}
\end{equation*}
$$

Then, we establish the relationship between $\gamma_{\mathrm{SR}}$ and $p_{f}$. Since $p_{f}$ only depends on the quality of the $\mathrm{S}-\mathrm{R}$ link, according to Shannon's lossy source channel separation theorem [74], we have

$$
\begin{equation*}
R(\mathscr{D}) \cdot R_{\mathrm{SR}}^{c} \leq C_{\mathrm{SR}}\left(\gamma_{\mathrm{SR}}\right), \tag{16}
\end{equation*}
$$

where $R(\mathscr{D})$ denotes the source rate-distortion function with the distortion measure $\mathscr{D}$.
In the case of binary transmission, the Hamming distortion between a source bit $x$ and its estimate $\hat{x}$ given by

$$
d(x, \hat{x})= \begin{cases}0 & \text { if } x=\hat{x},  \tag{17}\\ 1 & \text { if } x \neq \hat{x}\end{cases}
$$



Fig. 6. S-R link Hamming distortion $p_{f}$ versus $\operatorname{SNR}$, where $E_{n}=2$ and $R_{\mathrm{SR}}^{c}=1$.
is used as the distortion measure.
With the bit-wise Hamming distortion measure, the sequence-wise distortion measure $\mathscr{D}$ is equivalent to the crossover probability $p_{f}$, since both $p_{f}$ and $\mathscr{D}$ can be regarded as the theoretical BER of the S-R transmission with long enough sequence length. The rate-distortion function is represented as $R(\mathscr{D})=1-H(\mathscr{D})$ for i.i.d binary source [75].

Since we assume that Gaussian codebook is used for the S-R link transmission, the relationship between the required instantaneous channel SNR $\gamma_{\text {SR }}$ and its corresponding source rate $R(\mathscr{D})$ with distortion $\mathscr{D}$ is given by

$$
\begin{equation*}
\gamma_{\mathrm{SR}} \geq \frac{E^{n}}{2}\left(2^{\frac{2 R(\mathscr{O}) R_{S R}^{c}}{E^{n}}}-1\right) \tag{18}
\end{equation*}
$$

Then, we can obtain the relationship between $p_{f}$ and $\gamma_{\mathrm{SR}}$ as

$$
\begin{equation*}
p_{f}=H^{-1}\left(1-\frac{\frac{E^{n}}{2} \log \left(1+\frac{2 \gamma_{\mathrm{SR}}}{E^{n}}\right)}{R_{\mathrm{SR}}^{c}}\right) \tag{19}
\end{equation*}
$$

with $H^{-1}(\cdot)$ denoting the inverse function of $H(\cdot)$. The relationship between the S-R link SNR and $p_{f}$ is shown in Fig. 6. It is found from Fig. 6 that the value of error probability $p_{f}$ decreases as the SNR of the S-R link increases. Note that $p_{f}$ stays
constant within each block duration, but changes block-by-block, because of the block fading assumption.

With the Nakagami-m block fading assumption, the outage probability expressions with LF relaying in (8) can be expressed as

$$
\begin{align*}
P_{\mathrm{A}}= & \operatorname{Pr}\left[\gamma_{\mathrm{SR}} \geq \Theta^{-1}(1), \Theta^{-1}(0) \leq \gamma_{\mathrm{SD}} \leq \Theta^{-1}(1), \Theta^{-1}(0) \leq \gamma_{\mathrm{RD}}<\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)\right] \\
= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(1)}^{\Theta^{-1}(\infty)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)} p\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p\left(\gamma_{\mathrm{RD}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \frac{1}{\gamma_{\mathrm{SD}}}\left(1-\left[\frac{\gamma\left(m_{\mathrm{SR}}, m_{\mathrm{SR}} \frac{1}{\gamma_{\mathrm{SR}}}\right)}{\Gamma\left(m_{\mathrm{SR}}\right)}\right]\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \cdot\left[\frac{\gamma\left(m_{\mathrm{RD}}, m_{\mathrm{RD}} \frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)}{\Gamma\left(m_{\mathrm{RD}}\right)}\right] d \gamma_{\mathrm{SD}},  \tag{20}\\
P_{\mathrm{B}}= & \operatorname{Pr}\left[\Theta^{-1}(0) \leq \gamma_{\mathrm{SR}} \leq \Theta^{-1}(1), \Theta^{-1}(0) \leq \gamma_{\mathrm{SD}} \leq \Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right), \gamma_{\mathrm{RD}} \geq \Theta^{-1}(0)\right] \\
= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(\infty)} p\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p\left(\gamma_{\mathrm{RD}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \frac{m_{\mathrm{SR}} m_{\mathrm{SR}}\left(\gamma_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}-1}}{\left(\bar{\gamma}_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}} \Gamma\left(m_{\mathrm{SR}}\right)} \exp \left(-\frac{m_{\mathrm{SR}} \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right)\left[1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right] d \gamma_{\mathrm{SR}} \tag{21}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{C}}= & \operatorname{Pr}\left\{\Theta^{-1}(0) \leq \gamma_{\mathrm{SR}} \leq \Theta^{-1}(1), \Theta^{-1}\left[1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right]\right. \\
& \left.\leq \gamma_{\mathrm{SD}} \leq \Theta^{-1}(1), \Theta^{-1}(0) \leq \gamma_{\mathrm{RD}} \leq \Theta^{-1}\left[\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)\right]\right\} \\
= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left[\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)\right]} p\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p\left(\gamma_{\mathrm{RD}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \exp \left(\frac{-\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \frac{m_{\mathrm{SR}}^{m}\left(\gamma_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}-1}}{\left(\bar{\gamma}_{\mathrm{SR}}\right)^{m}{ }^{m} \Gamma\left(m_{\mathrm{SR}}\right)} \exp \left(-\frac{m_{\mathrm{SR}} \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) \\
& \cdot\left[\frac{\gamma\left(m_{\mathrm{RD}}, m_{\mathrm{RD}} \frac{\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)}{\Gamma\left(m_{\mathrm{RD}}\right)}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{SR}}, \tag{22}
\end{align*}
$$

where $\xi\left(\gamma_{\text {SD }}, \gamma_{\mathrm{SR}}\right)=H\left\{H^{-1}\left[1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right] * H^{-1}\left[1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right]\right\}$ and $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function. Since we assume i.i.d. binary sequences, $H\left(b_{\mathrm{S}}\right)=H\left(b_{\mathrm{R}}\right)=$ 1.


Fig. 7. Outage probability comparison among the LF, DF, and S-R-D transmission with $m_{\mathrm{SD}}=$ 1.

Fig. 7 presents the outage probabilities of DF and LF relaying, denoted as $P_{\text {out }}^{\mathrm{DF}}$ and $P_{\text {out }}^{\mathrm{LF}}$. It is found that LF achieves lower outage probability than DF. This is because LF always forwards information sequences from the relay to the destination. The system can be regarded as a distributed turbo code. Note that the theoretical outage probabilities well match their corresponding simulation results obtained by Monte Carlo method. As a reference, the outage probability of a S-R-D transmission without direct S-D link, denoted as $P_{\text {out }}^{S-R-D}$, is also plotted in the figure. With $m_{\mathrm{SD}}=m_{\mathrm{SR}}=m_{\mathrm{RD}}=1$, i.e., all the links suffer from Rayleigh fading, $P_{\text {out }}^{S-R-D}$ only achieves first-order diversity since only one copy of the original information arrive at the destination and hence no diversity gain achieved. With $m_{\mathrm{SR}}=m_{\mathrm{RD}}=5$, i.e., the S-R and R-D links experience milder fading than the S-D link, $P_{\text {out }}^{\text {S-R-D }}$ can achieve almost the same decay in the outage curve as $P_{\text {out }}^{\mathrm{DF}}$ and $P_{\mathrm{out}}^{\mathrm{LF}}$. This is because with large $m_{\mathrm{SR}}$ and $m_{\mathrm{RD}}$, the S-R and R-D links with mild fading dominate the system performance. However, the outage probabilities of relaying with the direct S-D link, i.e., $P_{\text {out }}^{\mathrm{DF}}$ and $P_{\text {out }}^{\mathrm{LF}}$, are always lower than that without the direct S-D link, i.e., $P_{\text {out }}^{S-R-D}$, due to the spatial diversity gain provided by the S-D link.


Fig. 8. Comparison of the Gaussian codebook capacity and constellation constrained capacity based outage probability of LF relaying, where $m_{\mathrm{SR}}=m_{\mathrm{RD}}, m_{\mathrm{SD}}=1$, and $G_{\mathrm{SD}}=G_{\mathrm{RD}}=G_{\mathrm{SR}}$.

## (B) Outage Derivation with Constellation Constrained Capacity

According to Shannon's source channel separation theorem, if $H\left(b_{i}\right) \cdot R_{i \mathrm{D}}^{c} \leq R_{i} \cdot R_{i \mathrm{D}}^{c} \leq$ $C_{c c}\left(\gamma_{i \mathrm{D}}\right),(i \in \mathrm{~S}, \mathrm{R})$ is satisfied, the error probability can be made arbitrarily small at the destination, where $C_{c c}\left(\gamma_{\mathrm{i}}\right)$ is the channel constellation constrained capacity with the instantaneous SNR $\gamma_{i \mathrm{D}}$ of the $i$-D link. Since the constellation constrained capacity has no closed-form expression, numerical evaluation yields a relationship between $\gamma_{i \mathrm{D}}$ and $R_{i}$ as $\gamma_{i \mathrm{D}} \geq C_{c c}^{-1}\left(R_{i} \cdot R_{i \mathrm{D}}^{c}\right)$, where $C_{c c}^{-1}$ represents the inverse function of channel constellation constrained capacity.

Similarly, the relationship between $\gamma_{\mathrm{SR}}$ and $p_{f}$ is given by $R(\mathscr{D}) \cdot R_{\mathrm{SR}}^{c} \leq C_{c c}\left(\gamma_{\mathrm{SR}}\right)$ and $\gamma_{\mathrm{SR}} \geq C_{c c}^{-1}\left(R(\mathscr{D}) \cdot R_{\mathrm{SR}}^{c}\right)$, where the distortion $\mathscr{D}$ is equivalent to $p_{f}$ for a given instantaneous S-R link SNR $\gamma_{\text {SR }}$. Then, by using the method for calculating the outage probability with Gaussian codebook capacity, the outage probability of LF relaying with constellation constrained capacity can be calculated.

Fig. 8 compares the outage probabilities of LF relaying derived from Gaussian codebook capacity and constellation constrained capacity, denoted as $P_{\text {out }}^{\mathrm{LF}}, \mathrm{GCC}$ and
$P_{\text {out }}^{\mathrm{LF}}, \mathrm{CCC}$, respectively. ${ }^{4}$ The Monte Carlo simulation results of the outage probabilities with Gaussian codebook capacity are also presented. We can observe from Fig. 8 that the theoretical outage probabilities with LF relaying, calculated by using numerical integral in (20), (21), and (22), well match their corresponding Monte Carlo simulation results. It is also found that, the difference between $P_{\text {out }}^{\mathrm{LF}}, \mathrm{GCC}$ and $P_{\text {out }}^{\mathrm{LF}}, \mathrm{CCC}$ is negligible. This is because in low SNR regime the difference between Gaussian codebook capacity and constellation constrained capacity is very small. On the contrary, in high SNR region, even though the difference between Gaussian codebook capacity and constellation constrained capacity becomes large, $p_{f}$ approaches zero very quickly, resulting in lossless transmission over the S-R link. Therefore, the difference between Gaussian codebook capacity and constellation constrained capacity based outage probabilities is negligibly small.

### 2.1.3 Equivalent Diversity Order and Coding Gain

The equivalent diversity order (decay of the outage curve) and coding gain can be obtained by approximating (20), (21), and (22) at high SNR region. Invoking the series representation of incomplete gamma function $\gamma(a, x)=\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{a+n}}{n!(a+n)}$ [76, equation 8.354.1], together with the approximation $p\left(\gamma_{i j}\right) \approx \frac{m_{i j}\left(\gamma_{i j}\right)^{m_{i j}-1}}{\left(\bar{\gamma}_{i j}\right)^{m_{i j}} \Gamma\left(m_{i j}\right)}$ [50], after several steps of mathematical manipulations, the outage probability expression with LF relaying over Nakagami-m fading channels can be approximated as

$$
\begin{align*}
& P_{\mathrm{A}} \approx A \cdot A^{\prime} \cdot \bar{\gamma}_{\mathrm{SD}}^{\left(-m_{\mathrm{SD}}\right)} \bar{\gamma}_{\mathrm{RD}}^{\left(-m_{\mathrm{RD}}\right)},  \tag{23}\\
& P_{\mathrm{B}} \approx B \cdot \bar{\gamma}_{\mathrm{SD}}^{\left(-m_{\mathrm{SD}}\right)} \bar{\gamma}_{\mathrm{SR}}^{\left(-m_{\mathrm{SR}}\right)},  \tag{24}\\
& P_{\mathrm{C}} \approx C \cdot \bar{\gamma}_{\mathrm{SD}}^{\left(-m_{\mathrm{SD}}\right)} \bar{\gamma}_{\mathrm{RD}}^{\left(-m_{\mathrm{RD}}\right)} \bar{\gamma}_{\mathrm{SR}}^{\left(-m_{\mathrm{SR}}\right)}, \tag{25}
\end{align*}
$$

[^2]

Fig. 9. Comparison of outage curves obtained by using the numerical calculation via (20), (21), and (22) and approximation via (23), (24), and (25), where $m_{\mathrm{SD}}=1$ and $G_{\mathrm{SD}}=G_{\mathrm{RD}}=G_{\mathrm{SR}}$.
where

$$
\begin{align*}
A= & \frac{m_{\mathrm{SD}}^{m_{\mathrm{SD}}} m_{\mathrm{RD}}^{m_{\mathrm{RD}}-1}}{\Gamma\left(m_{\mathrm{SD}}\right) \Gamma\left(m_{\mathrm{RD}}\right)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \frac{\left(\frac{2}{1+\gamma_{\mathrm{SD}}}-1\right)^{m_{\mathrm{RD}}}}{\gamma_{\mathrm{SD}}^{1-m_{\mathrm{SD}}}} d \gamma_{\mathrm{SD}}  \tag{26}\\
B= & \frac{m_{\mathrm{SR}}^{m_{\mathrm{SR}}} m_{\mathrm{SD}}^{m_{\mathrm{SD}}-1}}{\Gamma\left(m_{\mathrm{SD}}\right) \Gamma\left(m_{\mathrm{SR}}\right)} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \frac{\left(\frac{2}{1+\gamma_{\mathrm{SR}}}-1\right)^{m_{\mathrm{SD}}}}{\gamma_{\mathrm{SR}}^{m_{\mathrm{SR}}-1}} d \gamma_{\mathrm{SR}}  \tag{27}\\
C= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\gamma_{\mathrm{SD}}^{m_{\mathrm{SD}}^{-1}} \gamma_{\mathrm{SR}}^{m_{\mathrm{SR}}-1}} \\
& \frac{m_{\mathrm{SR}}^{m_{\mathrm{SR}}} m_{\mathrm{SD}}^{m_{\mathrm{SD}}} m_{\mathrm{RD}}^{m_{\mathrm{RD}}^{-1}}}{\Gamma\left(m_{\mathrm{SD}}\right) \Gamma\left(m_{\mathrm{RD}}\right) \Gamma\left(m_{\mathrm{SR}}\right)} \xi^{m_{\mathrm{SD}}}\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} \tag{28}
\end{align*}
$$

$A^{\prime}=1-\frac{\gamma\left(m_{\mathrm{SR}}, m_{\mathrm{SR}} \frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)}{\Gamma\left(m_{\mathrm{SR}}\right)}$ indicates the probability of $p_{f}=0$ (i.e., no decoding error at R ).
Fig. 9 shows that at high SNR region the approximated outage curves obtained from (23), (24), and (25) well match the numerically calculated curves obtained from (20), (21), and (22). From this observation, it can be concluded that the approximation is sufficiently accurate to calculate the outage probability.

Since the exponent parts in (23), (24), and (25) stay unchanged even if we replace $\bar{\gamma}_{\mathrm{SD}}, \bar{\gamma}_{\mathrm{RD}}$, and $\bar{\gamma}_{\mathrm{SR}}$ by a certain representative value $\bar{\gamma}$, we have

$$
\begin{align*}
& P_{\mathrm{A}} \approx A \cdot A^{\prime} \cdot \bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)},  \tag{29}\\
& P_{\mathrm{B}} \approx B \cdot \bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)},  \tag{30}\\
& P_{\mathrm{C}} \approx C \cdot \bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}-m_{\mathrm{SR}}\right)} . \tag{31}
\end{align*}
$$

It is observed from (29), (30), and (31) that, $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}-m_{\mathrm{SR}}\right)}$ is upper bounded by $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)}$ or $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)}$ when $\bar{\gamma}$ increases. Moreover, it is not difficult to find that $A^{\prime}$ equals one asymptotically at high SNR regime. Therefore, the overall outage probability of LF relaying in Nakagami-m fading can be formulated as

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}}=\left(G_{c} \cdot \bar{\gamma}\right)^{\left(-G_{d}\right)}, \tag{32}
\end{equation*}
$$

where

$$
\begin{equation*}
G_{d}=m_{\mathrm{SD}}+\min \left(m_{\mathrm{SR}}, m_{\mathrm{RD}}\right) \tag{33}
\end{equation*}
$$

and

$$
G_{c}= \begin{cases}\frac{1}{\left(\sigma_{d \sqrt{ } \sqrt{B}}\right.}, & m_{\mathrm{SR}}<m_{\mathrm{RD}}  \tag{34}\\ \frac{1}{\left(\sigma_{d} \sqrt{A+B}\right.}, & m_{\mathrm{SR}}=m_{\mathrm{RD}} \\ \frac{1}{\left(\sigma_{d} \sqrt{A} \sqrt{A}\right.}, & m_{\mathrm{SR}}>m_{\mathrm{RD}}\end{cases}
$$

are the equivalent diversity order ${ }^{5}$ and coding gain [50] achieved by LF relaying, respectively. We can see from (33) that, the equivalent diversity order of the LF relaying system is restricted by the less reliable channel of either the S-R or the R-D link.

Since $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}-m_{\mathrm{SR}}\right)}$ decreases faster than that of $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)}$ and $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)}$, as $\bar{\gamma}$ increases, i.e., $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}-m_{\mathrm{SR}}\right)}$ approaches zero faster than $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)}$ and $\bar{\gamma}^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)}$ alone, the outage probability is asymptotically equal to

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}} \approx\left(\frac{1}{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right) / \sqrt{A}} \cdot \bar{\gamma}\right)^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)}+\left(\frac{1}{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}} / \sqrt{B}\right.} \cdot \bar{\gamma}\right)^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)} \tag{35}
\end{equation*}
$$

If $m_{\mathrm{SR}}<m_{\mathrm{RD}}$, the first term in (35) decreases faster than the second term. Therefore,

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}} \approx\left(\frac{1}{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)} \sqrt{B} \cdot \bar{\gamma}\right)^{\left(-m_{\mathrm{SD}}-m_{\mathrm{SR}}\right)} \tag{36}
\end{equation*}
$$

[^3]with $G_{c}=\frac{1}{\left(-m_{\mathrm{SD}}{ }^{-m_{\mathrm{SR}} \sqrt{B}}\right.}$ and $G_{d}=m_{\mathrm{SD}}+m_{\mathrm{SR}}$. If $m_{\mathrm{SR}}>m_{\mathrm{RD}}$, the second term in (35) decreases faster than the first term. Therefore,
\[

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}} \approx\left(\frac{1}{\left(-m_{\mathrm{SD}}{ }^{\left.-m_{\mathrm{RD}}\right)} \sqrt{A}\right.} \cdot \bar{\gamma}\right)^{\left(-m_{\mathrm{SD}}-m_{\mathrm{RD}}\right)} \tag{37}
\end{equation*}
$$

\]

with $G_{c}=\frac{1}{\left(-m_{\mathrm{SD}}{ }^{-m_{\mathrm{RD}} \sqrt{A}}\right.}$ and $G_{d}=m_{\mathrm{SD}}+m_{\mathrm{RD}}$. If $m_{\mathrm{SR}}=m_{\mathrm{RD}}=m_{0}$, the first term in (35) decreases at the same order as the first term. Therefore,

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{LF}} \approx\left(\frac{1}{\left(-m_{\mathrm{SD}}-m_{\mathrm{a}} / \sqrt{A+B}\right.} \cdot \bar{\gamma}\right)^{\left(-m_{\mathrm{SD}}-m_{0}\right)} \tag{38}
\end{equation*}
$$

with $G_{c}=\frac{1}{\left(-m_{\mathrm{SD}}-m_{0}\right)} \sqrt{A+B}$ and $G_{d}=m_{\mathrm{SD}}+m_{0}$.
The outage curves shown in Fig. 9 illustrate that the decay in outage curve cannot be obtained by only increasing the $m$ value of either the S-R or R-D link. The outage curves become sharper only when $m$ values of both the S-R and the R-D links increase simultaneously, which is consistent to the conclusion drawn in (33) regarding equivalent diversity order.

### 2.1.4 Optimal Relay Location

Let $d_{\mathrm{SD}}, d_{\mathrm{RD}}$, and $d_{\mathrm{SR}}$ denotes the distances between S and $\mathrm{D}, \mathrm{R}$ and D , and S and R , respectively. With $G_{\text {SD }}$ being normalized to the unity, $G_{\mathrm{SR}}$ and $G_{\mathrm{RD}}$ can be defined as $G_{\mathrm{SR}}=\left(\frac{d_{\mathrm{SD}}}{d_{\mathrm{SR}}}\right)^{\alpha}$ and $G_{\mathrm{RD}}=\left(\frac{d_{\mathrm{SD}}}{d_{\mathrm{RD}}}\right)^{\alpha}$, respectively, where $\alpha$ is the path loss exponent. Then, the average SNRs of the S-R and R-D links can be given as

$$
\begin{gather*}
\bar{\gamma}_{\mathrm{SR}}=\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(G_{\mathrm{SR}}\right)(\mathrm{dB}),  \tag{39}\\
\bar{\gamma}_{\mathrm{RD}}=\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(G_{\mathrm{RD}}\right)(\mathrm{dB}) . \tag{40}
\end{gather*}
$$

By substituting (39) and (40) into (20), (21), and (22), we can obtain the outage probability expressions with respect to the position of R.

Fig. 10 shows the impact of the relay location on the outage probability, with $\bar{\gamma}_{\mathrm{SR}}=1$ $\mathrm{dB} . \mathrm{R}$ is located on the line parallel to the line connecting S and D between $x=0$ and $x=1$, as shown in Fig. 2. The distance between the relay and the line connecting S and D is set at $\frac{1}{10}$ of the length of the S-D link ${ }^{6}$. The outage probability of DF relaying is

[^4]

Fig. 10. The optimal relay positions of LF and DF relaying where $m_{\mathrm{SD}}=1$ and $\alpha=3.52$. The vertical lines indicate the points where the outage probabilities are minimized.
also depicted as a reference. Joint decoding is considered for both LF and DF at the destination. With DF, the lowest outage probability can be achieved when the relay is located close to the source (left hand side), since the relay has to be located close to the source to guarantee the correct recovery of the information transmitted from the source. It is found that when the relay is close to the source, LF and DF show the same performance. This is because with reliable S-R transmission, the probability that error is detected after decoding at the relay becomes small. In this case, the relay in DF keeps active, which results in DF and LF relaying exhibiting the same outage performance. On the contrary, when the relay moves close to the destination, the probability that error is detected after decoding at the relay increases with the degraded S-R link quality. In the case, the relay in DF keeps silence which results in LF outperforming DF by always forward the decoder output to the destination regardless of whether error is detected after decoding in the information part or not. With LF, the lowest outage probability can be achieved when the relay is located at the midpoint as long as $m_{\mathrm{SR}}=m_{\mathrm{RD}}$. It is observed that the outage curves are symmetric with respect to the midpoint of the line connecting $S$ and $D$ with LF relaying. This is because with LF relaying, the errors due to the S-R link can be corrected at the destination, and therefore, the midpoint ( $d_{\mathrm{SR}}=d_{\mathrm{RD}}$ )
is the optimal point where the contributions of the S-R and R-D links are balanced. Moreover, while keeping the same or even lower outage probability, LF relaying has a larger range of search area for a relay (helper) than DF relaying.


Fig. 11. The optimal relay positions of LF and DF relaying where $m_{\mathrm{SD}}=m_{\mathrm{SR}}=1$ and $\alpha=3.52$. The vertical lines indicate the points where the outage probabilities are minimized.

Figs. 11 and 12 also show the impact of the relay location on the outage probability when either the value of $m_{\mathrm{SR}}$ or $m_{\mathrm{RD}}$ changes. It can be found that as the $m$ value of either the S-R link or the R-D link becomes large, the optimal relay location shifts close to the destination or the source, respectively. It is also found in Figs. 10, 11 and 12 that when the relay moves close to the source (left hand side), the outage performances of LF and DF with the same $m_{\mathrm{SR}}$ and $m_{\mathrm{RD}}$ values gradually become identical. This observation indicates that as the length of the S-R link reduces, it becomes less likely the errors happen in the S-R transmission, which results in DF performing the same as LF. Moreover, Fig. 11 shows that, when the relay is located close to the destination (right hand side), the outage probabilities of LF relaying with different $m_{\mathrm{RD}}$ values become identical. The same tendency can be found from the outage curves of DF relaying. This indicates that the S-R link with low transmission quality dominates the system outage probability. Note that in Fig. 12 the outage probabilities of LF and DF asymptotically
become identical as the relay moves toward to the source regardless of $m_{\mathrm{SR}}$ or $m_{\mathrm{RD}}$ values. This is because when the distance between the relay and the destination increase, the R-D link quality is degraded. Hence, the R-D link dominates the system outage probability. Note that while keeping the same or ever lower outage probability, the area for searching a relay (helper) can be increased by LF relaying compared to conventional DF relaying. For example, in Fig. 10, for achieving the outage probability not larger than $10^{-2}$, the area for the position of the relay indicated by the x -coordinate with LF is larger than that with DF when $m_{\mathrm{SR}}=m_{\mathrm{RD}}=3$.


Fig. 12. The optimal relay positions of LF and DF relaying where $m_{\mathrm{SD}}=m_{\mathrm{RD}}=1$ and $\alpha=3.52$. The vertical lines indicate the points where the outage probabilities are minimized.

### 2.2 Fading Correlations for Wireless Cooperative Communications: Diversity Order and Coding Gain

### 2.2.1 System and Channels Models

The wireless relay network model used in this section is shown in Fig. 13. A source S communicates with a destination D with the help of a relay R. Each terminal is equipped with a single antenna. Transmission phases are orthogonal and no multiple access channel is involved. The cooperation protocols considered in this section are LF, DF, and ADF relaying.


Fig. 13. The schematic scenario of one relay aided communication system.

### 2.2.2 Relaying

In LF relaying, $S$ broadcasts the coded information sequence to $D$ and $R$ in the first time slot. The information sequence, obtained as the result of decoding at R , is re-interleaved, re-encoded and transmitted to D in the second time slot, even if the decoder detects error.

In conventional DF relaying, S broadcasts the coded information sequence to D and R in the first time slot. R tries to fully recover the received information sequence. If it is successfully recovered, the information sequence is forwarded to D in the second time slot. R keeps silent if error is detected after decoding.

In ADF relaying, S broadcasts the coded information sequence to D and R in the first time slot. If the transmitted information is successfully recovered at R , the recovered
information sequence is re-interleaved, re-encoded and transmitted to D in the second time slot. If R detects error in the information part after decoding, R notifies S of the information recovery failure via a feedback link, and $S$ interleaves the information sequence, re-encodes, and retransmits the information sequence to D again.

## Correlated Channel Model

The S-D and R-D links are considered to be spatially correlated with the correlation coefficient $\rho_{s}\left(0 \leq \rho_{s} \leq 1\right)$ representing the spatial correlation between $h_{\mathrm{SD}}$ and $h_{\mathrm{RD}}$ at the first transmission time slot and the second, respectively. $h_{i j}(i \in\{\mathrm{~S}, \mathrm{R}\}, j \in\{\mathrm{R}, \mathrm{D}\}, i \neq j)$ denotes the complex channel gain of $i-j$ link. The channel gains of the two transmissions over the S-D link are also correlated in time with the correlation coefficient $\rho_{t}\left(0 \leq \rho_{t} \leq 1\right)$ representing the temporal correlation between $h_{\mathrm{SD}}$ and $h_{\mathrm{S}^{\prime} \mathrm{D}}$, where $h_{\mathrm{S}^{\prime} \mathrm{D}}$ denotes the complex channel gain of the S-D link in the second time slot. $\rho_{s}=0$ or $\rho_{t}=0$ indicate independent fading, and $\rho_{s}=1$ or $\rho_{t}=1$ for fully correlated case.

In this section, each link is assumed to suffer from frequency non-selective Rayleigh fading. The pdf of instantaneous SNR $\gamma_{i j}$ is given by

$$
\begin{equation*}
p\left(\gamma_{i j}\right)=\frac{1}{\bar{\gamma}_{i j}} \exp \left(-\frac{\gamma_{i j}}{\bar{\gamma}_{i j}}\right),(i \in\{\mathrm{~S}, \mathrm{R}\}, j \in\{\mathrm{R}, \mathrm{D}\}, i \neq j), \tag{41}
\end{equation*}
$$

### 2.2.3 Outage Analysis in Independent Fading

## Outage Behavior of LF in Independent Fading

Following the same technicals as those used for the outage derivation of LF relaying over Nakagami-m fading channel in the previous section, the outage probability of LF
relaying over independent Rayleigh channels can be expressed as

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LFF} \text { Ind }}= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \left(1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)\right) d \gamma_{\mathrm{SD}} \\
+ & \frac{1}{\bar{\gamma}_{\mathrm{SR}}} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)}\left[1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right] \\
& \cdot \exp \left(-\frac{\gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) d \gamma_{\mathrm{SR}}+\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \\
& \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \exp \left(-\frac{\gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) \\
& \cdot\left(1-\exp \left(-\frac{\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)\right) d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{SR}} \tag{42}
\end{align*}
$$

## Outage Behavior of DF in Independent Fading

In DF relaying, relay keeps silent if error is detected after decoding. When $p_{f}=0$, the outage probability of DF is the same as that of LF . When $p_{f} \neq 0$, the outage probability is equivalent to that of point-to-point S-D transmission. Therefore, the outage probability of DF relaying can be expressed as

$$
\begin{align*}
P_{\text {out }}^{\mathrm{DF}, \text { Ind }}= & \operatorname{Pr}\left[p_{f}=0,0 \leq R_{\mathrm{S}} \leq 1,0 \leq R_{\mathrm{R}} \leq H\left(p_{f}^{\prime}\right)\right]+\operatorname{Pr}\left[0<p_{f} \leq 0.5,0 \leq R_{\mathrm{S}} \leq 1\right] \\
= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \left(1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)\right) d \gamma_{\mathrm{SD}} \\
& +\left(1-\exp \left(-\frac{\Theta^{-1}(1)}{\bar{\gamma}_{\mathrm{SR}}}\right)\right)\left(1-\exp \left(-\frac{\Theta^{-1}(1)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right) \tag{43}
\end{align*}
$$

## Outage Behavior of ADF in Independent Fading

With ADF, S retransmits an interleaved and re-encoded version of the information, if it is notified of the decoding failure via a feedback link from R. The outage probability of ADF is the same as that of DF and LF, When $p_{f}=0$. When $p_{f} \neq 0$, the rate region of ADF is shown in Fig. 14. Similarly, the outage probability of ADF when $p_{f} \neq 0$ is defined as the probability that the source rate pair ( $R_{\mathrm{S}}, R_{\mathrm{S}}$ ), fall into the inadmissible


Fig. 14. Rate region for the two transmissions over the S-D link of ADF relaying, when $p_{f} \neq 0$.
area E shown in Fig. 14. $R_{\mathrm{S}}$, is the rate of retransmitted information sequence from S . Let $P_{\mathrm{E}}$ denotes the probability that ( $R_{\mathrm{S}}, R_{\mathrm{S}}$ ) fall into the inadmissible area E . The outage probability of ADF relaying is then given by

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \operatorname{Ind}}= & P_{\mathrm{A}}+P_{\mathrm{E}} \\
= & \operatorname{Pr}\left[p_{f}=0,0 \leq R_{\mathrm{S}} \leq 1,0 \leq R_{\mathrm{R}} \leq H\left(p_{f}\right)\right] \\
+ & \operatorname{Pr}\left[0<p_{f} \leq 0.5,0 \leq R_{\mathrm{S}} \leq 1,0 \leq R_{\mathrm{S}^{\prime}} \leq H\left(p_{f}\right)\right] \\
= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \left(1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)\right) d \gamma_{\mathrm{SD}} \\
& +\frac{1}{\bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}}}\left(1-\exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}}}\right) \\
& \left(1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD})}\right)\right.}{\bar{\gamma}_{\mathrm{SD}}}\right)\right) d \gamma_{\mathrm{SD}}, \tag{44}
\end{align*}
$$

where $P_{\mathrm{A}}$ is equal to that in (8). The derivations for the explicit expressions of (42), (43), and (44) may not be possible. We use a numerical method [77] to evaluate $P_{\text {out }}^{\mathrm{LF}}$ Ind , $P_{\text {out }}^{\mathrm{DF}, \text { Ind }}$, and $P_{\text {out }}^{\mathrm{ADF}, \text { Ind }}$.

## Approximations

By invoking the property of exponential function $e^{-x} \approx 1-x$ for small $x$, corresponding to high SNR regime, the outage probabilities of LF, DF, and ADF relaying, respectively, can be approximated by the closed-form expressions, as,

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LFF} \text { Ind }} \approx & \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) \\
& +\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}\right),  \tag{45}\\
P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }} \approx & \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) \\
& +\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}, \tag{46}
\end{align*}
$$

and

$$
\begin{align*}
P_{\text {out }}^{\mathrm{ADFF}, \text { Ind }} \approx & \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) \\
& +\frac{1}{\bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) . \tag{47}
\end{align*}
$$

It is found from Fig. 15 that in spatially or temporary independent fading (i.e., $\rho_{s}=0$, and $\rho_{t}=0$ ), the approximated outage curves obtained from (45), (46), and (47) well match the numerically calculated outage curves from (42), (43), and (44). This observation indicates that the approximation is accurate to the exact outage probability.

## Diversity Order and Coding Gain

By setting the geometric gain of each link to be identical and replacing $\bar{\gamma}_{i j}$ with a generic $\bar{\gamma}$, corresponding to equilateral triangle node locations (i.e., $\mathrm{S}, \mathrm{R}$, and D are located to the vertex of an equilateral triangle), (45), (46), and (47) are reduced to


Fig. 15. Comparison between the exact and approximated outage probabilities for LF, DF, and ADF relaying over independent and spatial correlated channels, where the S-R, S-D, and R-D links have equal distances and $\rho_{t}=0$.

$$
\begin{gather*}
P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Ind}}=\frac{2 \ln (4)-2}{(\bar{\gamma})^{2}}+\frac{8 \ln (2)-2 \ln (4)-4}{2(\bar{\gamma})^{3}}-\frac{4 \ln (2)-3}{2(\bar{\gamma})^{4}},  \tag{48}\\
P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Ind}}=\frac{\ln (4)}{(\bar{\gamma})^{2}}+\frac{4 \ln (2)-2 \ln (4)-2}{2(\bar{\gamma})^{3}}-\frac{4 \ln (2)-3}{2(\bar{\gamma})^{4}}, \tag{49}
\end{gather*}
$$

and

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}=\frac{\ln (4)-1}{(\bar{\gamma})^{2}}+\frac{4 \ln (2)-3}{2(\bar{\gamma})^{3}} \tag{50}
\end{equation*}
$$

respectively.
Then, in high average SNR regime, (48), (49), and (50) can further be expressed as

$$
\begin{align*}
& P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}=\left(G_{c}^{\mathrm{LF}} \cdot \bar{\gamma}\right)^{-G_{d}^{\mathrm{LF}}},  \tag{51}\\
& P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}=\left(G_{c}^{\mathrm{DF}} \cdot \bar{\gamma}\right)^{-G_{d}^{\mathrm{DF}}}, \tag{52}
\end{align*}
$$

and

$$
\begin{equation*}
P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Ind}}=\left(G_{c}^{\mathrm{ADF}} \cdot \bar{\gamma}\right)^{-G_{d}^{\mathrm{ADF}}}, \tag{53}
\end{equation*}
$$

where $G_{d}^{\mathrm{LF}}=G_{d}^{\mathrm{DF}}=G_{d}^{\mathrm{ADF}}=2$ are the diversity orders of LF, DF , and ADF relaying. This is consistent to the decay shown in the outage curves in Fig. 15 (i.e., those with $\rho_{s}=0$ ). $G_{c}^{\mathrm{LF}}=\frac{1}{\sqrt{2 \ln (4)-2}}, G_{c}^{\mathrm{DF}}=\frac{1}{\sqrt{\ln (4)}}$, and $G_{c}^{\mathrm{ADF}}=\frac{1}{\sqrt{\ln (4)-1}}$ are the coding gains of LF, DF, and ADF relaying, respectively [50]. It is easy to find that $G_{c}^{\mathrm{ADF}}>G_{c}^{\mathrm{LF}}>G_{c}^{\mathrm{DF}}$, which is consistent to the relationship shown in the outage curves in Fig. 15, i.e., with spatially and temporally independent channels $\left(\rho_{s}=0\right.$ and $\left.\rho_{t}=0\right), P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}>P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}>P_{\mathrm{out}}^{\mathrm{ADF}}$, Ind .

Theorem 2. The outage probability of LF is smaller than that of DF, i.e., $P_{\text {out }}^{L F \text { Ind }}<$ $P_{\text {out }}^{\text {DF Ind }}$.

Proof. The difference between $P_{\text {out }}^{\mathrm{DF} \text {, Ind }}$ in (45) and $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}$ in (46), is

$$
\begin{equation*}
P_{\text {out }}^{\mathrm{LF}, \text { Ind }}-P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}=\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-2+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}\right) . \tag{54}
\end{equation*}
$$

Let $\Delta=\left(\ln (4)-2+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}\right)$. Since

$$
\begin{equation*}
\lim _{\gamma_{\mathrm{SR}} \rightarrow \infty} \Delta=-1.3069<0 \tag{55}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\mathrm{d} \Delta}{\mathrm{~d} \bar{\gamma}_{\mathrm{SR}}}=\frac{3-4 \ln (2)}{\bar{\gamma}_{\mathrm{SR}}^{2}}>0, \tag{56}
\end{equation*}
$$

$\Delta$ increases monotonically as $\bar{\gamma}_{\mathrm{SR}}$ increases and limited to -1.3069 . Therefore, it proves that (54) always has negative value for any given SNR, which indicates that the $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}$ is smaller than $P_{\text {out }}^{\mathrm{DF}, \text { Ind }}$.

Theorem 3. The outage probability of LF is larger than that of ADF, i.e., $P_{\mathrm{out}}^{L E \text { Ind }}>$ $P_{\text {out }}^{A D F, \text { Ind }}$.

Proof. The difference between $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}$ in (45) and $P_{\text {out }}^{\mathrm{ADF}}$, Ind in (47) is

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}-P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }} & =\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}\right) \\
& -\frac{1}{\bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) . \tag{57}
\end{align*}
$$

We assume $\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}} \leq \frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}$ with $\bar{\gamma}_{\mathrm{SD}}>1^{7}$. Since $\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)>0$, we have

$$
\begin{align*}
& \frac{1}{\bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) \\
& <\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right) \\
& \leq \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SR}}}\right), \tag{58}
\end{align*}
$$

which indicates (57) is greater than 0 . This proves $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}>P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}$.

### 2.2.4 Outage Analysis in Correlated Fading

Let the correlation of two complex channel gains be represented by a correlation coefficient $\rho$. The joint pdf of two signal amplitudes of correlated Rayleigh fading channels is given by [78]. It is not difficult to convert the amplitude joint pdf into that of two SNRs, $\gamma_{1}$ and $\gamma_{2}$, as

$$
\begin{equation*}
p\left(\gamma_{1}, \gamma_{2}\right)=\frac{1}{\bar{\gamma}_{1} \bar{\gamma}_{2}(1-\rho)} I_{0}\left(\frac{2}{(1-\rho)} \sqrt{\frac{\rho \gamma_{1} \gamma_{2}}{\bar{\gamma}_{1} \bar{\gamma}_{2}}}\right) \exp \left[-\frac{\frac{\gamma_{1}}{\bar{\gamma}_{1}}+\frac{\gamma_{2}}{\gamma_{2}}}{1-\rho}\right], \tag{59}
\end{equation*}
$$

where $I_{0}(\cdot)$ is the zero-th order modified Bessel's function of the first kind.

## Outage Behavior of LF in Correlated Fading

We follow the same techniques as those used for calculating the outage probability in independent fading in (42). The multiple integral with the constraints (9), (10) and (11) over the joint pdf (59) leads to the outage probability expressions of LF relaying in

[^5]spatially correlated S-D and R-D links, as
\[

$$
\begin{align*}
& P_{\text {out }}^{\mathrm{LF}, \text { Cor }}=\exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)} I_{0}\left(\frac{2\left|\rho_{s}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{RD}}}{\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\left|\rho_{s}\right|^{2}\right)}}\right) \\
& \cdot \exp \left[-\frac{\frac{\gamma_{\mathrm{SD}}}{\gamma_{\mathrm{RD}}}+\frac{\gamma_{\mathrm{RD}}}{\gamma_{\mathrm{RD}}}}{1-\left|\rho_{s}\right|^{2}}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
& +\int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(\infty)} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SR}}} \exp \left(-\frac{\gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) I_{0}\left(\frac{2\left|\rho_{s}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{RD}}}{\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}\left(1-\left|\rho_{s}\right|^{2}\right)}\right) \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)} \exp \left[-\frac{\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{RD}}}+\frac{\gamma_{\mathrm{\gamma D}}}{\bar{\gamma}_{\mathrm{RD}}}}{1-\left|\rho_{s}\right|^{2}}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} d \gamma_{\mathrm{SR}}, \\
& +\int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left[\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)\right]} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SR}}} \exp \left(-\frac{\gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) I_{0}\left(\frac{2\left|\rho_{s}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{RD}}}{\left.\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\left|\rho_{s}\right|^{2}\right)}\right)}\right. \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)} \exp \left[-\frac{\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{RD}}}+\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}{1-\left|\rho_{s}\right|^{2}}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} d \gamma_{\mathrm{SR}}, \tag{60}
\end{align*}
$$
\]

where the pdfs in (42) have been replaced by the joint pdf in (59).

## Outage Behavior of DF in Correlated Fading

Similarly, following the techniques for the outage derivation of DF in independent fading in (43), the outage probability expression of DF relaying in the correlated S-D
and R-D links can be expressed as,

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{DF}, \operatorname{Cor}}= & \exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)} I_{0}\left(\frac{2\left|\rho_{s}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{RD}}}{\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\left|\rho_{s}\right|^{2}\right)}}\right) \\
& \cdot \exp \left[-\frac{\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{RD}}}+\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}{1-\left|\rho_{s}\right|^{2}}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
+ & \left(1-\exp \left(-\frac{\Theta^{-1}(1)}{\bar{\gamma}_{\mathrm{SR}}}\right)\right)\left(1-\exp \left(-\frac{\Theta^{-1}(1)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right) \tag{61}
\end{align*}
$$

where the pdfs in (43) have been replaced by the joint pdf in (59).

## Outage Behavior of ADF in Correlated Fading

When analyzing the outage probability of ADF, two correlation coefficients $\rho_{s}$ and $\rho_{t}$ are taken into account. The outage probability expression of ADF relaying over the links with spatially and temporally correlated fading can be derived by following the techniques for calculating the outage probability in independent fading in (44), as

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}= & \exp \left(-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)} I_{0}\left(\frac{2\left|\rho_{s}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{RD}}}{\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}\left(1-\left|\rho_{s}\right|^{2}\right)}\right) \\
& \exp \left[-\frac{\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{RD}}}+\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}{1-\left|\rho_{s}\right|^{2}}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}}, \\
+ & \left(1-\exp \left(\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)} \int_{\gamma_{\mathrm{S}^{\prime} \mathrm{D}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \\
& \frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}}\left(1-\rho_{t}^{2}\right)} I_{0}\left(\frac{2\left|\rho_{t}\right| \gamma_{\mathrm{SD}} \gamma_{\mathrm{S}^{\prime} \mathrm{D}}}{\sqrt{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}}}\left(1-\left|\rho_{t}\right|^{2}\right)}\right) \\
& \exp \left[-\frac{\frac{\gamma_{\mathrm{YDD}}}{\bar{\gamma}_{\mathrm{SD}}}+\frac{\gamma_{\mathrm{S}_{\mathrm{SD}} \mathrm{D}}}{1-\left|\rho_{t}\right|^{2}}}{}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{S}^{\prime} \mathrm{D}}, \tag{62}
\end{align*}
$$

where the pdfs in (44) have been replaced by the joint pdf in (59). The pdfs of the S-D and R-D links are replaced by the joint pdf having spatial correlation coefficient $\rho_{s}$ and the pdfs of the two S-D links are replaced by the joint pdf having temporal correlation
coefficient $\rho_{t}$. Note that, the outage expressions (60), (61), and (62) in correlated fading reduce to those in independent links (42), (43), and (44) when $\rho_{s}=0$ and $\rho_{t}=0$.

Fig. 15 also plots the outage curves with high spatial correlation $\rho_{s}$, where all the S-D, S-R, and R-D links have the same average SNR. It is found from Fig. 15 that with independent S-D and R-D transmission, $\rho_{s}=0$, ADF outperforms LF and DF in terms of the outage performance. However, ADF does not show obvious superiority compared to LF and DF with high spatial correlation, $\rho_{s}=0.99$, even the two S-D transmissions are independent, $\rho_{t}=0$. This is because, since in the high SNR regime, the relay can recover the original information sequence with small probability of error. Hence, R forwards the decoded, re-interleaved, and re-encoded sequence to D , and it less likely uses the S-D link for retransmission in ADF in high S-R link average SNR regime. Therefore, the temporal correlation $\rho_{t}$ has less impact on the outage performance than the spatial correlation $\rho_{s}$. In this case, the value of spatial correlation $\rho_{s}$ of the complex fading gains $h_{\mathrm{SD}}$ and $h_{\mathrm{RD}}$ in the first and the second time slot, respectively, dominates the outage probability. On the contrary, in the low SNR regime, ADF can achieve better outage performance than LF and DF but slightly, since the errors may not be well eliminated in every link with low SNR, resulting in that the admissible rate region is not satisfied.

The impact of spatial correlation $\rho_{s}$ on the outage probabilities of LF, DF and ADF relaying are demonstrated in Fig. 16. As can be seen from the figure, over the entire $\rho_{s}$ value region, LF exhibits superior outage performance to DF . This is because with LF, the relay system can be seen as a distributed turbo code. It is also found that ADF achieves lower outage probability than LF. This is because the inadmissible rate region with ADF is made smaller than LF by the feedback information, as shown in Fig. 5 and Fig. 14. It is observed that for all the three relaying schemes, the outage probabilities increase as the spatial correlation $\rho_{s}$ between the channel complex gains with the S-D and R-D links in the two transmission time slots becomes large, since the correlation degrades the outage performances. Intuitively, spatial correlation decreases the statistical independence of fading variations in the S-D and R-D links. It is also found that the gaps among the outage curves with $\mathrm{DF}, \mathrm{LF}$, and ADF relaying decrease as the value of $\rho_{s}$ increases. This is because the larger the spatial correlation, the smaller the contribution provided by the R-D link transmission. Hence, the system performances largely depend on the S-D link, which results in DF, LF, and ADF relaying exhibiting the similar performance.


Fig. 16. Outage probability versus the spatial correlation $\rho_{s}$, where $\rho_{t}=0$ and $\bar{\gamma}_{\mathbf{S D}}=16$.

Fig. 17 shows the outage performance of ADF relaying with the temporal correlation $\rho_{t}$ and spatial correlation $\rho_{s}$ as parameters. As can be seen from the Fig. 17, the higher the $\rho_{s}$ or $\rho_{t}$ values, the larger the outage probability. The impact of $\rho_{s}$ on outage performance is almost the same as that of $\rho_{t}$. This is because the average SNR of the S-R link is set at $\bar{\gamma}_{\mathrm{SR}}=1.6 \mathrm{~dB}$. With such low average, the probability that the S-R link instantaneous SNR above the outage threshold is equal to that below the outage threshold, which is $50 \%$. It means that the probability of the source retransmit signal equals to the probability of the relay forwards signal, which results in the impact of $\rho_{s}$ and $\rho_{t}$ on outage performance being the same. ${ }^{8}$

[^6]

Fig. 17. Impacts of temporal correlation $\rho_{t}$ and spatial correlation $\rho_{s}$ on the outage probability of ADF, $\bar{\gamma}_{\mathbf{S R}}=1.6$.

## Approximations

The Bessel function of the first kind can be expressed with its series expansion by Frobenius method [79, 9.1.10], as:

$$
\begin{equation*}
I_{0}(x)=\sum_{m=0}^{\infty} \frac{(-1)^{m}}{m!\Gamma(m+1)}\left(\frac{x}{2}\right)^{2 m} \tag{63}
\end{equation*}
$$

Then, (60), (61), and (62) can be, respectively, approximated as

$$
\begin{align*}
P_{\text {out }}^{\mathrm{LF}, \text { Cor }} & \approx\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{2 \ln (4)-3}{2 \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}^{2}}+\frac{2 \ln (4)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{SR}}} \\
& +\frac{\ln (4)-1}{2 \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}+\frac{4 \ln (16)-11}{4 \bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{SR}}^{2}}, \tag{64}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{DF}, \text { Cor }} & \approx\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{SR}}}, \tag{65}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}} & \approx\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}} \bar{\gamma}_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}} \bar{\gamma}_{\mathrm{SR}}\left(1-\rho_{t}^{2}\right)} \\
& +\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{S}^{\prime} \mathrm{D}} \bar{\gamma}_{\mathrm{RD}} \bar{\gamma}_{\mathrm{SR}}\left(1-\rho_{t}^{2}\right)}+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}^{2} \bar{\gamma}_{\mathrm{SR}}\left(1-\rho_{t}^{2}\right)} . \tag{66}
\end{align*}
$$

The accuracy of the approximations is demonstrated in Fig. 18 in section 2.2.5.

## Diversity Order

The diversity order $d$ is defined as [21]

$$
\begin{equation*}
d=-\lim _{\gamma \rightarrow \infty} \frac{\log \left(P_{\mathrm{out}}\right)}{\log (\bar{\gamma})} \tag{67}
\end{equation*}
$$

According to (67), the diversity order of LF, DF, ADF over correlated channels can be expressed as

$$
\begin{align*}
d_{\mathrm{LF}}= & -\lim _{\bar{\gamma} \rightarrow \infty} \frac{\log \left(P_{\mathrm{out}}^{\mathrm{LF}, \text { Cor }}\right)}{\log (\bar{\gamma})} \\
= & -\lim _{\bar{\gamma} \rightarrow \infty}\left\{\operatorname { l o g } \left[\frac{5 \ln (4)-7}{2 \bar{\gamma}^{3}}+\frac{4 \ln (16)-11}{4 \bar{\gamma}^{4}}+\left(1-\frac{1}{\bar{\gamma}}\right)\right.\right. \\
& \left.\left.\cdot\left(\frac{\ln (4)-1}{\bar{\gamma}^{2}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-2 \ln (4)-6}{2 \bar{\gamma}^{3}\left(1-\rho_{s}^{2}\right)}\right)\right] / \log (\bar{\gamma})\right\}=2, \rho_{s} \neq 1,  \tag{68}\\
d_{\mathrm{DF}}= & -\lim _{\bar{\gamma} \rightarrow \infty} \frac{\log \left(P_{\mathrm{out}}^{\mathrm{DF}, \operatorname{Cor}}\right)}{\log (\bar{\gamma})} \\
= & -\lim _{\bar{\gamma} \rightarrow \infty}\left\{\operatorname { l o g } \left[\frac{2 \ln (4)-3}{\bar{\gamma}^{3}}+\frac{\ln (4)-1}{2 \bar{\gamma}^{2}}+\frac{4 \ln (16)-11}{4 \bar{\gamma}^{4}}\right.\right. \\
+ & \left.\left.\left(1-\frac{1}{\bar{\gamma}}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}^{2}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)+2 \ln (4)-6}{2 \bar{\gamma}^{3}\left(1-\rho_{s}^{2}\right)}\right)\right] / \log (\bar{\gamma})\right\}=2, \rho_{s} \neq 1, \tag{69}
\end{align*}
$$

and

$$
\begin{align*}
d_{\mathrm{ADF}}= & -\lim _{\bar{\gamma} \rightarrow \infty} \frac{\log \left(P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}\right)}{\log (\bar{\gamma})} \\
= & -\lim _{\gamma \rightarrow \infty}\left\{\operatorname { l o g } \left[\frac{\ln (4)-1}{\bar{\gamma}^{3}\left(1-\rho_{t}^{2}\right)}+\frac{4 \ln (2)+2 \ln (4)-6}{2 \bar{\gamma}^{4}\left(1-\rho_{t}^{2}\right)}\right.\right. \\
& \left.\left.+\left(1-\frac{1}{\bar{\gamma}}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}^{2}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 \bar{\gamma}^{3}\left(1-\rho_{s}^{2}\right)}+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}^{3}\left(1-\rho_{s}^{2}\right)}\right)\right] / \log (\bar{\gamma})\right\}=2, \\
& \rho_{s} \neq 1, \rho_{t} \neq 1 \tag{70}
\end{align*}
$$

where $\bar{\gamma}_{\mathrm{SD}}, \bar{\gamma}_{\mathrm{RD}}$, and $\bar{\gamma}_{\mathrm{SR}}$ are represented by a generic symbol $\bar{\gamma}$ under the equilateral triangle nodes location assumption. It is shown that the full diversity gain can be achieved by LF, DF , and ADF relaying as long as $\rho_{s} \neq 1$ for LF and DF, and $\rho_{s} \neq 1$ and $\rho_{t} \neq 1$ for ADF. When $\rho_{s}=1$ and $\rho_{t}=1$, i.e., in fully correlated fading, the diversity order reduces to one.

### 2.2.5 Optimal Relay Locations for Minimizing the Outage Probability

In this subsection, we investigate the optimal relay locations which minimize the outage probabilities of LF, DF, and ADF relaying. The outage expressions can be rewritten so that they are functions of the position of the relay by taking the geometric gain into consideration. It is shown that the optimization for the relay location can be formulated as a convex optimization problem.

## Optimal Relay Locations in Independent Fading

Substituting (39) and (40) in section 2.1.4, which represent the relationships between the average SNRs and the geometric gains, into (45), (46), and (47), yields the outage probability expressions of LF, DF, and ADF relaying over the links suffering from
independent fading with respect to the position of R as

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right) \\
& \cdot\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)+\frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)} \\
& \cdot\left(\ln (4)-1+\frac{4 \ln (2)-3}{2\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\right),  \tag{71}\\
P_{\mathrm{out}}^{\mathrm{DFF} \text { Ind }}= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right) \\
& \cdot\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)+\frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}, \tag{72}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right) \\
& \cdot\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)+\frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}^{2}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)} \\
& \cdot\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}}\right), \tag{73}
\end{align*}
$$

under the assumption that R moves along the line between S and D , with which $d_{\mathrm{SR}}=d$ and $d_{\mathrm{RD}}=1-d$.

The general optimization problem with regard to $d$ can be formulated as

$$
\begin{array}{ll}
d^{*}=\arg \min _{d} & P_{\text {out }}(d) \\
\text { subject to: } & d-1<0,  \tag{74}\\
& -d<0
\end{array}
$$

The Karush-Kuhn-Tucker (KKT) condition for (74) can be written as

$$
\begin{align*}
& \frac{\partial P_{\text {out }}(d)}{\partial d}+\mu_{1}-\mu_{2}, \\
& \mu_{1}>0, \\
& \mu_{2}>0 \\
& \mu_{1}(d-1)<0,  \tag{75}\\
& \mu_{2}(-d)<0, \\
& d-1<0, \\
& -d<0
\end{align*}
$$

where $\mu_{1}$ and $\mu_{2}$ are the constraint coefficients.
Proposition 4. Outage probability expressions of LF, DF, and ADF relaying in (71), (72), and (73), are convex with respect to $d \in(0,1)$.

Proof. See Appendix 1.
Taking the first-order derivative of $P_{\text {out }}^{\mathrm{LF}}$, Ind,$P_{\text {out }}^{\mathrm{DF}, \text { Ind }}$, and $P_{\text {out }}^{\mathrm{ADF}}$, Ind , respectively, in (71), (72), and (73) with respect to $d$ and setting the derivative results to zero, we have

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Ind}}}{\partial d} & =\frac{\partial \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)}{\partial d} \\
& +\frac{\partial \frac{1}{\overline{\gamma_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\right)}}{\partial d}=0, \tag{76}
\end{align*}
$$

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Ind}}}{\partial d} & =\frac{\partial \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}}\right)}{\partial d} \\
& +\frac{\partial \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}}{\partial d}=0, \tag{77}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}}{\partial d} & =\frac{\partial \frac{1}{\overline{\gamma_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)}\left(1-\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \gamma_{\mathrm{SD}}}\right)}}{\partial d} \\
& +\frac{\partial \frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}^{2}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}}\right)}{\partial d}=0 . \tag{78}
\end{align*}
$$

It may be excessively complex to derive the explicit expression for $d^{*}$ from (76), (77), and (78). Hence, the Newton-Raphson method [80, Chapter 9] is used to numerically calculate the solution of $d^{*}$.

## Optimal Relay Locations in Correlated Fading

Similar to the independent fading case, the outage probability expressions of LF, DF and ADF relaying in correlated fading with respect to the position of R can be expressed as (79), (80), and (81).

$$
\begin{align*}
P_{\text {out }}^{\text {LF, Cor }}= & \left(1-\frac{1}{\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)^{2}\left(1-\rho_{s}^{2}\right)}\right) \\
& +\frac{2 \ln (4)-3}{2 \bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)^{2}}+\frac{2 \ln (4)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}+\frac{\ln (4)-1}{2 \bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)} \\
& +\frac{4 \ln (16)-11}{4 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)^{2}}, \tag{79}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{DF}, \text { Cor }}= & \left(1-\frac{1}{\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}\right. \\
+ & \left.\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)^{2}\left(1-\rho_{s}^{2}\right)}\right) \\
& +\frac{1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}, \tag{80}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}} & =\left(1-\frac{1}{\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)}\right)\left(\frac{\ln (4)-1}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{2 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(1-\rho_{s}^{2}\right)}+\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)^{2}\left(1-\rho_{s}^{2}\right)}\right) \\
& +\frac{4 \ln (2)-3}{2 \bar{\gamma}_{\mathrm{SD}}^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)\left(1-\rho_{t}^{2}\right)} \\
& +\frac{\ln (4)-\frac{3}{2}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{1-d}\right)^{\alpha}\right)^{2}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d}\right)^{\alpha}\right)\left(1-\rho_{t}^{2}\right)} \tag{81}
\end{align*}
$$

The optimization problem for the correlated fading channels' case can also be formulated by (74) for LF, DF, and ADF relaying.

Proposition 5. Outage probability expressions of $L F, D F$, and $A D F$ relaying, respectively, given by (79), (80), and (81), are convex with respect to $d \in(0,1)$.


Fig. 18. Outage probabilities versus relay locations for LF, DF, and ADF relaying over spatially independent and correlated channels, where $\rho_{t}=0$.

Proof. The convexity of (79), (80), and (81) can be proven by taking second-order partial derivative of $P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Cor}}, P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Cor}}$, and $P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}$ in (79), (80), and (81) with respect to $d$, and showing that the derivative results are positive in the range $d \in(0,1)^{9}$.

Take the first-order derivative of $P_{\text {out }}^{\mathrm{LF}, \mathrm{Cor}}, P_{\text {out }}^{\mathrm{DF}, \mathrm{Cor}}$, and $P_{\text {out }}^{\mathrm{ADF}, \text { Cor }}$, respectively, in (79), (80), and (81) with respect to $d$ and set the derivative result to zero, $\frac{\partial P_{\mathrm{ot}}^{\mathrm{LF}} \mathrm{Cor}}{\partial d}=0$, $\frac{\partial P_{\text {out }}^{\mathrm{PF}, \mathrm{Cor}}}{\partial d}=0$, and $\frac{\partial P_{\text {out }}^{\mathrm{ADP}, ~ C o r ~}}{\partial d}=0$. As in the independent fading case, the optimal relay location $d^{*}$ for LF, DF , and ADF relaying over correlated fading channels can also be obtained by utilizing an iterative root-finding algorithm [80].

Fig. 18 depicts the impact of the relay location on outage performances of LF, DF and ADF relaying in spatially independent $\left(\rho_{s}=0\right)$ and correlated ( $\rho_{s}=0.99$ ) cases. The relay is assumed to move along the line between $\mathrm{S}(x=0)$ and $\mathrm{D}(x=1)$. Both the numerically calculated outage probabilities and approximated outage probabilities are plotted. It is found from the figure that the approximated and numerically calculated outage probability curves are consistent with each other. This observation indicates the

[^7]

Fig. 19. Impact of temporal correlation $\rho_{t}$ and spatial correlation $\rho_{s}$ on optimal relay location $d^{*}$, where $\bar{\gamma}_{\text {SD }}=2(\mathrm{~dB})$.
accuracy of the approximations. It is found from Fig. 18 that in spatially and temporally independent fading ( $\rho_{s}=0, \rho_{t}=0$ ) the optimal relay locations for LF, DF, and ADF that achieve the lowest probabilities are $d^{*}=0.5, d^{*}<0.5$ and, $d^{*}>0.5$, respectively. In spatially correlated fading ( $\rho_{s}=0.99$ ), the optimal relay locations for LF, DF, and ADF relaying shift closer to the destination compared to the case in independent fading. This is because the fading correlation reduces the diversity gain provided by the R-D link transmission, which results in the optimal position of the relay shifting close to the destination to keep the average SNR of the R-D link high.

The impact of the temporal correlation $\rho_{t}$ and the spatial correlation $\rho_{s}$ on the optimal relay location $d^{*}$ of LF, DF, ADF relaying is depicted in Fig. 19. For DF, LF, and ADF relaying, the higher the $\rho_{s}$, the larger the $d^{*}$, which indicates that the optimal relay locations shift close to the destination as the spatial correlation increases. This is because the average SNR of the R-D link need to be kept high to compensate the loss caused by the fading correlation. Obviously, $\rho_{t}$ has no impact on $d^{*}$ in LF and DF relaying. With ADF, it is found that the value of $d^{*}$ becomes small as $\rho_{t}$ increases. The
time diversity gain achieved by using the S-D link twice is reduced by the temporal correlation, which leads to that the relay should be located close to the source in order to reduce the S-R transmission error. This decreases the probability of retransmission from the source.

### 2.2.6 Optimal Power Allocation for Minimizing the Outage Probability

In this section, our aim is to minimize the outage probabilities for LF, DF, and ADF relaying by adjusting power allocated to $S$ and $R$. We consider a transmit power sharing strategy. The source (or the relay) can use the transmit power that is allocated to the system when the power is not used by the relay (or the source). The source and the relay can control its transmit power properly under total transmit power constraint. In this work, we do not consider the practical implementation of the transmit power sharing strategy and only focus on the theoretical analysis. We assume that the CSI is only available at the receiver side. Let the power allocated to S and R be denoted as $P_{T} k$ and $P_{T}(1-k)$, respectively, where $P_{T}$ represents the total transmit power and $k(0<k<1)$ is the power allocation ratio. With the noise variance of each link being normalized to the unity, the geometric gain times transmit power is equivalent to their corresponding average SNR. Then the average SNRs of each link can be expressed as functions of $k$ as

$$
\begin{align*}
\bar{\gamma}_{\mathrm{SD}} & =P_{T} k G_{\mathrm{SD}},  \tag{82}\\
\bar{\gamma}_{\mathrm{RD}} & =P_{T}(1-k) G_{\mathrm{RD}},  \tag{83}\\
\bar{\gamma}_{\mathrm{SR}} & =P_{T} k G_{\mathrm{SR}} . \tag{84}
\end{align*}
$$

## Optimal Power Allocation in Independent Fading

By substituting (82), (83), and (84) into (45), (46), and (47) in section 2.2.3, the outage expressions of LF, DF, and ADF relaying in independent fading with respect to $k$ can be written as

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }} & =\frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right) \\
& +\frac{1}{P_{T}^{2} k^{2} G_{\mathrm{SD}} G_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SR}}}\right) \tag{85}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Ind}} & =\frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right) \\
& +\frac{1}{P_{T}^{2} k^{2} G_{\mathrm{SD}} G_{\mathrm{SR}}}, \tag{86}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }} & =\frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right) \\
& +\frac{1}{P_{T}^{3} k^{3} G_{\mathrm{SR}} G_{\mathrm{SD}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right), \tag{87}
\end{align*}
$$

respectively.
The optimization problem with regards to $k$ can be formulated as

$$
\begin{array}{ll}
k^{*}=\arg \min _{k} & P_{\text {out }}(k)  \tag{88}\\
\text { subject to: } & k-1<0, \quad-k<0, .
\end{array}
$$

The KKT condition for (88) can be written as

$$
\begin{align*}
& \frac{\partial P_{\text {out }}(k)}{\partial k}+\mu_{3}-\mu_{4}, \\
& \mu_{3}>0, \\
& \mu_{4}>0 \\
& \mu_{3}(k-1)<0,  \tag{89}\\
& \mu_{4}(-k)<0, \\
& k-1<0, \\
& -k<0
\end{align*}
$$

where $\mu_{3}$ and $\mu_{4}$ are the constraint coefficients.
Proposition 6. Outage probability expressions of $L F, D F$, and $A D F$ relaying, respectively, in (85), (86), and (87), are convex with respect to $k \in(0,1)$.

Proof. See Appendix 2.
Taking the first-order derivative of $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}, P_{\text {out }}^{\mathrm{DF}, \text { Ind }}$, and $P_{\text {out }}^{\mathrm{ADF}, \text { Ind }}$, respectively in (85), (86), and (87) with respect to $k$ and setting the derivative results to zero, we have,

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}}{\partial d} & =\frac{\partial \frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right)}{\partial k} \\
& +\frac{\partial \frac{1}{P_{T}^{2} k^{2} G_{\mathrm{SD}} G_{\mathrm{SR}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SR}}}\right)}{\partial k}=0, \tag{90}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Cor}} & =\left(1-\frac{1}{P_{T} k G_{\mathrm{SD}}}\right)\left(\frac{\ln (4)-1}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}^{2} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{2 \ln (4)-3}{2 P_{T} k G_{\mathrm{SD}} P_{T}^{2} k^{2} G_{\mathrm{SR}}^{2}}+\frac{2 \ln (4)-3}{2 P_{T}^{2} k^{2} G_{\mathrm{SD}}^{2} P_{T} k G_{\mathrm{SD}}} \\
& +\frac{\ln (4)-1}{2 P_{T} k G_{\mathrm{SD}} P_{T} k G_{\mathrm{SD}}}+\frac{4 \ln (16)-11}{4 P_{T}^{2} k^{2} G_{\mathrm{SD}}^{2} P_{T}^{2} k^{2} G_{\mathrm{SR}}^{2}} \tag{93}
\end{align*}
$$

$$
\begin{align*}
P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Cor}} & =\left(1-\frac{1}{P_{T} k G_{\mathrm{SD}}}\right)\left(\frac{\ln (4)-1}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}^{2} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{1}{P_{T} k G_{\mathrm{SD}} P_{T} k G_{\mathrm{SD}}} \tag{94}
\end{align*}
$$

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}}{\partial d} & =\frac{\partial \frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right)}{\partial k} \\
& +\frac{\partial \frac{1}{P_{T}^{2} k^{2} G_{\mathrm{SD}} G_{\mathrm{SR}}}}{\partial k}=0 \tag{91}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\partial P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}}{\partial d} & =\frac{\partial \frac{1}{P_{T}^{2} k(1-k) G_{\mathrm{SD}} G_{\mathrm{RD}}}\left(1-\frac{1}{P_{T} k G_{\mathrm{SR}}}\right)\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right)}{\partial k} \\
& +\frac{\partial \frac{1}{P_{T}^{3} k^{3} G_{\mathrm{SR}} G_{\mathrm{SD}}}\left(\ln (4)-1+\frac{4 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}}\right)}{\partial k}=0 . \tag{92}
\end{align*}
$$

The optimal power allocation ratio $k^{*}$ can be obtained by numerically solving ( 90 ), (91), and (92).

## Optimal Power Allocation in Correlated Fading

The outage expressions of LF, DF, and ADF in correlated fading can be written as (93), (94), and (95), respectively. The optimization problem in correlated fading can also be formulated by (88) for LF, DF, and ADF relaying.

Proposition 7. Outage probability expressions of $L F, D F$, and $A D F$ relaying, respectively, in (93), (94), and (95), are convex with respect to $k \in(0,1)$.

$$
\begin{aligned}
P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}= & \left(1-\frac{1}{P_{T} k G_{\mathrm{SD}}}\right)\left(\frac{\ln (4)-1}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}+\frac{2 \ln (2)-3}{2 P_{T} k G_{\mathrm{SD}}^{2} P_{T}(1-k) G_{\mathrm{RD}}\left(1-\rho_{s}^{2}\right)}\right. \\
& \left.+\frac{\ln (4)-\frac{3}{2}}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}}^{2}\left(1-\rho_{s}^{2}\right)}\right)+\frac{\ln (4)-1}{P_{T} k G_{\mathrm{SD}} P_{T}(1-k) G_{\mathrm{RD}} P_{T} k G_{\mathrm{SD}}\left(1-\rho_{t}^{2}\right)} \\
& +\frac{\ln (2)-3}{2 P_{T}^{2} k^{2} G_{\mathrm{SD}}^{2} P_{T}(1-k) G_{\mathrm{RD}} P_{T} k G_{\mathrm{SD}}\left(1-\rho_{t}^{2}\right)}+\frac{3}{P_{T} k G_{\mathrm{SD}} P_{T}^{2}(1-k)^{2} G_{\mathrm{RD}}^{2} P_{T} k G_{\mathrm{SD}}\left(1-\rho_{t}^{2}\right)} .
\end{aligned}
$$

Proof. The convexity of (93), (94), and (95) can be proven by taking second-order partial derivative of $P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Cor}}, P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Cor}}$, and $P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}$ in (93), (94), and (95) with respect to $k$, and showing that the derivative results are positive in the range $k \in(0,1)^{10}$.

By taking the first-order derivative of $P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Cor}}, P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Cor}}$, and $P_{\mathrm{out}}^{\mathrm{ADF}}$, Cor , respectively, in (93), (94), and (95) with respect to $k$ and setting the derivative results to zero, $\frac{\partial P_{\text {out }}^{\mathrm{LE}} \mathrm{Cor}}{\partial k}=0, \frac{\partial P_{\text {out }}^{\mathrm{DF}} \mathrm{CH}}{\partial k}=0$, and $\frac{\partial P_{\text {out }}^{\mathrm{ADF}} \mathrm{Cor}}{\partial k}=0$. Similar to the independent fading case, the optimal power allocation ratio $k^{*}$ of LF, DF, and ADF relaying over correlated fading channels can be numerically obtained.

Fig. 20 presents the impact of the power allocation ratio to $S$ and $R$ on outage probability. We normalize the geometric gain of the S-D link to the unity under the assumption that the relay is located at the midpoint of the S-D link. The total transmit power is set at $2(\mathrm{~dB})$. It can be observed from Fig. 20 that, the optimal power ratio $k^{*}$ of LF is larger than that of ADF, and smaller than that of DF, which indicates that LF needs more power for the source than ADF, and needs less power than DF. This is because in DF relaying, the source needs more power to keep the quality of the S-D links for reducing the error probability. With ADF relaying, even though error occurs in the S-R link due to the deep fade of the channel, the source still can retransmit the information sequence to the destination, yielding the time diversity gain. It is also found that when more transmit power is allocated to the source (larger $k$ ), the outage curves of DF, LF, and ADF relaying with the same spatial correlation $\rho_{s}$ asymptotically merge. This indicates that with more transmit power for $S$, the probability of error occurring in the S-R link reduces, resulting in that R always forwards the received information sequence. Therefore, the outage curves with DF, LF, and ADF become almost the same. Moreover, the larger the spatial correlation $\rho_{s}$, the higher the outage

[^8]

Fig. 20. Outage probabilities versus power allocations for LF, DF, and, ADF relying over independent and correlated channels, where $P_{T}=2(\mathbf{d B})$ and $\rho_{t}=0$.
probability with the LF, DF, and ADF relaying schemes, however, more transmit power needs to be allocated to R (smaller $k$ ) for achieving the lower outage probabilities. This is because correlations reduce the diversity gain provided by the R-D link and R needs more transmit power to maintain the quality of the $\mathrm{R}-\mathrm{D}$ link. In practice, to keep a certain outage, one can allocate larger range of the transmit power with LF than DF under a total power constraint.

Fig. 21 shows the impact of $\rho_{t}$ and $\rho_{s}$ on optimal power allocation ratio $k^{*}$ for LF, $\mathrm{DF}, \mathrm{ADF}$ relaying, where the total transmit power for S and R is set at $P_{T}=2(\mathrm{~dB})$. It can be seen from Fig. 21 that the larger the spatial correlation $\rho_{s}$, the larger the optimal $k^{*}$ value with DF, LF, and ADF. Since the spatial correlation reduce the diversity gain provided by the R-D link, more power needs to be allocated to the source to increase the quality of the $S-D$ transmission. It is also found that, as the temporal correlation $\rho_{t}$ in ADF increases, the value of optimal $k^{*}$ decreases, in order to achieve the smallest outage probability. This is because the higher the temporal correlation, the lower the diversity gain provided by the $S$-D retransmission, which results in that more power should be allocated to the relay to increase the quality of the R-D transmission.


Fig. 21. Impact of temporal correlation $\rho_{t}$ and spatial correlation $\rho_{s}$ on optimal power allocation ratio $k^{*}$, where $P_{T}=2(\mathbf{d B})$.

### 2.3 Impact Analysis of Line-of-Sight Components in Lossy-Forward Relaying over Fading Channels Having Different Statistical Properties

### 2.3.1 Channel Model

The S-D link is assumed to experience frequency non-selective block Rayleigh fading which only has NLOS components. Both the S-R and R-D links are assumed to suffer from block fading variation having LOS component, following either Rician or Nakagami-m distribution. The pdf of the instantaneous SNR $\gamma_{i j}(i j \in(\operatorname{SR}, \mathrm{RD}))$ following Rician distribution is

$$
\begin{equation*}
p^{R i c i}\left(\gamma_{i j}\right)=\left(\frac{\left(1+K_{i j}\right) e^{-K_{i j}}}{\bar{\gamma}_{i j}}\right) \exp \left(-\frac{\left(1+K_{i j}\right) \gamma_{i j}}{\bar{\gamma}_{i j}}\right) I_{0}\left(2 \sqrt{\frac{K_{i j}\left(1+K_{i j}\right) \gamma_{i j}}{\bar{\gamma}_{i j}}}\right), \tag{96}
\end{equation*}
$$

where $K_{i j}$ denotes the ratio of the LOS component power-to-NLOS components average power of the corresponding link.


Fig. 22. Single relay LF relaying network. The S-R and R-D links experiences either Rician or Nakagami-m fading.

The Nakagami- m fading with factor $m$ is approximated by Rician fading with factor $K$ [20], [81], as

$$
\begin{equation*}
m=\frac{(K+1)^{2}}{2 K+1} \tag{97}
\end{equation*}
$$

### 2.3.2 Outage Probability Derivation

Solving (9), (10) and (11) in section 2.1.2 with the pdf (96), given above, of the instantaneous SNR, the outage probability of LF relaying with the fading variations of the S-R and R-D links following Rician distribution can be expressed as

$$
\begin{align*}
P_{\mathrm{A}}^{R i c i}= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(1)}^{\Theta^{-1}(\infty)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)} p^{R i c i}\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p^{R i c i}\left(\gamma_{\mathrm{RD}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} Q_{1}\left(\sqrt{2 K_{\mathrm{SR}}}, \sqrt{\frac{2\left(1+K_{\mathrm{SR}}\right)}{\bar{\gamma}_{\mathrm{SR}}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \cdot\left[1-Q_{1}\left(\sqrt{2 K_{\mathrm{RD}}}, \sqrt{2\left(1+K_{\mathrm{RD}}\right) \frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}}\right)\right] d \gamma_{\mathrm{SD}},  \tag{98}\\
P_{\mathrm{B}}^{R i c i}= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}(\infty)} p^{R i c i}\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p^{R i c i}\left(\gamma_{\mathrm{RD}}\right) d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\exp \left(-\frac{\left(1+K_{\mathrm{SR}}\right) \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right)\left(\frac{\left(1+K_{\mathrm{SR}}\right) e^{-K_{\mathrm{SR}}}}{\bar{\gamma}_{\mathrm{SR}}}\right)} \\
& \cdot I_{0}\left(2 \sqrt{\left.\frac{K_{\mathrm{SR}}\left(1+K_{\mathrm{SR}}\right) \gamma_{\mathrm{SR}}}{\gamma_{\mathrm{SR}}}\right)\left[1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right] d \gamma_{\mathrm{SR}}}\right. \tag{99}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{C}}^{R i c i}= & \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR})}\right)\right.}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{RD}}=\Theta^{-1}(0)}^{\Theta^{-1}\left[\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)\right]} p^{R i c i}\left(\gamma_{\mathrm{SR}}\right) p\left(\gamma_{\mathrm{SD}}\right) p^{R i c i}\left(\gamma_{\mathrm{RD}}\right) \\
& d \gamma_{\mathrm{SR}} d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{RD}} \\
= & \frac{1}{\bar{\gamma}_{\mathrm{SD}}}\left(\frac{\left(1+K_{\mathrm{SR}}\right) e^{-K_{\mathrm{SR}}}}{\bar{\gamma}_{\mathrm{SR}}}\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(\frac{-\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \cdot \exp \left(-\frac{\left(1+K_{\mathrm{SR}}\right) \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) I_{0}\left(2 \sqrt{\frac{K_{\mathrm{SR}}\left(1+K_{\mathrm{SR}}\right) \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}}\right) \\
& \cdot\left[1-Q_{1}\left(\sqrt{2 K_{\mathrm{RD}}}, \sqrt{2\left(1+K_{\mathrm{RD}}\right) \frac{\Theta^{-1}\left[\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)\right]}{\bar{\gamma}_{\mathrm{RD}}}}\right)\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{SR}} \tag{100}
\end{align*}
$$

where $Q_{1}(\cdot, \cdot)$ is the Marcum $Q$ function.

The outage probability of LF relaying with the $\mathrm{S}-\mathrm{R}$ and $\mathrm{R}-\mathrm{D}$ links suffering from Nakagami-m fading can be derived with the pdf of the instantaneous SNR following Nakagami-m distribution, given in (5) in section 2.1.1, as

$$
\begin{align*}
& P_{\mathrm{A}}^{\text {Naka }}= \frac{1}{\bar{\gamma}_{\mathrm{SD}}}\left(1-\left[\frac{\gamma\left(m_{\mathrm{SR}}, m_{\mathrm{SR}} \frac{1}{\bar{\gamma}_{\mathrm{SR}}}\right)}{\Gamma\left(m_{\mathrm{SR}}\right)}\right]\right) \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \exp \left(-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \\
& \cdot {\left[\frac{\gamma\left(m_{\mathrm{RD}}, m_{\mathrm{RD}} \frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SD}}\right)\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)}{\Gamma\left(m_{\mathrm{RD}}\right)}\right] d \gamma_{\mathrm{SD}}, }  \tag{101}\\
& P_{\mathrm{B}}^{\text {Naka }}= \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \frac{\left.m_{\mathrm{SR}}^{m_{\mathrm{SR}}}\left(\gamma_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}-1}\right)^{\bar{\gamma}_{\mathrm{SR}}} \Gamma\left(m_{\mathrm{SR}}\right)}{\exp }\left(-\frac{m_{\mathrm{SR}} \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right) \\
& \cdot\left[1-\exp \left(-\frac{\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}{\bar{\gamma}_{\mathrm{SD}}}\right)\right] d \gamma_{\mathrm{SR}}, \tag{102}
\end{align*}
$$

and

$$
\begin{align*}
P_{\mathrm{C}}^{\text {Naka }=} & \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \int_{\gamma_{\mathrm{SR}}=\Theta^{-1}(0)}^{\Theta^{-1}(1)} \int_{\gamma_{\mathrm{SD}}=\Theta^{-1}\left(1-\Theta\left(\gamma_{\mathrm{SR}}\right)\right)}^{\Theta^{-1}(1)} \exp \left(\frac{-\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}\right) \frac{m_{\mathrm{SR}}^{m_{\mathrm{SR}}}\left(\gamma_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}-1}}{\left(\bar{\gamma}_{\mathrm{SR}}\right)^{m_{\mathrm{SR}}} \Gamma\left(m_{\mathrm{SR}}\right)} \\
& \cdot \exp \left(-\frac{m_{\mathrm{SR}} \gamma_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{SR}}}\right)\left[\frac{\gamma\left(m_{\mathrm{RD}}, m_{\mathrm{RD}} \frac{\xi\left(\gamma_{\mathrm{SD}}, \gamma_{\mathrm{SR}}\right)}{\bar{\gamma}_{\mathrm{RD}}}\right)}{\Gamma\left(m_{\mathrm{RD}}\right)}\right] d \gamma_{\mathrm{SD}} d \gamma_{\mathrm{SR}}, \tag{103}
\end{align*}
$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function. Note that since the factor $K$ of Rician fading is connected to the factor $m$ of Nakagami-m fading by (97), the impact of the difference in the statistical characteristics with a fair parameter setting between Rician and Nakagami-m fading on the outage performance can be evaluated by adjusting the factor $K$ in Rician fading and the factor $m$ in Nakagami-m fading.

### 2.3.3 Kullback-Leibler divergence and Jensen-Shannon divergence

The Kullback-Leibler divergence (KLD) is used to measure the difference between probability distributions. Based on Nakagami-m pdf, (5) in section 2.1.1, and Rician pdf, (96) in section 2.3.1, the KLD of Rician relative to Nakagami-m distribution is
given as [82]

$$
\begin{equation*}
d_{\mathrm{KL}}\left(p^{\mathrm{Rici}}(\gamma) \| p^{\text {Naka }}(\gamma)\right)=\int_{\gamma} p^{\mathrm{Rici}}(\gamma) \ln \frac{p^{\mathrm{Rici}}(\gamma)}{p^{\text {Naka }}(\gamma)} d \gamma \tag{104}
\end{equation*}
$$

The KLD of Nakagami-m relative to Rician distribution is defined as

$$
\begin{equation*}
d_{\mathrm{KL}}\left(p^{\mathrm{Naka}}(\gamma) \| p^{\mathrm{Rici}}(\gamma)\right)=\int_{\gamma} p^{\mathrm{Naka}}(\gamma) \ln \frac{p^{\mathrm{Naka}}(\gamma)}{p^{\mathrm{Rici}}(\gamma)} d \gamma \tag{105}
\end{equation*}
$$

Obviously, since (104) and (105) have different value, and hence KLD does not satisfy the definition of "distance". Nevertheless, it is used to measure the information distance between Rician and Nakagami-m distributions [82]. When $K=0$ and $m=1$, $d_{\mathrm{KL}}\left(p^{\mathrm{Rici}}(\gamma) \| p^{\mathrm{Naka}}(\gamma)\right)=0$ and $d_{\mathrm{KL}}\left(p^{\mathrm{Naka}}(\gamma) \| p^{\mathrm{Rici}}(\gamma)\right)=0$, which indicates that the Rician and Nakagami-m distributions converge to the identical Rayleigh distribution.

Jensen-Shannon divergence (JSD) is another method of measuring how similar/different two probability distributions are. The JSD is defined by

$$
\begin{align*}
d_{\mathrm{JD}}\left(p^{\mathrm{Rici}}(\gamma) \| p^{\mathrm{Naka}}(\gamma)\right) & =\frac{1}{2}\left\{d_{\mathrm{KL}}\left[p^{\mathrm{Rici}}(\gamma) \| \frac{1}{2}\left(p^{\mathrm{Rici}}(\gamma)+p^{\text {Naka }}(\gamma)\right)\right]\right\} \\
& +\frac{1}{2}\left\{d_{\mathrm{KL}}\left[p^{\text {Naka }}(\gamma) \| \frac{1}{2}\left(p^{\text {Naka }}(\gamma)+p^{\mathrm{Rici}}(\gamma)\right)\right]\right\} . \tag{106}
\end{align*}
$$

Fig. 23 shows the KLD and JSD curves, $d_{\mathrm{KL}}\left(p^{\mathrm{Rici}}(\gamma) \| p^{\text {Naka }}(\gamma)\right), d_{\mathrm{KL}}\left(p^{\mathrm{Naka}}(\gamma) \| p^{\mathrm{Rici}}(\gamma)\right)$ and $d_{\mathrm{JD}}\left(p^{\mathrm{Rici}}(\gamma) \| p^{\mathrm{Naka}}(\gamma)\right)$, all as a function of the factor $K$ (or its corresponding $m$ value by (97)). We can easily find that $d_{\mathrm{KL}}\left(p^{\text {Rici }}(\gamma) \| p^{\text {Naka }}(\gamma)\right)$ and $d_{\mathrm{KL}}\left(p^{\text {Naka }}(\gamma) \| p^{\text {Rici }}(\gamma)\right)$ are not identical to each other, because of the asymmetricity of KLD. We can also see from Fig. 23 that the values of KLD and JSD first increase and then gradually decrease as $K$ (and hence $m$ ) becomes large. This observation is used to verify the outage difference of LF relaying over Rician and Nakagami-m fading channels in the next subsection.

### 2.3.4 Numerical Results

The theoretical outage probabilities of LF relaying with the S-R and R-D links suffering from Rician fading, which is denoted as $P_{\text {out }}^{\text {Rici }}$, and that suffering from Nakagami-m fading, which is denoted as $P_{\text {out }}^{\text {Naka }}$, are presented in Fig. 24. The relay is located on a line parallel to the line connecting S and D with the distant between the relay and the line connecting S and D is set at $\frac{1}{2}$ of the length of the S-D link. The value of $K_{\mathrm{SR}}$ and


Fig. 23. KLD and JSD between Rician and Nakagami-m distributions.
$m_{\mathrm{SR}}$ is set at $K_{\mathrm{SR}}=0$ and $m_{\mathrm{SR}}=1$, respectively, which indicates that the S-R link also suffering from Rayleigh fading. It is found from Fig. 24 that the $P_{\text {out }}^{\text {Rici }}$ and $P_{\text {out }}^{\text {Naka }}$ curves show the same tendency: the larger the $K_{\mathrm{RD}}$ (or $m_{\mathrm{RD}}$ ) value is, the smaller the outage probability, for a given average SNR value. This indicates that as the channel variation of the R-D link becomes mild, low outage probability can be achieved. However, the decay in outage curves keeps the same when $K_{\mathrm{RD}}$ (or $m_{\mathrm{RD}}$ ) value increases. This is because that, even the LOS component power in the R-D link increases, the equivalent diversity order (decay of the outage curve) of the LF relying system does not change. The equivalent diversity order of the system is determined by either the S-R link or the R-D link which achieves small equivalent diversity order.

Fig. 25 shows the theoretical outage probabilities $P_{\text {out }}^{\text {Rici }}$ and $P_{\text {out }}^{\text {Naka }}$ versus the average SNR of the S-D link, where $K_{\mathrm{SR}}=K_{\mathrm{RD}}\left(m_{\mathrm{SR}}=m_{\mathrm{RD}}\right)$. It is found that the outage curves can achieve sharper decay than that with 2nd order diversity, when the values of $K(m)$ of both the S-R and R-D links increase simultaneously. This is because the increased LOS component power in the S-R and R-D links increase the equivalent diversity order of the LF relaying system.

From Fig. 25 we also found that, when $K_{\mathrm{SR}}=K_{\mathrm{RD}}=0\left(m_{\mathrm{SR}}=m_{\mathrm{RD}}=1\right), P_{\mathrm{out}}^{\mathrm{Rici}}$ and $P_{\text {out }}^{\text {Naka }}$ show the same outage probability. This is because obviously, the S-R and


Fig. 24. Comparison of the outage probability of LF relaying and in Rician and Nakagami-m fading. $K_{\mathrm{SR}}=0\left(m_{\mathrm{SR}}=1\right)$.

R-D links reduce to Rayleigh fading. However, when the $K_{\mathrm{SR}}$ and $K_{\mathrm{RD}}$ ( $m_{\mathrm{RD}}$ and $m_{\mathrm{RD}}$ ) value increase, the outage curves in Rician and Nakagami-m fading exhibit different outage performances. Again, the difference diminishes when $K_{\mathrm{SR}}$ and $K_{\mathrm{RD}}$ ( $m_{\mathrm{SR}}$ and $m_{\mathrm{RD}}$ ) become large, which is consistent to the KLD and JSD analyses provided in the previous subsection. This observation indicates that even with a specific parameter setting yielding the same LOS ratio, Rician is not equivalent to Nakagami-m model for representing the shape of the entire portion of distribution.

### 2.4 Summary

In this chapter, the outage probability of three-node one-way relaying has been analyzed over fading channels on each link having different statistical properties.

First of all, for LF relaying over Nakagami-m fading channels, the exact outage probability expressions for arbitrary values of the shape factor $m$ were derived in (20), (21), and (22) in section [2.1.2 Outage Probability Analysis]-[Outage Event of LF relaying]. The equivalent diversity order and coding gain of LF relaying were defined and derived in (33) and (34) in section [2.1.3 Equivalent Diversity Order and Coding

Gain] through yet accurate enough approximation of the outage expressions (23), (24), and (25). It is found from (33) as well as from Fig. 9 that, with the increased LOS component power, the lower outage probability can be achieved. However, the outage curve decay is dominated by the less reliable channel of either the S-R or R-D link. It can be concluded from the numerical results provided in section [2.1.2 Outage Probability Analysis] and from Fig. 7 that LF relaying achieve better performance than conventional DF relaying. Since the relay in LF always forwards the decoder output to the destination, where re-interleaving performed for the decoded sequence, the whole system can be regarded as a distributed turbo code. It is also found from Figs.10, 11, and 12 that LF relaying not only reduces the outage probability, but also expands the search area for a relay (helper) while keeping the outage probability the same as that with DF or even lower.

Then, in the outage analysis of LF, DF, and ADF relaying over Rayleigh fading channels, the diversity order and coding gain was derived in section [2.2.3 Outage Analysis in Independent Fading], with yet accurate approximations (45), (46), and (47), in independent fading. It is proven by Theorems 2 and 3 that the coding gain of LF is larger than DF but smaller ADF since ADF utilize the error-free feedback from the relay which results in the situation that errors are not included in the encoders of both the source and the relay. The diversity orders for LF, DF, and ADF relaying were also obtained in (68), (69), and (70) in correlated fading in section [2.2.4 Outage Analysis in Correlated Fading]-[Diversity Order]. It is found from (68), (69), and (70) that the full diversity order can be achieved by LF, DF, and ADF relaying as long as the channels are not spatially or temporary fully correlated. The observations from numerical results provided in sections [2.2.5 Optimal Relay Locations for Minimizing the Outage Probability] and [2.2.6 Optimal Power Allocation for Minimizing the Outage Probability] suggest that compared to the independent fading case, the relay should be located close to the destination, or more transmit power should be allocated to the relay to reduce the diversity gain loss caused by the fading correlation, and hence can achieve the lowest outage probabilities, in correlated fading.

Finally, we analyzed the impact of different statistical properties of LOS component, i.e., Rician and Nakagami-m distributions, on outage performance. It is found from section [2.3.3 Kullback-Leibler divergence and Jensen-Shannon divergence] that the KLD and JSD between Nakagami-m and Rician distributions does not change monotonically as $K$ or $m$ becomes large. Instead, it first increases and then decreases. The KLD and JSD analyses, and the numerical results in Figs. 24 and 25 indicate


Fig. 25. Comparison of the outage probability of LF relaying and in Rician and Nakagami-m fading. $K_{\mathrm{SR}}=K_{\mathrm{RD}}, m_{\mathrm{SR}}=m_{\mathrm{RD}}$.
that even with a specific parameter setting yielding the same LOS ratio, Rician is not equivalent to Nakagami-m model for representing the shape of the entire portion of the distribution.

## 3 Performance Analysis for Two-Way Lossy-Forward Relaying with Random Rician $K$-factor

In this chapter, we derive the outage probability upper bound for a two-way LF relay system over Rician fading channels with random $K$-factor. The $K$-factor is assumed to follow empirical distributions, logistic and normal, derived from measurement data. It is found that two-way relaying with LF is superior to that with DF in terms of outage performance, regardless of either logistic or normal distribution is used to represent the variation of $K$-factor.

### 3.1 System Model

The system model is shown in Fig. 26, where two sources, A and B, exchange their i.i.d. binary information with the help of a relay R. All nodes are equipped with a single antenna. For achieving high spectral efficiency, we consider non-orthogonal two-way relaying in this Chapter. The entire transmission cycle is divided into two time slots, hence compared to three time slots systems in the orthogonal two-way relaying, $\frac{3}{2}$ times the spectrum efficiency gain is expected. In the first time slot, $A$ and $B$ encode their own information sequence $\mathrm{u}_{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{B}}$ separately, and send them to R simultaneously over a MAC. $\mathrm{u}_{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{B}}$ are also broadcast to B and A , respectively. Since the relay is not interested in the original information sequence sent from the sources, in the second time slot, the relay first decodes the information from A and B and then conducts the exclusive-or (XOR) based on the estimated binary information sequences $\hat{u}_{\mathrm{A}}$ and $\hat{u}_{B}$. Then, regardless of whether the estimates are correct or not, the relay re-encodes, modulates the XOR-ed version of the estimates, and broadcasts the resulting sequence to A and B over a broadcast channel. ${ }^{11}$

The channel coefficient $h_{i, j}$ between two nodes, $i$ and $j$, follows the Rician distribution with the $K$-factor $K_{i, j}(i, j \in(\mathrm{~A}, \mathrm{~B}, \mathrm{R}), i \neq j)$. In this section, we set

[^9]

Fig. 26. Two-way relaying transmission.
$K_{\mathrm{A}, \mathrm{B}}=K_{\mathrm{B}, \mathrm{A}}=0^{12}$. The joint pdf of the instantaneous SNR $\gamma_{i, j}$ and the random $K$-factor with the Rician distribution is denoted as $p_{\gamma_{i, j}}\left(\gamma_{i, j}, K_{i, j}\right)=p_{\gamma_{i, j}}\left(\gamma_{i, j} \mid K_{i, j}\right) p_{K_{i, j}}\left(K_{i, j}\right)$. The channel coefficient $h_{i, j}$ and $K$-factor vary independently block-by-block and stay the same over one block duration due to the block fading assumption.

After the integration over the instantaneous SNR, the cumulative distribution function (cdf) of instantaneous SNR with Rician distribution is obtained and represented by $F_{\gamma_{i, j}}\left(\gamma_{i, j}, K_{i, j}\right)^{13}$. The geometric gains between every two nodes are assumed to be the same.

The distribution of $K$-factor is represented by either logistic distribution $p_{\log }(K)=$ $\frac{\exp \left(-\frac{K-\mu}{\sigma}\right)}{\sigma\left[1+\exp \left(-\frac{K-\mu}{\sigma}\right)\right]^{2}}$ or normal distribution $p_{\text {nor }}(K)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left(\frac{-(K-\mu)^{2}}{2 \sigma^{2}}\right)$, which are found to be well-fit models in [52]; $\mu$ and $\sigma^{2}$ denote mean and variance of the distributions, respectively.

[^10]

Fig. 27. Rate region for $A-R$ and $B-R$ transmissions.

### 3.2 Outage Probability

Two-way LF relaying fails in achieving reliable communication if at least one of destination nodes (A or B) is in outage. The outage event $\mathscr{E}_{\mathrm{O}}$ of the two-way LF relaying system is, therefore, defined as $\mathscr{E}_{\mathrm{O}}=\mathscr{E}_{\mathrm{OA}} \cup \mathscr{E}_{\mathrm{OB}}$, where $\mathscr{E}_{\mathrm{OA}}$ and $\mathscr{E}_{\mathrm{OB}}$ represents the outage event for A and B, respectively. Without loss of generality, we only focus on the derivation of $\mathscr{E}_{\mathrm{OA}}$, since, even though parameters that determine $p_{\gamma_{i, j}}\left(\gamma_{i, j}, K_{i, j}\right)$ may be different for A and B , the mathematical outage expressions for A and B are the same.

First of all, we calculate the outage probability of $u_{A}$ and $u_{B}$ occurring in the first MAC time slot. It is obvious that for the first time slot transmission, the achievable region for $\{A, B\}$-to-R transmission is determined by the MAC rate region as shown in Fig. 27. The rate region outside the MAC achievable region can be divided into three sub-regions, $\mathrm{D}, \mathrm{G}$, and F , with $P_{\mathrm{D}}, P_{\mathrm{G}}$, and $P_{\mathrm{F}}$ denoting the probability that the rate pair $\left(R_{\mathrm{A}}^{c}, R_{\mathrm{B}}^{c}\right)$ falls into $\mathrm{D}, \mathrm{G}$, and F in Fig. 27 , respectively. $R_{\mathrm{A}}^{c}$ and $R_{\mathrm{B}}^{c}$ denote the spectrum efficiency of A-R and B-R channel, respectively, including the channel coding rate and the modulation multiplicity. When the rate pair $\left(R_{\mathrm{A}}^{c}, R_{\mathrm{B}}^{c}\right)$, falls in the achievable region, two sources can be recovered with arbitrarily small error probability at the relay. Neither of the information sequence from the two sources can be successfully recovered when the rate pair falls in the region F . When $\left(R_{\mathrm{A}}^{c}, R_{\mathrm{B}}^{c}\right)$ falls into the regions D or G , only one
of the information sequence can be perfectly recovered while the other probabilistically fails.

The probabilities that the rate pair $\left(R_{\mathrm{A}}^{c}, R_{\mathrm{B}}^{c}\right)$ falls into regions $\mathrm{D}, \mathrm{G}$, and F can be expressed as [43, 83]

$$
\begin{align*}
P_{\mathrm{D}} & =\operatorname{Pr}\left[R_{\mathrm{A}}^{c}>C\left(\gamma_{\mathrm{A}, \mathrm{R}}\right), R_{\mathrm{B}}^{c} \leq C\left(\frac{\gamma_{\mathrm{B}, \mathrm{R}}}{1+\gamma_{\mathrm{A}, \mathrm{R}}}\right)\right]  \tag{107}\\
P_{\mathrm{G}} & =\operatorname{Pr}\left[R_{B}^{c}>C\left(\gamma_{B, R}\right), R_{A}^{c} \leq C\left(\frac{\gamma_{\mathrm{A}, R}}{1+\gamma_{B, R}}\right)\right]  \tag{108}\\
P_{\mathrm{F}} & =\operatorname{Pr}\left[R_{\mathrm{A}}^{c}>C\left(\frac{\gamma_{\mathrm{A}, \mathrm{R}}}{1+\gamma_{\mathrm{B}, \mathrm{R}}}\right), R_{\mathrm{B}}^{c}>C\left(\frac{\gamma_{\mathrm{B}, \mathrm{R}}}{1+\gamma_{\mathrm{A}, \mathrm{R}}}\right), R_{\mathrm{A}}^{c}+R_{\mathrm{B}}^{c}>\log _{2}\left(1+\gamma_{\mathrm{A}, \mathrm{R}}+\gamma_{\mathrm{B}, \mathrm{R}}\right)\right] \tag{109}
\end{align*}
$$

respectively.
With the random Rician $K$-factor, $P_{\mathrm{D}}$ and $P_{\mathrm{F}}$ are expressed as a set of integrals with respect to the pdfs of the instantaneous SNRs and $K$, as,

$$
\begin{align*}
P_{D}= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{B}, \mathrm{R}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=0}^{2_{\mathrm{A}}^{R_{\mathrm{A}}^{c}-1}} \int_{\gamma_{\mathrm{B}, \mathrm{R}}}^{\infty}=\left(2^{2 R_{\mathrm{B}}^{c}-1}\right)\left(1+\gamma_{\mathrm{A}, \mathrm{R}}\right) \\
& \cdot p_{\gamma_{\mathrm{B}, \mathrm{R}}}\left(\gamma_{\mathrm{B}, \mathrm{R}} \mid K_{\mathrm{B}, \mathrm{R}}\right) p_{\log }\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{B}, \mathrm{R}}\right) d \gamma_{\mathrm{A}, \mathrm{R}} d \gamma_{\mathrm{B}, \mathrm{R}} d_{K_{\mathrm{A}, \mathrm{R}}} d_{K_{\mathrm{B}, \mathrm{R}}} \\
= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{B}, \mathrm{R}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=0}^{2_{\mathrm{A}}^{R_{\mathrm{A}}^{c}-1}} p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{B}, \mathrm{R}}\right) \\
& \cdot\left(1-F_{\gamma_{\mathrm{B}, \mathrm{R}}}\left[\left(2^{2 R_{\mathrm{B}}^{c}}-1\right)\left(1+\gamma_{\mathrm{A}, \mathrm{R}}\right) \mid K_{\mathrm{B}, \mathrm{R}}\right]\right) d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{A}, \mathrm{R}}} d_{K_{\mathrm{B}, \mathrm{R}}} \tag{110}
\end{align*}
$$

and

$$
\begin{align*}
P_{F}= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{B}, \mathrm{R}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}}^{2^{R_{\mathrm{A}}^{c}-1}} p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{B}, \mathrm{R}}\right) \\
& \cdot F_{\gamma_{\mathrm{B}, \mathrm{R}}}\left[\left(2^{2 R_{\mathrm{B}}^{c}}-1\right)\left(1+\gamma_{\mathrm{A}, \mathrm{R}}\right) \mid K_{\mathrm{B}, \mathrm{R}}\right] d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{A}, \mathrm{R}}} d_{K_{\mathrm{B}, \mathrm{R}}} \\
+ & \left.\int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{B}, \mathrm{R}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=2^{2_{\mathrm{A}}^{c}-1}}^{2^{R_{\mathrm{B}}^{c}}\left(2^{R_{\mathrm{A}}^{c}}-1\right.}\right) p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) \\
& \cdot p_{\log }\left(K_{\mathrm{B}, \mathrm{R}}\right)\left\{F_{\gamma_{\mathrm{B}, \mathrm{R}}}\left[\left(2^{R_{\mathrm{A}}^{c}+R_{\mathrm{B}}^{c}}-1-\gamma_{\mathrm{A}, \mathrm{R}}\right) \mid K_{\mathrm{B}, \mathrm{R}}\right]\right. \\
& \left.-F_{\gamma_{\mathrm{B}, \mathrm{R}}}\left[\left.\frac{\gamma_{\mathrm{A}, \mathrm{R}}}{2^{R_{\mathrm{A}}^{c}}-1} \right\rvert\, K_{\mathrm{B}, \mathrm{R}}\right]\right\} d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{A}, \mathrm{R}}} d_{K_{\mathrm{B}, \mathrm{R}}} \tag{111}
\end{align*}
$$

for the logistic-distributed $K$-factor. With the normal-distributed $K$-factor, the expressions of $P_{\mathrm{D}}$ and $P_{\mathrm{F}}$ can be straightforwardly obtained by replacing $p_{\log }(K)$ with $p_{\text {nor }}(K)$ in (110) and (111). $P_{\mathrm{G}}$ can be calculated similarly as $P_{\mathrm{D}}$ in (110).

Based on the decoding results for $\mathrm{u}_{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{B}}$ at R , the calculation of the outage probability of $\mathscr{E}_{\mathrm{OA}}, \operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}}\right)$, can be further classified into three cases as: Case 1 : $\left(\hat{\mathrm{u}}_{\mathrm{A}} \neq \mathrm{u}_{\mathrm{A}}\right.$ and $\left.\hat{\mathrm{u}}_{\mathrm{B}}=\mathrm{u}_{\mathrm{B}}\right)$ or $\left(\hat{\mathrm{u}}_{\mathrm{A}}=\mathrm{u}_{\mathrm{A}}\right.$ and $\left.\hat{\mathrm{u}}_{\mathrm{B}} \neq \mathrm{u}_{\mathrm{B}}\right)$ with which $\operatorname{Pr}($ Case 1$)=P_{\mathrm{D}}+P_{\mathrm{G}}$; Case 2: $\hat{\mathrm{u}}_{\mathrm{A}} \neq \mathrm{u}_{\mathrm{A}}$ and $\hat{\mathrm{u}}_{\mathrm{B}} \neq \mathrm{u}_{\mathrm{B}}, \operatorname{Pr}\left(\right.$ Case 2) $=P_{\mathrm{F}} ;$ Case 3: $\hat{\mathrm{u}}_{\mathrm{A}}=\mathrm{u}_{\mathrm{A}}$ and $\hat{\mathrm{u}}_{\mathrm{B}}=\mathrm{u}_{\mathrm{B}}, \operatorname{Pr}($ Case 3$)=$ $1-P_{\mathrm{D}}-P_{\mathrm{G}}-P_{\mathrm{F}}$.

### 3.2.1 $\quad \operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 1$)$

In Case $1, \mathrm{R}$ can decode either $\mathrm{u}_{\mathrm{A}}$ or $\mathrm{u}_{\mathrm{B}}$ correctly. However, the XOR-ed sequence is correlated with the original information of A and B . Therefore, the problem can be regarded as source coding with a helper where the information sequence transmitted from the relay is regarded as a helper.

After canceling the successfully decoded information sequence, the calculation of the Hamming distortion of the other information sequence becomes the same as that in a point-to-point ( P 2 P ) transmission. The relationship between the XOR-ed version of the estimates $\hat{\mathrm{u}}_{\mathrm{A} \oplus \mathrm{B}}=\hat{\mathrm{u}}_{\mathrm{A}} \oplus \hat{\mathrm{u}}_{\mathrm{B}}$ and its original one (denoted by $\mathrm{u}_{\mathrm{A} \oplus \mathrm{B}}=\mathrm{u}_{\mathrm{A}} \oplus \mathrm{u}_{\mathrm{B}}$ ) can be modeled by bit-flipping model with bit-flipping probability $p_{f}$, $\hat{\mathrm{u}}_{\mathrm{A} \oplus \mathrm{B}}=\mathrm{u}_{\mathrm{A} \oplus \mathrm{B}} \oplus E$ where $\operatorname{Pr}(E=1)=p_{f}$. Therefore, the Hamming distortion can be obtained by using the Shannon's lossy source-channel separation theorem as in section 2.1.2.

The admissible rate region in this case is illustrated in Fig. 28. The outage probability becomes $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 1$)=P_{M}+P_{L}$, where $P_{M}$ and $P_{L}$ are the probabilities that the source rate pair of A and $\mathrm{R}\left(R_{\mathrm{A}}^{s}, R_{\mathrm{R}}^{s}\right)$ falls into regions M and L in Fig. 28, respectively. With the relay-destination correlation also being represented by a bit-flipping model with probability $p_{f}^{\prime}, P_{M}$ and $P_{L}$ can then be written as ${ }^{14}$

[^11]

Fig. 28. Rate region for Case 1 according to the source coding with a helper theorem.

$$
\begin{align*}
P_{M}= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{R}, \mathrm{~B}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=0}^{1} \int_{\gamma_{\mathrm{R}, \mathrm{~B}}=1}^{\infty} \int_{\gamma_{\mathrm{A}, \mathrm{~B}}=0}^{2^{H\left(p_{f}\right)}-1} p_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(\gamma_{\mathrm{A}, \mathrm{~B}} \mid K_{\mathrm{A}, \mathrm{~B}}\right) \\
& \cdot p_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(\gamma_{\mathrm{R}, \mathrm{~B}} \mid K_{\mathrm{R}, \mathrm{~B}}\right) p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{R}, \mathrm{~B}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) \\
& d \gamma_{\mathrm{A}, \mathrm{~B}} d \gamma_{\mathrm{R}, \mathrm{~B}} d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}} \\
= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{R}, \mathrm{~B}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=0}^{1} F_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(2^{H\left(p_{f}\right)}-1\right)\left[1-F_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(1, K_{\mathrm{R}, \mathrm{~B}}\right)\right] \\
& \cdot p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{R}, \mathrm{~B}}\right) d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}} \tag{112}
\end{align*}
$$

and

$$
\begin{align*}
& P_{L}= \int_{K_{A, R}} \int_{K_{R, B}} \int_{\gamma_{A, R}=0}^{1} \int_{\gamma_{,, B}=0}^{1} \int_{\gamma_{A, B}=0}^{2} 2^{H\left(p_{f} * p_{f}^{\prime}\right)}-1 \\
& \gamma_{\gamma_{A, B}}\left(\gamma_{\mathrm{A}, \mathrm{~B}} \mid K_{\mathrm{A}, \mathrm{~B}}\right) \\
& \cdot p_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(\gamma_{\mathrm{R}, \mathrm{~B}} \mid K_{\mathrm{R}, \mathrm{~B}}\right) p_{\log }\left(K_{\mathrm{R}, \mathrm{~B}}\right) d \gamma_{\mathrm{A}, \mathrm{~B}} d \gamma_{\mathrm{R}, \mathrm{~B}} d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}}, \\
&= \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{R}, \mathrm{~B}}} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=0}^{1} \int_{\gamma_{\mathrm{R}, \mathrm{~B}}=0}^{1} F_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(2^{H\left(p_{f} * p_{f}^{\prime}\right)}-1\right)  \tag{113}\\
& \cdot p_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(\gamma_{\mathrm{R}, \mathrm{~B}} \mid K_{\mathrm{R}, \mathrm{~B}}\right) p_{\log }\left(K_{\mathrm{R}, \mathrm{~B}}\right) d \gamma_{\mathrm{R}, \mathrm{~B}} d \gamma_{\mathrm{A}, \mathrm{R}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}} .
\end{align*}
$$

### 3.2.2 $\quad \operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 2$)$

In Case 2, neither $u_{A}$ nor $u_{B}$ can be decoded without error at $R$. Therefore, we ignore the influence of the helper. Then, outage probability only depends on the direct A-B link transmission, as

$$
\begin{equation*}
\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid \text { Case } 2\right)=\int_{\gamma_{\mathrm{A}, \mathrm{~B}}=0}^{2_{\mathrm{A}}^{R_{\mathrm{A}}^{c}-1}} p_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(\gamma_{\mathrm{A}, \mathrm{~B}} \mid K_{\mathrm{A}, \mathrm{~B}}\right) d \gamma_{\mathrm{A}, \mathrm{~B}} \tag{114}
\end{equation*}
$$

Since in Case 2, we ignore the signal from the relay, resulting that the obtained outage probability is an upper bound.

### 3.2.3 $\quad \operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 3$)$

In Case $3, \mathrm{R}$ can fully recover $\mathrm{u}_{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{B}}$. The admissible rate region of $R_{\mathrm{A}}^{s}$ and $R_{\mathrm{R}}^{s}$ is shown in Fig. 29.


Fig. 29. Rate region for $R_{\mathrm{A}}^{S}$ and $R_{\mathrm{R}}^{S}$ in Case 2.

The outage occurs when $\left(R_{\mathrm{A}}^{s}, R_{\mathrm{R}}^{s}\right)$ falls into the inadmissible rate region J in Fig. 29, with probability $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 3$)=P_{J}$. Therefore, we have

$$
\begin{align*}
P_{J}= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{R}, \mathrm{~B}}} \int_{\gamma_{\mathrm{R}, \mathrm{~B}}=0}^{1} \int_{\gamma_{\mathrm{A}, \mathrm{~B}}=0}^{2^{1-C\left(\gamma_{\mathrm{\gamma}, \mathrm{~B}}\right)}-1} \int_{\gamma_{\mathrm{A}, \mathrm{R}}=1}^{\infty} p_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(\gamma_{\mathrm{A}, \mathrm{~B}} \mid K_{\mathrm{A}, \mathrm{~B}}\right) \\
& \cdot p_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(\gamma_{\mathrm{A}, \mathrm{R}} \mid K_{\mathrm{A}, \mathrm{R}}\right) p_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(\gamma_{\mathrm{R}, \mathrm{~B}} \mid K_{\mathrm{R}, \mathrm{~B}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) p_{\log }\left(K_{\mathrm{R}, \mathrm{~B}}\right) \\
& d \gamma_{\mathrm{A}, \mathrm{R}} d \gamma_{\mathrm{A}, \mathrm{~B}} d \gamma_{\mathrm{R}, \mathrm{~B}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}} \\
= & \int_{K_{\mathrm{A}, \mathrm{R}}} \int_{K_{\mathrm{R}, \mathrm{~B}}} \int_{\gamma_{\mathrm{R}, \mathrm{~B}}=0}^{1} F_{\gamma_{\mathrm{A}, \mathrm{~B}}}\left(2^{1-C\left(\gamma_{\mathrm{R}, \mathrm{~B}}\right)}-1\right)\left[1-F_{\gamma_{\mathrm{A}, \mathrm{R}}}\left(1, K_{\mathrm{A}, \mathrm{R}}\right)\right] \\
& \cdot p_{\gamma_{\mathrm{R}, \mathrm{~B}}}\left(\gamma_{\mathrm{R}, \mathrm{~B}} \mid K_{\mathrm{R}, \mathrm{~B}}\right) p_{\log }\left(K_{\mathrm{A}, \mathrm{R}}\right) p_{\mathrm{log}}\left(K_{\mathrm{R}, \mathrm{~B}}\right) d \gamma_{\mathrm{R}, \mathrm{~B}} d_{K_{\mathrm{R}, \mathrm{~B}}} d_{K_{\mathrm{A}, \mathrm{R}}} . \tag{115}
\end{align*}
$$

The outage probability of $\mathscr{E}_{\text {OA }}$ is given as $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}}\right) \leq \operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 1$) \operatorname{Pr}($ Case 1$)+$ $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 2$) \operatorname{Pr}($ Case 2$)+\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}} \mid\right.$ Case 3$) \operatorname{Pr}($ Case 3$)$. Then, the outage probability of the two-way LF system can be expressed as $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{O}}\right) \leq 1-\left[1-\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}}\right)\right]\left[1-\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OB}}\right)\right]$, where $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OB}}\right)$ can be obtained by following the similar calculations as those for $\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}}\right)$. It should be noted here that the sequences both A and B receive from the relay are correlated. Nevertheless, we calculate the outage probability by $1-$ $\left[1-\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OA}}\right)\right]\left[1-\operatorname{Pr}\left(\mathscr{E}_{\mathrm{OB}}\right)\right]$, assuming that they the outage probabilities occurred for A and B are independent.

### 3.3 Numerical Results

In this subsection, we present the outage probabilities of two-way LF relaying with the random Rician $K$-factor following the normal and logistic distributions. The outage probabilities of two-way DF relaying are also provided as the references. With two-way DF relaying, the relay keeps silent if error is detected after decoding in the information sequence sent from either A or B . The mean $\mu$ and variance $\sigma^{2}$ of the normal and logistic distributions follow the measurement data from urban, suburban, and rural areas, respectively [52], as shown in Table 7.

Fig. 30 compares the outage performance of two-way LF and two-way DF relaying. Since the outage probabilities of two-way DF in urban, suburban, and rural areas are almost the same, we only plot the outage curves of two-way DF relaying in rural area with Rican $K$-factor following the normal and logistic distributions, respectively. It is


Fig. 30. Comparison of the outage probability of the two-way LF and two-way DF relaying, where $R_{x}^{c}=1$ (bits/s/Hz) $(x \in(\mathbf{A}, \mathbf{B}))$.
Table 7. Measurement data of mean $\mu$ and variance $\sigma^{2}$.

|  | Urban Area |  | Suburban Area |  | Rural Area |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mu(\mathrm{dB})$ | $\sigma^{2}(\mathrm{~dB})$ | $\mu(\mathrm{dB})$ | $\sigma^{2}(\mathrm{~dB})$ | $\mu(\mathrm{dB})$ | $\sigma^{2}(\mathrm{~dB})$ |
|  | 1.88 | 4.13 | 2.41 | 3.84 | 2.63 | 3.82 |
| $p_{\text {log }}(K)$ | 1.85 | 4.15 | 2.6 | 3.88 | 3 | 3.88 |

found that the outage probability of two-way LF relaying is superior to that of two-way DF with either normal-distributed or logistic-distributed $K$-factor.

### 3.4 Summary

We theoretically analyzed the outage probability of a two-way LF transmission system over Rician fading channels with the $K$-factor of the Rician fading being a random variable. The variation of $K$-factor follows either logistic or normal distribution estimated from measurement data. It is found that, compared to two-way DF relaying, two-way LF relaying achieves lower outage probability, regardless of either logistic or normal distribution is used to represent the variation of $K$-factor, as shown in Fig. 30. This is because with two-way LF relaying, the relay always broadcast the decoder output regardless of whether error is detected after decoding in the information part or not, and
joint decoding is performed at the destination that compensate the decoding errors at the relay.

## 4 Performance Analysis for Two-Source Two-Relay Transmission over $\kappa-\mu$ Fading <br> Channels

In this chapter, we derive the outage probability of a two-source two-relay transmission system, where all the links experience $\kappa-\mu$ fading variations. The source-relay links are assumed to be non-orthogonal MACs. Two transmission schemes are considered for relay-destination transmission, i.e., non-orthogonal maximum ratio transmission and orthogonal transmission with joint-decoding at the destination. It is found that, regardless of whether the LOS component exists in the channel or not, the outage performance of the system with orthogonal transmission with joint-decoding is superior to that with maximum ratio transmission.

### 4.1 System Model

In this chapter, we consider two sources $S_{1}$ and $S_{2}$ communicate with one common destination $D$ with the help of two relays $R_{1}$ and $R_{2}$, as illustrated in Fig. 31, which has a symmetric topology, i.e., $S_{1}$ and $S_{2}$ as well as $R_{1}$ and $R_{2}$ are symmetric along the horizontal line through D . The relays are shared to both sources and no direct link exists between the sources and the destination. All nodes are equipped with a single antenna and operate in a half-duplex mode. The transmission is divided into two hops. In the first hop, $S_{1}$ and $S_{2}$ broadcast binary i.i.d. information sequence to $R_{1}$ and $R_{2}$ simultaneously over MACs. In the second hop, the relays decode the information sequences from both sources, re-encode, and forward them to the destination. We consider two schemes for the second hop transmission: 1) Non-orthogonal maximum ratio transmission scheme; 2) Orthogonal transmission with joint-decoding at $D$.

With the index $\{1,2, \ldots, 6\}$ representing the corresponding links shown in Fig. 31, the signal received at $\mathrm{R}_{i}(i \in\{1,2\})$ can be written as

$$
\begin{align*}
& y_{\mathrm{R}_{1}}=\sqrt{G_{1}} h_{1} s_{1}+\sqrt{G_{2}} h_{2} s_{2}+n_{\mathrm{R}_{1}}  \tag{116}\\
& y_{\mathrm{R}_{2}}=\sqrt{G_{3}} h_{3} s_{1}+\sqrt{G_{4}} h_{4} s_{2}+n_{\mathrm{R}_{2}} \tag{117}
\end{align*}
$$



Fig. 31. Two-source two-relay transmission system.
respectively, where $G_{i}$ is the geometric gain due to the distance of each link, $h_{i}$ is the complex channel gain, and $n_{\mathrm{R} i}$ is the zero-mean AWGN with the variance $N_{0} / 2$ per dimension. The modulated symbols of $S_{1}$ and $S_{2}$ are denoted by $s_{1}$ and $s_{2}$, respectively.

With the non-orthogonal transmission in the relay-destination link, the signal received at D in one time slot can be written as

$$
\begin{equation*}
y_{\mathrm{D}}=\sqrt{G_{5}} h_{5} s_{\mathrm{R} 1}+\sqrt{G_{6}} h_{6} s_{\mathrm{R} 2}+n_{\mathrm{D}}, \tag{118}
\end{equation*}
$$

where $s_{\mathrm{R} 1}$ and $s_{\mathrm{R} 2}$ denote the modulated symbols transmitted from $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$, respectively.

With the orthogonal transmission in the relay-destination link, the signal received at D in two time slots can be written as

$$
\begin{align*}
& y_{\mathrm{D}_{1}}=\sqrt{G_{5}} h_{5} s_{\mathrm{R} 1}+n_{\mathrm{D}_{1}},  \tag{119}\\
& y_{\mathrm{D}_{2}}=\sqrt{G_{6}} h_{6} s_{\mathrm{R} 2}+n_{\mathrm{D}_{2}}, \tag{120}
\end{align*}
$$

respectively, where $n_{\mathrm{D}_{1}}$ and $n_{\mathrm{D}_{2}}$ denote the AWGN noise in two time slots with identical distributions.

All the links are assumed to suffer from block $\kappa-\mu$ fading. The pdf of instantaneous SNR of link $i, \gamma_{i}$, is given by [25]

$$
\begin{array}{r}
f_{\kappa \mu}\left(\gamma_{i}\right)=\frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} \cdot \gamma_{i}^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp (\mu \kappa)_{i}^{\frac{\mu+1}{2}}} \cdot \exp \left(-\frac{\mu(1+\kappa) \gamma_{i}}{\bar{\gamma}_{i}}\right) \\
\quad \cdot I_{\mu-1}\left[2 \mu \cdot \sqrt{\frac{\kappa(1+\kappa) \mu}{\bar{\gamma}_{i}}}\right],(i \in\{1,2, \ldots, 6\}), \tag{121}
\end{array}
$$

where $\bar{\gamma}_{i}$ represents the average SNR of the link $i$. The parameter $\kappa$ is related to the ratio of the total power of the dominant components to the total power of the scattered waves. The parameter $\mu$ represents the number of multipath clusters ${ }^{15}$, and $I_{\mu-1}$ is the $(\mu-1)$ th order modified Bessel function of the first kind. All the links are assumed mutually i.i.d. The cdf of $\kappa-\mu$ distribution is given by [25],

$$
\begin{equation*}
F_{\kappa \mu}\left(\gamma_{i}\right)=1-Q_{1}\left(\sqrt{2 \kappa \mu}, \frac{\sqrt{2 \kappa(1+\kappa)} \gamma_{i}}{\bar{\gamma}_{i}}\right) . \tag{122}
\end{equation*}
$$

We set the horizontal distance between the sources and the destination to $d_{0}$, the distance between $S_{1}$ and $S_{2}$ as well as the distance between $R_{1}$ and $R_{2}$ to $d_{0} / 2$. With the geometric gain of the horizontal distance between the sources and the destination, $G_{0}$, being normalized to one, the geometric gains of the link $i$ with length $d_{i}$ can be defined as $G_{i}=\left(d_{0} / d_{i}\right)^{\alpha}$ [84], where $\alpha$ is the path loss factor.

### 4.2 Outage Probability

In this section, we provide the definition and derivation of the outage probability for the two-source two-relay system. The outage probabilities for $S_{1}$ and $S_{2}$ are the same due to the symmetry of the system topology. Therefore, we only focus on the derivation of the outage probability for $S_{1}$.

Since the transmission includes two hops, the overall outage probability of $S_{1}$ is calculated based on the law of Bayes' rule as,

$$
\begin{align*}
\mathrm{P}_{\text {out }} & =\operatorname{Pr}(\text { outage } \mid \text { Case I }) \operatorname{Pr}(\text { Case I }) \\
& +\operatorname{Pr}(\text { outage } \mid \text { Case II }) \operatorname{Pr}(\text { Case II }) \\
& +\operatorname{Pr}(\text { outage } \mid \text { Case III }) \operatorname{Pr}(\text { Case III }), \tag{123}
\end{align*}
$$

[^12]

Fig. 32. MAC rate region for source-relay transmission.
where Case I, II, and III indicate that: In Case I, the information of $S_{1}$ cannot be decoded error-freely at both the relays; In Case II, information of $S_{1}$ can be recovered at both the relays without error; In Case III, only one of the relays can decode the information of $\mathrm{S}_{1}$ with an arbitrary low error probability.

### 4.2.1 Outage Probability Calculation in Case I

For the source-relay transmission hop, the MAC rate region for $\left(S_{1}-R_{1}, S_{2}-R_{1}\right)$ links is shown in Fig. 32. If the rate pair $\left(R_{c 1}, R_{c 2}\right)^{16}$ falls into the region A or B shown in Fig. 32, error-free transmission for $S_{1}$ cannot be guaranteed. Therefore,

$$
\begin{equation*}
\operatorname{Pr}(\text { Case } \mathrm{I})=\left(\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}\right)\left(\mathrm{P}_{\mathrm{A}}^{\prime}+\mathrm{P}_{\mathrm{B}}^{\prime}\right) \tag{124}
\end{equation*}
$$

where $P_{A}$ and $P_{B}$ denotes the probability that $\left(R_{c 1}, R_{c 2}\right)$ falls into the regions $A$ and $B$, respectively. $\mathrm{P}_{A}^{\prime}$ and $\mathrm{P}_{\mathrm{B}}^{\prime}$ denotes the probability that rate pair for $\left(\mathrm{S}_{1}-\mathrm{R}_{2}, \mathrm{~S}_{2}-\mathrm{R}_{2}\right)$ links falls into their corresponding MAC rate regions, respectively.

[^13]$P_{A}$ and $P_{B}$ can be expressed as [83]
\[

$$
\begin{align*}
\mathrm{P}_{\mathrm{A}}= & \operatorname{Pr}\left[R_{c 1}>\log \left(1+\gamma_{1}\right), R_{c 2} \leq \log \left(1+\frac{\gamma_{2}}{1+\gamma_{1}}\right)\right],  \tag{125}\\
\mathrm{P}_{\mathrm{B}}=\operatorname{Pr}[ & {\left[R_{c 1}>\log \left(1+\frac{\gamma_{1}}{1+\gamma_{2}}\right), R_{c 2}>\log \left(1+\frac{\gamma_{2}}{1+\gamma_{1}}\right),\right.} \\
& \left.R_{c 1}+R_{c 2}>\log \left(1+\gamma_{1}+\gamma_{2}\right)\right] . \tag{126}
\end{align*}
$$
\]

With the assumption that all links suffer from statistically independent block $\kappa-\mu$ fading, $\mathrm{P}_{\mathrm{A}}$ and $\mathrm{P}_{\mathrm{B}}$ can be calculated by integral with respect to the pdf of the instantaneous SNRs of the corresponding links, as,

$$
\begin{align*}
\mathrm{P}_{\mathrm{A}} & =\int_{\gamma_{1}=0}^{2^{R_{c 1}-1}} f_{\kappa \mu}\left(\gamma_{1}\right) d \gamma_{1} \int_{\gamma_{2}=\left(2^{\left.R_{c 2}-1\right)\left(1+\gamma_{1}\right)}\right.}^{\infty} f_{\kappa \mu}\left(\gamma_{2}\right) d \gamma_{2} \\
& =\int_{\gamma_{1}=0}^{2^{R_{c 1}-1}} f_{\kappa \mu}\left(\gamma_{1}\right) \cdot\left(1-F_{\kappa \mu}\left[\left(2^{R_{c 2}}-1\right)\left(1+\gamma_{1}\right)\right]\right) d \gamma_{1} \tag{127}
\end{align*}
$$

and

$$
\begin{align*}
& \mathrm{P}_{\mathrm{B}}= \int_{\gamma_{1}=0}^{2^{R_{c 1}-1}} f_{\kappa \mu}\left(\gamma_{1}\right) d \gamma_{1} \int_{\gamma_{2}=0}^{\left(2^{\left.R_{c 2}-1\right)\left(1+\gamma_{1}\right)} f_{\kappa \mu}\left(\gamma_{2}\right) d \gamma_{2}\right.} \\
&+\int_{\gamma_{1}=2^{R_{c 1}-1}}^{2^{R_{c 2}\left(2^{R_{c 1}}-1\right)}} f_{\kappa \mu}\left(\gamma_{1}\right) d \gamma_{1} \int_{\gamma_{2}=\frac{\gamma_{1}}{2^{2 R_{c 1}-1}}-1}^{2\left(R_{c 1}+R_{c 2}\right)}-1-\gamma_{1} \\
& f_{\kappa \mu}\left(\gamma_{2}\right) d \gamma_{2} \\
&= \int_{\gamma_{1}=0}^{2^{R_{c 1}-1}} f_{\kappa \mu}\left(\gamma_{1}\right) \cdot F_{\kappa \mu}\left[\left(2^{R_{c 2}}-1\right)\left(1+\gamma_{1}\right)\right] \\
&+ \int_{\gamma_{1}=2^{R_{c 1}}-1}^{2_{c 2}^{\left.R_{c 2} R_{c 1}-1\right)}} f_{\kappa \mu}\left(\gamma_{1}\right)\left\{F_{\kappa \mu}\left(\frac{\gamma_{1}}{2^{R_{c 1}}-1}-1\right)\right.  \tag{128}\\
&\left.-F_{\kappa \mu}\left(2^{\left(R_{c 1}+R_{c 2}\right)}-1-\gamma_{1}\right)\right\} d \gamma_{1} .
\end{align*}
$$

$\mathrm{P}_{\mathrm{A}}^{\prime}$ and $\mathrm{P}_{\mathrm{B}}^{\prime}$ can be calculated by following the similar technique based on their corresponding MAC rate region.

Since in Case I, neither $R_{1}$ nor $R_{2}$ can fully decode the information of $S_{1}$. Therefore, the decoded information sequences of $S_{1}$ at the two relays can be regarded as noisy versions of the original ones. In this case, the outage analysis falls into the chief executive officer (CEO) problem [85] in the relay-destination hop. For the purpose of simplicity, we set $\operatorname{Pr}$ (outage $\mid$ Case I$)=1$, and therefore, the calculate outage curve represents theoretical upper bound.

### 4.2.2 Outage Probability Calculation in Case II

For Case II where the information of the source $S_{1}$ is fully recovered at $R_{1}$ and $R_{2}$, it is easy to have

$$
\begin{equation*}
\operatorname{Pr}(\text { Case II })=\left(1-\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{B}}\right)\left(1-\mathrm{P}_{\mathrm{A}}^{\prime}-\mathrm{P}_{\mathrm{B}}^{\prime}\right) . \tag{129}
\end{equation*}
$$

We consider two transmit schemes for the second hop (relay-destination) in Case II, i.e, non-orthogonal maximum ratio transmission and orthogonal transmission with joint-decoding.

## Maximum Ratio Transmission Scheme

With maximum ratio transmission, the outage probability is given by

$$
\begin{align*}
\operatorname{Pr}(\text { outage } \mid \text { Case II }) & =\operatorname{Pr}\left[R_{c}>\log \left(1+\gamma_{5}+\gamma_{6}\right)\right] \\
& =\int_{\gamma_{5}=0}^{2^{R_{c 5}-1}} f_{\kappa \mu}\left(\gamma_{5}\right) d \gamma_{5} \int_{\gamma_{6}=0}^{\left(2^{\left.R_{c 6}-1-\gamma_{5}\right)}\right.} f_{\kappa \mu}\left(\gamma_{6}\right) d \gamma_{6} \\
& =\int_{\gamma_{5}=0}^{2^{R_{c 5}-1}} f_{\kappa \mu}\left(\gamma_{5}\right) F_{\kappa \mu}\left(2^{\left.R_{c 6}-1-\gamma_{5}\right) d \gamma_{5} .}\right. \tag{130}
\end{align*}
$$

## Orthogonal Transmission with Joint-decoding Scheme

Let $b_{R 1}$ and $b_{R 2}$ denotes the successfully recovered information sequences of $S_{1}$ at the $R_{1}$ and $R_{2}$, respectively. The admissible rate region of the $R_{1}-D$ and $R_{2}-D$ links are shown in Fig. 33.

The outage happens when the information coding rate pair of $\mathrm{R}_{1}$ and $\mathrm{R}_{2}\left(R_{\mathrm{R} 1}, R_{\mathrm{R} 2}\right)$ falls into the inadmissible area F shown in Fig. 33, with probability $\mathrm{P}_{\mathrm{F}}$. Therefore, we have

$$
\begin{align*}
& \operatorname{Pr}(\text { outage } \mid \text { Case II })=\mathrm{P}_{\mathrm{F}} \\
& =\int_{\gamma_{5}=0}^{2^{R_{c 5}-1}} f_{\kappa \mu}\left(\gamma_{5}\right)\left(1-F_{\kappa \mu}(1)\right) \cdot F_{\kappa \mu}\left(2^{\left(1-\frac{\log \left(1+\gamma_{5}\right)}{R_{c 5}}\right) R_{c 6}}-1\right) d \gamma_{5} . \tag{131}
\end{align*}
$$



Fig. 33. Rate region for $R_{R 1}$ and $R_{R 2}$ in Case II for orthogonal transmission with jointdecoding scheme.

### 4.2.3 Outage Probability Calculation in Case III

In Case III, only one of the relays, either $\mathrm{R}_{1}$ or $\mathrm{R}_{2}$ can successfully decode the information sent from $S_{1}$. Therefore, the possibility of Case III is given as

$$
\begin{align*}
\operatorname{Pr}(\text { Case III }) & =\left(1-\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{B}}\right)\left(\mathrm{P}_{\mathrm{A}}^{\prime}+\mathrm{P}_{\mathrm{B}}^{\prime}\right) \\
& +\left(1-\mathrm{P}_{\mathrm{A}}^{\prime}-\mathrm{P}_{\mathrm{B}}^{\prime}\right)\left(\mathrm{P}_{\mathrm{A}}+\mathrm{P}_{\mathrm{B}}\right) . \tag{132}
\end{align*}
$$

The fully decoded information sequence at one relay is exactly the same as the original one. Even though decoding error is detected at the other relay, the erroneously decoded information sequence is correlated with the source sequence. Therefore, the decoded sequences at the two relays are correlated. Hence, the problem falls into the category of source coding with a helper. The outage probability is defined as the probability that $\left(R_{\mathrm{R} 1}, R_{\mathrm{R} 2}\right)$ falls into the inadmissible areas J and K in Fig. 34, denoted as $\mathrm{P}_{\mathrm{J}}$ and $\mathrm{P}_{\mathrm{K}}$, respectively. $p_{f}$ represents the bit flipping probability between the information sequences obtained after decoding at $R_{1}$ and $R_{2}$.


Fig. 34. Rate region for $R_{R 1}$ and $R_{R 2}$ in Case III.

We assume the recovered information of $S_{1}$ at $R_{2}$ is erroneous and the message are fully recovered at $\mathrm{R}_{1}$. According to Shannon's lossless source channel separation theorem [73], the relationship between $\gamma_{5}$ and $R_{\mathrm{R} 1}$ is established as $\gamma_{5} \geq\left(2^{R_{\mathrm{R} 1} R_{c 5}}-1\right)$.

Following the similar techniques in section 2.1.2, with the Hamming distortion measure for binary sources, the relationship between $p_{f}$ and $\gamma_{3}$ can be established as $p_{f}=H^{-1}\left(1-\log _{2}\left(1+\gamma_{3}\right)\right)$, according to Shannon's lossy source channel separation theorem [86].

Then, the outage probability in Case III can be written as

$$
\begin{align*}
& \operatorname{Pr}(\text { outage } \mid \text { Case IIII })=\mathrm{P}_{\mathrm{J}}+\mathrm{P}_{\mathrm{K}}=\int_{\gamma_{3}=0}^{2^{R_{c 3}-1}} f_{\kappa \mu}\left(\gamma_{3}\right) \\
& \cdot F_{\kappa \mu}\left(2^{\left(1-\frac{\log \left(1+\gamma_{3}\right)}{R_{c 5}}\right) R_{c 5}}-1\right) d \gamma_{3} \\
& +\int_{\gamma_{3}=0}^{1} \int_{\gamma_{5}=2^{\left[1-H_{2}^{-1}\left(1-\log \left(1+\gamma_{3}\right)\right)\right]}-1}^{1} f_{\kappa \mu}\left(\gamma_{3}\right) f_{\kappa \mu}\left(\gamma_{5}\right) \\
& \cdot F_{\kappa \mu}\left(2^{\xi\left(\gamma_{3}, \gamma_{5}\right)}-1\right) d \gamma_{3} d \gamma_{5} . \tag{133}
\end{align*}
$$



Fig. 35. Outage probabilities of the two-source two-relay system with maximum ratio transmission and orthogonal transmission with joint-decoding schemes; $\mu=1$.

### 4.3 Numerical Results

The outage performance of the two-source two-relay system is illustrated in this section. The path loss factor is set at $\alpha=3.52$ [84]. We assume the $\kappa$ and $\mu$ values in each link are identical and represented by generic symbols $\kappa$ and $\mu$.

Fig. 35 shows the theoretical outage probabilities of the two-source two-relay system for $S_{1}$, the same results can be obtained for $S_{2}$ due to the symmetric topology. The outage probabilities are calculated with maximum ratio transmission and orthogonal transmission with joint-decoding schemes in the relay-destination hop, denoted as "MSMR MRT" and "MSMR JD" in Fig. 35, respectively. It can be found that the outage probability curves exhibit the tendency that the larger the $\kappa$ values, the smaller the outage probability. With $\mu=1$, the $\kappa-\mu$ fading becomes Rician fading, where $\kappa$ equals to Rician factor $K$ [87, Chapter 19]. Therefore, as the channel LOS component power increases (large $\kappa$ ), low outage probability can be achieved. It is also found that the outage probability with orthogonal transmission with joint-decoding is superior to that with maximum ratio transmission scheme, with $(\kappa>1)$ or without $(\kappa=1)$ the LOS component in the channels. This is because maximum ratio transmission performs signal


Fig. 36. Outage probabilities of the two-source two-relay system with maximum ratio transmission, and with joint-decoding; $\kappa=1$.
level combining before decoding, whereas orthogonal transmission with joint-decoding is information level combining after decoding. ${ }^{17}$

Fig. 36 also compares the outage performance of the two-source two-relay system with orthogonal transmission with joint-decoding and maximum ratio transmission schemes with different $\mu$ (number of multipath clusters). It can be observed from Fig. 36 that, the higher the $\mu$ value, the lower the outage probability for a given SNR. However, the outage performance of the two-source two-relay system with orthogonal transmission with joint-decoding always outperforms that with maximum ratio transmission.

### 4.4 Summary

We derived the outage probability for a two-source two-relay cooperative communications system over $\kappa-\mu$ fading channels, of which result is shown in (123) in section [4.2

[^14]Outage Probability]. Both non-orthogonal maximum ratio transmission and orthogonal transmission with joint-decoding are considered for the relay-destination transmission. The theoretical results, demonstrated in Fig. 35 and Fig. 36, show that the two-source two-relay system with orthogonal transmission with joint-decoding scheme outperforms maximum ratio transmission scheme in terms of outage probability.

## 5 Conclusions and Further Work

### 5.1 Conclusions

We have investigated cooperative wireless communications by focusing on exploiting the statistical nature of channel variations. The major contributions of this dissertation are summarized as follows.

First of all, we analyzed the outage performance for a three-node one-way LF relaying system. Based on the source coding with a helper theorem, the outage probability of LF relaying over independent block Nakagami-m fading channels with arbitrary values of the shape factor $m$ was derived. The equivalent diversity order (defined as the decay of the outage curve) and coding gain of LF relaying was identified. It is shown that LF relaying is superior to conventional DF relaying in terms of the outage probability. Moreover, with LF, while keeping the outage probability the same or even lower, the search area for a relay (helper) can be increased over conventional DF relaying. Diversity orders were also derived for LF, DF, and ADF relaying assuming each link suffers from statistically independent or correlated Rayleigh fading. It is found that compared to independent fading, in correlated fading, to achieve the lowest outage probability, the relay should be located closer to the destination, or more transmit power should be allocated to the relay, both to recover the gain loss caused by the fading correlation. Furthermore, Rician is found not equivalent to Nakagami-m model for representing the entire shape of the distributions, even with a specific parameter setting yielding the same LOS components power ratio.

Then, we evaluated the outage performance of a two-way LF relaying system over Rician fading channels with random $K$-factor. The $K$-factor is assumed to follow empirical distributions, i.e., logistic or normal, derived from measurement data. It is found that two-way relaying with LF is superior to that with DF in terms of outage performance, regardless of either logistic or normal distribution is used to represent the variation of $K$-factor.

Finally, the work was extended to a two-source two-relay transmission system, where all the links experience $\kappa-\mu$ fading variations. It is found that, with or without the LOS component in channels, the outage probability of the two-source two-relay transmission system with orthogonal transmission with joint-decoding is lower than with non-orthogonal maximum ratio transmission.

## $5.2 \quad$ Future Work

There are several interesting and challenging issues left as future work, which are listed below:

- In the impact analysis of the spatial and temporal correlations of the fading variations on the outage performance in Chapter 2, only the simple three-node relay system was considered. The work can be extended to more complex network topologies, such as multi-relay systems. However, in many cases of the structures, exact information theoretic bounds are still unknown. For example, one-source, one-destination, and multi-relay case falls into the source coding with multiple helper problem, which is one of the open problems in the network information theory and the rate region has not yet been known.
- Also in the impact analysis of the spatial and temporal correlations of the fading variations on the outage performance in Chapter 2, the Rayleigh fading is assumed for performance analyses. More generic channel models, such as Nakagami-m, $\kappa-\mu$, or $\eta-\mu$ fading should be considered. However, for more generic channel models, the joint pdf for even two realizations following correlated distributions need to be obtained first.
- In the outage analysis for two-way LF relaying in Chapter 3, the errors in the two source-to-relay links are ignored in the case when the information of neither of the source can be decoded at the relay without errors. Hence, the outage probability in this case only depends on the direct source-destination link transmission, which results in the calculated outage probability being an upper bound. However, even though neither of the source information can be decoded without error at the relay, there is still a possibility that the errors can be corrected at the destination by joint decoding the two information sequences, one received via the source-destination link, and the other via the relay-destination link. A tighter bound of the theoretical outage probability can be derived, if the relationship between the error probabilities and the SNRs of two source-to-relay links can be explicitly defined.
- Also in Chapter 3, perfect self-interference cancellation due to the full duplex setting is assumed in the two-way LF transmission. In practice, self-interference may not be canceled completely, and it causes inter-symbol interference. Therefore, the work can be extended to the rate region analysis, as well as the outage probability calculation with inter-symbol interference being taken into consideration, which is also left as future work.
- Even though the source coding theory, e.g. the source coding with a helper theorem, is utilized for outage calculation. However, practical source coding is not considered in the performance analysis. Combining source coding, e.g., the Huffman coding, and channel coding is left as a important future study.
- So far, we focused on frequency flat fading only. The system performance in frequency selective fading is not yet known and therefore, needs to be analyzed with channel estimation and equalization.
- This dissertation considered the Rayleigh fading model for NLOS scenario. However, in vehicle-to-vehicle communications, the amplitude distribution of the received signal will be a product of two Rayleigh random processes if both the source and destination nodes in the network are moving, hence giving rise to the double-Rayleigh amplitude distribution [88]. This is also left as future work.


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## Appendix 1 Proof of Convexity of (71), (72), and (73)

By taking second-order partial derivative of $P_{\text {out }}^{\mathrm{LF}, \text { Ind }}, P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}$, and $P_{\text {out }}^{\mathrm{ADF}, \text { Ind }}$ in (71), (72), and (73) with respect to $d$, respectively, we have

$$
\begin{align*}
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{LF}, \text { Ind }}}{\partial d^{2}} & =\frac{9 \alpha d^{\alpha}(\alpha+1)}{4 \bar{\gamma}_{\mathrm{SD}} d^{(\alpha+2)}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{3}} \\
& +\frac{90 \alpha^{2} d^{2 \alpha}}{\bar{\gamma}_{\mathrm{SD}} d^{(2 \alpha+2)}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{4}},  \tag{134}\\
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{DF}, \text { Ind }}}{\partial d^{2}} & =\frac{20 \alpha^{2} d^{2 \alpha}}{\bar{\gamma}_{\mathrm{SD}} d^{(2 \alpha+2)}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{3}} \\
& +\frac{10 \alpha^{2} d^{(\alpha-1)}}{\bar{\gamma}_{\mathrm{SD}} d^{(\alpha+1)}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{2}}, \tag{135}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{ADF}, \text { Ind }}}{\partial d^{2}} & =\frac{20 \alpha^{2}\left(\frac{1}{\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)}-1\right)(1-d)^{2 \alpha}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{3}(1-d)^{(2 \alpha+2)}} \\
& +\frac{\alpha^{2} d^{(\alpha-1)}}{\bar{\gamma}_{\mathrm{SD}}\left(\bar{\gamma}_{\mathrm{SD}}+10 \lg \left(\frac{1}{d^{\alpha}}\right)\right)^{2} d^{(\alpha+1)}} \tag{136}
\end{align*}
$$

Obviously, (134), (135), and (136) are positive in the range $0<d<1$ which indicates that the objective functions are convex with respect to $0<d<1$.

## Appendix 2 Proof of Convexity of (85), (86), and (87)

Taking the second-order derivative of $P_{\text {out }}^{\mathrm{LF}, \mathrm{Cor}}, P_{\text {out }}^{\mathrm{DF}, ~ C o r ~}$, and $P_{\text {out }}^{\mathrm{ADF}}$, Cor in (85), (86), and (87) with respect to $k$, respectively, we have

$$
\begin{align*}
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{LF}, \mathrm{Cor}}}{\partial k^{2}} & =\frac{\left(\frac{1}{P^{T} G_{\mathrm{SR}} k}+1\right)}{5 P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} k^{3}(1-k)^{2}}+\frac{6\left(\frac{1}{10 P^{T} G_{\mathrm{SR}} k}+\frac{19}{50}\right)}{P_{T}^{2} G_{\mathrm{SD}} G_{\mathrm{RD}} k^{4}} \\
& +\frac{17}{25 P_{T}^{3} G_{\mathrm{SD}} G_{\mathrm{SR}}^{2} k^{5}}+\frac{9\left(\frac{1}{P^{T} G_{\mathrm{SR}} k}+1\right)}{20 P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} G_{\mathrm{SR}} k^{4}(1-k)},  \tag{137}\\
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{DF}, \mathrm{Cor}}}{\partial k^{2}} & =\frac{9\left(\frac{1}{P^{T} G_{\mathrm{SR}} k}+1\right)}{2 P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} k^{4}(1-k)}+\frac{5\left(\frac{1}{P^{T} G_{\mathrm{SR}} k}+1\right)}{P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} k^{3}(1-k)^{2}} \\
& +\frac{6}{P_{T}^{2} G_{\mathrm{SD}} G_{\mathrm{SR}} k^{4}}+\frac{5}{2 P_{T}^{4} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} G_{\mathrm{SR}} k^{5}(1-k)}, \tag{138}
\end{align*}
$$

and

$$
\begin{align*}
\frac{\partial^{2} P_{\mathrm{out}}^{\mathrm{ADF}, \mathrm{Cor}}}{\partial k^{2}} & =\frac{\left(\frac{1}{P^{T} G_{\mathrm{SR}} k}+1\right)}{5 P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} k^{3}(1-k)^{2}}+\frac{12\left(\frac{1}{10 P^{T} G_{\mathrm{SR}} k}+\frac{19}{50}\right)}{P_{T}^{3} G_{\mathrm{SD}} G_{\mathrm{RD}} k^{5}} \\
& +\frac{9}{10 P_{T}^{4} G_{\mathrm{SD}}^{2} G_{\mathrm{SR}} k^{6}}+\frac{9\left(\frac{1}{P^{T} G_{\mathrm{SR}}}+1\right)}{20 P_{T}^{3} G_{\mathrm{SD}}^{2} G_{\mathrm{RD}} G_{\mathrm{SR}} k^{4}(1-k)} \tag{139}
\end{align*}
$$

It is not difficult to find that (138), (137), and (139) are positive in the range $0<k<1$ which indicates that the objective functions are convex with respect to $0<k<1$.


[^0]:    ${ }^{1}$ We only assume frequency non-selective fading. The impact of the frequency selectivity in relaying system is left as a future work, as described in Chapter 5
    ${ }^{2}$ Strictly speaking, the first order statistic of vehicle-to-vehicle communication is described by double Rayleigh distribution, if there are non-line-of-sight (NLOS) components. However, this dissertation assumes the simplest NLOS components model, Rayleigh fading.

[^1]:    ${ }^{3}$ The symbol indexes are omitted in (2), (3), and (4) for conciseness.

[^2]:    ${ }^{4}$ In the numerical calculation, $E^{n}=2$. By assuming a half rate channel code and QPSK modulation for each channel, $R_{i j}^{c}=1(i \in(\mathrm{~S}, \mathrm{R}), j \in(\mathrm{R}, \mathrm{D}), i \neq j)$.

[^3]:    ${ }^{5} G_{d}$ indicates the decay of the outage probability curve versus average SNR $G_{i j} j \frac{E_{s}}{N_{0}}$ because $G_{d}$ appears in the exponent part in (32). Therefore, the decay of the outage curve is concerned and it has the equivalent meaning as the diversity order. However, according to the definition, $G_{d}$ can take arbitrary real number which is related to the shape factor $m_{i j}$. In this sense, we call the decay of the curve equivalent diversity order, which does not follow the standard definition of the diversity order.

[^4]:    ${ }^{6}$ With longer (or shorter) distant between the relay and the line connecting S and D , outage probabilities increase (or decrease). However the outage probabilities show the same tendencies as those in Fig. 10, Fig. 11, and Fig. 12

[^5]:    ${ }^{7}$ Note that the approximated outage expressions (45), (46), and (47) are obtained at high SNR regime. Therefore, it is reasonable to give proof for $\bar{\gamma}_{\mathrm{SD}}>1$ and $\bar{\gamma}_{\mathrm{SR}}>1$.

[^6]:    ${ }^{8}$ For showing $\rho_{t}$ has the same impact as $\rho_{s}$, the average SNR of the S-R link is set at $\bar{\gamma}_{\mathrm{SR}}=1.6 \mathrm{~dB}$. In such low SNR region, the outage probability varies very slightly with $\rho_{s}$ and $\rho_{t}$. For clearly showing the outage surface in Fig. 17, we use linear scale in Z-coordinate.

[^7]:    ${ }^{9}$ The details of the proof are straightforward but lengthy, and therefore are omitted here for brevity

[^8]:    ${ }^{10}$ The details of the proof are straightforward but lengthy, and therefore are omitted here for brevity.

[^9]:    ${ }^{11}$ The self-interference due to the full duplex setting in sources A and B may not be perfectly cancelated in practical, however, in our theoretical analysis, we ignore the impact of self-interference and assumed that full-duplex wireless communication is feasible.

[^10]:    ${ }^{12}$ In cooperative networks, a reasonable assumption is that since the A-B (B-A) channel suffers from severe fading, $B(A)$ needs the help of a relay via the R-B (R-A) channel suffering from moderate fading. Hence, it is reasonable that we set the $K$-factor of the A-B (B-A) channel at $K_{\mathrm{A}, \mathrm{B}}=K_{\mathrm{B}, \mathrm{A}}=0$, which corresponds to Rayleigh fading.
    ${ }^{13}$ Refer to [21, Section 2.3.2] for the pdf and cdf expressions of Rician fading.

[^11]:    ${ }^{14}$ Details of the relationship between the source rate and instantaneous SNR can be found in section 2.1.2.

[^12]:    ${ }^{15}$ Signal arrives at a receiver clustered in time and in direction, and there may be several clusters.

[^13]:    $\overline{{ }^{16} R_{\mathrm{c} i} \text { denotes the spectrum efficiency of link }} i$ including the channel coding rate and the modulation multiplicity.

[^14]:    ${ }^{17}$ Since cooperative communication is a diversity technique seeking for coverage extension and degrees of freedom enhancement, the spectrum efficiency loss is not taken into account in the performance comparison between maximum ratio transmission and orthogonal transmission with joint-decoding schemes in this section. Note that, CSI is not needed for orthogonal transmission with joint-decoding scheme at the transmitter side, however, maximum ratio transmission requires full CSI at the transmitter side for implementation.

