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A stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities

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Abstract Nowadays, with the increasing of aesthetic products, it becomes more and more important and quite difficult for consumers to choose their preferred products, especially the ones whose artistic and aesthetic aspects play a crucial role in consumer purchase decisions. Taking Kansei as one quality aspect of products, consumer-oriented Kansei evaluation focuses on evaluation of existing commercial products based on consumers' Kansei preferences. This paper proposes a stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities. Particularly, given a consumer's preferences, the concept of stochastic dominance is used to build an evaluation function for each Kansei attribute. Then, the importance weights captured by a priority hierarchy of Kansei attributes, together with the fuzzy majority, are incorporated into the aggregation of individual stochastic dominance degrees into an overall one. An application to the hand-painted Kutani cups in Ishikawa, Japan, is conducted to illustrate the effectiveness and efficiency of the proposed approach. It is seen that the proposed approach outperforms the existing research in terms of easy of use and better decision-support to the consumers.

Keywords Consumer-oriented Kansei evaluation · Stochastic dominance · Priority hierarchy · Fuzzy majority · Hand-painted Kutani cups.

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1 Introduction

Nowadays, it is important for manufacturers to have a consumer-oriented approach to improve the attractiveness of their products, which should not only satisfy the functional requirements of products, defined objectively; but also the aesthetic needs, by essence subjective (Oztekin et al, 2013; Petiot and Yannou, 2004). The aesthetic aspects of products have become key factors in consumer purchase decisions (Schütte, 2005) and actually received much attention since 1970s from the community of consumer-focused design and Kansei engineering. Kansei engineering has been developed as a new product development methodology to "translate the technology of a consumer's feeling and image for a product into the design elements of the product" (Nagamachi, 2002). As quoted from Schütte et al (2004), Kansei is an individual subjective impression from a certain artifact, environment or situation using all senses of *sight*, *hearing*, *feeling*, *smell*, *taste* [and *sense of balance*] as well as their *recognition*. Kansei engineering has been widely applied to new product design in industries such as automotive, home electronics, office machines, cosmetics, building products, and other sectors (Chen et al, 2015; Chuan et al, 2013; Grimsæth, 2005; Okudan et al, 2013); in Japan and Korea, and later in Europe (Schütte, 2005). Such studies, referred to as *Kansei design*, aim at developing new product prototypes that generate specific aesthetics of products by discovering the relationships between design elements and Kansei attributes (Chen and Chuang, 2008). It has been concluded that the aesthetic quality of a design can greatly enhance the desirability of a product and influence consumer satisfaction (Chen and Chang, 2009; Ishihara, 2014; Lu and Petiot, 2014; Yadav and Goel, 2008; Yang, 2011).

With the increasing of aesthetic products designed by Kansei engineering, it may become more and more important and quite difficult for consumers to choose their preferred products, especially the ones whose artistic and aesthetic aspects play a crucial role in consumer purchase decisions, e.g., the traditional crafts. The *consumer-oriented Kansei evaluation* (Huynh et al, 2010; Yan et al, 2012) regards Kansei as one aspect of quality of products and focuses on evaluation of existing commercial products based on consumers' Kansei preferences. Accordingly, the consumer-oriented Kansei evaluation may provide a decision-support to consumers for selecting aesthetic products based on their Kansei preferences, and thus would be helpful for marketing or recommendation purposes (Kudo et al, 2006), particularly important in the era of e-commerce where recommender systems have become an important research area (Adomavicius and Tuzhilin, 2005). On the other hand, in the era of e-commerce, consumers' Kansei preferences and preferred aesthetic products may be discovered from the navigation history with the help of recommender systems. In this sense, by integrating with the relationship between design elements and Kansei attributes, consumer-oriented Kansei evaluation may provide a decision-support for Kansei design, since designers are able to design new products best satisfying consumers' Kansei preferences. It should be emphasized that many studies of Kansei design have involved a Kansei evaluation process in which a design could be selected for production (Chen and Chuang, 2008; Imai et al, 2013). Also note that evaluations for ranking and selection are two closely related, common facets of human decision-making activities, in practice. So far, decision analysis approaches have been widely applied to a variety of evaluation problems in the literature (e.g., Chang, 2016; Nureize et al, 2014; Sotirov and Krasteva, 1994; Wu et al, 2015; Zeng et al, 2008). In this paper, we aim at proposing a Kansei evaluation approach for aesthetics products based on consumers' Kansei preferences, especially to be used at online stores in the era of e-commerce.

The multi-attribute nature of consumers' Kansei (Chen and Chuang, 2008) has led many researchers to study (consumer-oriented) Kansei evaluation from the perspective of multi-attribute decision-making (Nishizaki et al, 2016). There are generally three typical types of multi-attribute approaches to consumer-oriented Kansei evaluation (Yan et al, 2008, 2012). By treating the qualitative Kansei information as crisp numerical data, most Kansei related studies use additive multi-attribute utility approach to rate and rank the new product prototypes based on the mean scale ratings of Kansei assessments (e.g., Okudan et al, 2013; Petiot and Yannou, 2004). Due to the vagueness in the evaluation of aesthetics, it may not be appropriate to perform Kansei evaluation solely based on crisp numerical data (Nakamori and Ryoke, 2006). In a different but similar context, Martínez (2007) has proposed a sensory evaluation model based on the linguistic 2-tuple representation model (Herrera and Martínez, 2000), which considers the evaluation problem as a multi-expert/multi-attribute decision-making problem. The linguistic 2-tuple approach assumes a consistent order relation on the qualitative scale treated as the linguistic term set of a linguistic variable (Zadeh, 1975). Despite its advantage in modeling the vagueness of Kansei data, the linguistic 2-tuple approach is still based on the mean scale ratings (Yan et al, 2012). Since the evaluation of aesthetics is quite subjective and highly individualistic, it may be not appropriate to perform Kansei evaluation solely based on mean scale ratings without further considering the variation in consumer evaluations (Chen and Chuang, 2008; Nakamori and Ryoke, 2006). To solve both the vagueness and the variation in consumer-oriented Kansei evaluation, the target-based approach (Huynh et al, 2010) seems to be prominent in the sense that an uncertain Kansei profile is expressed in terms of a probability distribution over the qualitative scale by considering the vagueness and variation of Kansei assessments simultaneously. Based on the uncertain profiles, Huynh et al (2010) have proposed a fuzzy target-based approach to consumer-oriented Kansei evaluation, in which consumer's Kansei preferences are expressed in terms of fuzzy targets. They first calculate the probability of meeting each target, and then aggregate the individual probabilities by the ordered weighted averaging (OWA) operator (Yager, 1988). Taking the target-based approach a further step, Yan et al (2012) have proposed a non-additive multi-attribute consumer-oriented Kansei evaluation approach based on the non-additive target-based decision analysis (Grabisch and Labreuche, 2010; Yan et al, 2013a), in which the fuzzy targets can be mutually dependent.

Despite the great prominence in consumer-oriented Kansei evaluation, there are still two problems to be solved in the target-based approaches. Firstly, existing approaches have conducted (consumer-oriented) Kansei evaluation function by quantifying consumers' Kansei preferences in terms of fuzzy sets. However, the quantification is in fact the process of transforming the ordinal (preference) information into a cardinal scale that represents an "arbitrary passage", which may sometimes be dangerous (Yan and Ma, 2015; Yan et al, 2013b, 2014). Even if the quantification in terms of fuzzy sets is rational, the fuzzy-set-based semantics is often defined subjectively and context-dependently, which may sensitively influence the final recommendation results. In this sense, direct computations based on the preference orders may provide a better solution to the consumer-oriented Kansei evaluation function. In addition, our second motivation comes from the multi-attribute nature of consumer-oriented Kansei evaluation. On one hand, a consumer's Kansei preferences may not be fully satisfied at the same time. Thus, fuzzy majority, which may be expressed in terms of linguistic quantifiers, e.g. "as many as possible", seems to be a suitable aggregation rule in consumer-oriented Kansei evaluation. In Huynh et al (2010)' work, the fuzzy majority is modeled by the OWA aggregation operator (Yager, 1988). On the other hand, it is important for the consumers to consider different weights for their Kansei preferences as some Kansei preferences are more important than others. In this case, the consumers may associate different weights with different Kansei attributes. Note the consumer-oriented Kansei evaluation focuses mainly on personalized recommendation in the era of e-commerce (Huynh et al, 2010). It is thus quite difficult and time-consuming for him to assign exact importance weights for Kansei attributes in a dynamic environment, since a consumer may choose different Kansei attributes. It is necessary to provide a convenient way for consumers to express the importance weights of the Kansei attributes. As pointed out by Torra (1997), the weights induced by the OWA operator reflect the weights of values; whereas the weights of attributes reflect the reliabilities of information sources. It is thus necessary to incorporate both the fuzzy majority and the weights of Kansei attributes into consumer-oriented Kansei evaluation.

Due to the above two observations, this paper tries to propose an alternative approach to consumer-oriented Kansei evaluation. To do so, uncertain Kansei profiles in terms of probability distributions over the qualitative scale are first derived. Secondly, according to a consumer's Kansei preferences toward selected Kansei attributes, the preference orders on the Kansei scale are defined. Then, the concept of stochastic dominance degree (Yan et al, 2013b) is utilized to build an evaluation function, which derives a fuzzy preference matrix of all the products for each selected Kansei attribute. As we shall see, there is no need to quantify the consumer's Kansei preferences. Finally, the importance weights of Kansei attributes are captured by a prioritization of these attributes, which may be easily specified by the consumer or defined according to the sequence of selecting Kansei attributes by the consumers. By taking the concept of fuzzy majority into account, the priority weighted OWA aggregation operator is proposed to obtain the global dominance degrees for the products, which incorporates both the importance weights of Kansei attributes and the fuzzy majority into our consumer-oriented Kansei evaluation and thus may provide a better decision-support to the consumers.

The rest of this paper is as follows. Section 2 introduces the necessary and preparatory procedure of the Kansei experiments and formulates our research problems. Section 3 proposes a stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities, which incorporates both the importance weights of Kansei attributes and the concept of fuzzy majority. Section 4 conducts an application of our proposed approach to the hand-painted Kutani cups in Ishikawa, Japan. A comparative study with existing research is also given. Finally, this paper is concluded with some remarks in Section 5.

2 Problem formulation

2.1 The necessary and preparatory Kansei experiment

Kansei experiment is a necessary and preparatory procedure in the community of Kansei research (Kansei design and consumer-oriented Kansei evaluation). The first step in Kansei experiment is to select a product domain and collect product samples. It is easy to collect product images from the marketplace such as web-sites, producers, catalogs, and magazines (Grimsæth, 2005). Researchers then need to eliminate duplicate or similar ones. The refined products are representative experimental samples representing the product domain investigated. The minimal sample size must represent the targeted market segment at a statistical significance (Oztekin et al, 2013). In the literature, the number of product samples usually ranges from 15 to 112 (e.g., Grimsæth, 2005; Llinares and Page, 2011; Yan et al, 2008, 2012; Yang, 2011). Formally, let $\mathbb{O} = \{O_1, O_2, \ldots, O_M\}$ be a set of representative products to be evaluated.

Secondly, we have to identify and measure the Kansei attributes used by people to express their psychological feelings for the products. Usually, the Kansei attributes are identified by a panel of experts familiar with Kansei Engineering and the product domain via a brainstorming process (Schütte et al, 2004). There are different ways of measuring the Kansei such as *words* (e.g. Yan et al, 2012), *physiological response* (e.g. Balters and Steinert, 2015; Kanoh et al, 2011), *people's behaviors and actions* (e.g. Kang et al, 2008), and *facial and body expressions* (e.g. Elokla et al, 2010). Most Kansei related studies, which have been published in English, use words when measuring the Kansei. The words are external descriptions of the Kansei within a person's mind and will be used to measure the Kansei in this study.

The Kansei attributes are bipolar pairs of words that describe the product domain and can be collected from all available sources such as *magazines*, *manuals*, *product reviews*, and *users* (Grimsæth, 2005). Researchers then need to eliminate duplicate or similar Kansei attributes. The refined set of Kansei attributes describes the semantic space, the number of which can be generally as high as 600 (Nagamachi, 1995). In order to reduce the burden of Kansei evaluation, about 20-30 Kansei attributes are usually used (Grimsæth, 2005). Formally, the set of refined Kansei attributes is expressed as follows:

- let $\mathbb{X} = \{X_1, X_2, \ldots, X_N\}$ be a set of refined Kansei attributes;
- let $\mathrm{KW}_n = \langle \mathrm{kw}_n^-, \mathrm{kw}_n^+ \rangle$ be the bipolar pair of Kansei words regarding Kansei attribute X_n, where kw_n^- and kw_n^+ are the left and right Kansei words, respectively;
- let $\mathbb{KW}$ be the set of bipolar pairs of Kansei words such that $\mathbb{KW} = \left\{ \langle \mathrm{kw}_n^-, \mathrm{kw}_n^+ \rangle \,|n = 1, \ldots, N \right\}$.

Thirdly, the semantic differential (SD) method (Osgood et al, 1957) is used to design a questionnaire consisting of listing the set of N Kansei attributes, each of which corresponds to a bipolar pair of Kansei words with a G-point odd qualitative scale. In our model, the qualitative scale is treated as a categorical one and symbolically denoted by $\mathbb{V} = \{V_1, V_2, \ldots, V_G\}$, where $g = 1, \ldots, G$. The left-most hand point V_1 and the right-most hand point V_G stand for left Kansei word kw^- and right Kansei word kw^+, respectively. In practice, people can reasonably manage to keep about seven points in mind (Miller, 1956). Therefore, most Kansei related studies always use a 7-point qualitative scale (e.g. Yan et al, 2012) such that $\mathbb{V} = \{1, 2, 3, 4, 5, 6, 7\}$.

Finally, the questionnaire is distributed to a population of subjects $\mathbb{E} = \{E_1, E_2, \ldots, E_K\}$, who are selected and asked to express their subjective assessments for the products in $\mathbb{O}$ on Kansei attributes in $\mathbb{X}$ via the qualitative scale $\mathbb{V}$, simultaneously. In many papers regarding Kansei related studies, the population of subjects involved in the Kansei experiment ranges typically from 10 to 35 (Chang et al, 2006; Huang et al, 2011; Yang, 2011). Formally, the Kansei assessment provided by subject $E_k \in \mathbb{E}$ for product $O_m \in \mathbb{O}$ on Kansei attribute $X_n \in \mathbb{X}$ is denoted as x_{mn}^k, where $\forall x_{mn}^k \in \mathbb{V}, m = 1, \ldots, M, n = 1, \ldots, N$, and $k = 1, \ldots, K$; the Kansei database of product O_m is denoted by $\mathbf{DB}[O_m]$.

2.2 Research problems

Assume a potential consumer is interested in finding an aesthetic product that would meet his Kansei preferences given by a subset $\mathbf{KW}$ of the set $\mathbb{KW}$ with N bipolar pairs of Kansei words. In particular, we are concerned with the consumer-specified Kansei preferences (Huynh et al, 2010; Yan et al, 2012) that can be generally expressed as $\mathbf{KW} = \{\mathrm{kw}_{\mathbf{1}}^*, \mathrm{kw}_{\mathbf{2}}^*, \ldots, \mathrm{kw}_{\mathbf{N}}^*\}$, which corresponds to the selected Kansei attributes $\mathbf{X} = \{X_{\mathbf{1}}, X_{\mathbf{2}}, \ldots, X_{\mathbf{N}}\}$. Here, $*$ stands for $-$ or $+$, and $-,+$ represent *left Kansei word preference* and *right Kansei word preference*, respectively.

The problem now is how to evaluate the products in $\mathbb{O}$ using the Kansei database based on consumer-specified preferences $\mathbf{KW}$? In order to achieve this goal, we consider the following two problems.

- (1) To build an evaluation function without quantifying the consumer's Kansei preferences $\mathbf{KW}$.
- (2) A consumer's Kansei preferences may not be fully satisfied at the same time, fuzzy majority in terms of linguistic quantifier LQ will be specified by consumers. It is important for consumer to associate different importance weights with the selected Kansei attributes; here the importance weights are captured by a priority hierarchy of Kansei attributes, denoted by $\mathbb{H}$. Then we aim at incorporating both the fuzzy majority and the priority hierarchy into the Kansei evaluation function.

With these two problems, our problem can be expressed as follows.

$$[\mathbf{KW} = \{\mathrm{kw}_{\mathbf{1}}^*, \mathrm{kw}_{\mathbf{2}}^*, \ldots, \mathrm{kw}_{\mathbf{N}}^*\}, \mathbb{H}, LQ], \tag{1}$$

Table 1 Uncertain Kansei profiles of products

$\mathbb{O}$	Kansei attributes $\mathbb{X}$			
	X_1	X_2	$\ldots$	X_N
O_1	$[p_{11}(V_1), p_{11}(V_2), \ldots, p_{11}(V_G)]$	$[p_{12}(V_1), p_{12}(V_2), \ldots, p_{12}(V_G)]$	$\ldots$	$[p_{1N}(V_1), p_{1N}(V_2), \ldots, p_{1N}(V_G)]$
O_2	$[p_{21}(V_1), p_{21}(V_2), \ldots, p_{21}(V_G)]$	$[p_{22}(V_1), p_{22}(V_2), \ldots, p_{22}(V_G)]$	$\ldots$	$[p_{2N}(V_1), p_{2N}(V_2), \ldots, p_{2N}(V_G)]$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
O_M	$[p_{M1}(V_1), p_{M1}(V_2), \ldots, p_{M1}(V_G)]$	$[p_{M2}(V_1), p_{M2}(V_2), \ldots, p_{M2}(V_G)]$	$\ldots$	$[p_{MN}(V_1), p_{MN}(V_2), \ldots, p_{MN}(V_G)]$

which will be solved by a stochastic dominance based approach presented in the next section.

3 A stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities

In this section, we shall propose a stochastic dominance based approach to consumer-oriented Kansei evaluation, which basically consists of the following three steps. In the first step, uncertain Kansei profiles of the products are obtained in terms of probability distributions over the qualitative scale $\mathbb{V}$. Secondly, given the Kansei preferences **KW** specified by a consumer, a matrix of stochastic dominance degrees of all the products on each Kansei attribute is derived from the uncertain Kansei profiles. Accordingly, the stochastic dominance degree of a product on a Kansei attribute can be obtained. Finally, the individual stochastic dominance degrees are synthesized into an overall one by incorporating the priority weights of selected Kansei attributes and the concept of fuzzy majority.

3.1 Generation of uncertain Kansei profiles

With the Kansei database **DB** obtained, for each product O_m with its assessment data $\mathbf{DB}[O_m]$, a probability distribution of product O_m on Kansei attribute X_n over the qualitative scale $\mathbb{V}$ can be obtained based on our previous work (Huynh et al, 2010; Yan et al, 2008), expressed as follows:

$$\begin{aligned} p_{mn}(V_g) &= p_{\mathbb{V}}\left(V_g | O_m, X_n\right) \\ &= \frac{\left|\left\{E_k \in \mathbb{E} : x_{mn}^k = V_g\right\}\right|}{|\mathbb{E}|} \end{aligned} \tag{2}$$

where $g = 1, \ldots, G, k = 1, \ldots, K, n = 1, \ldots, N, m = 1, \ldots, M$, and $|\cdot|$ denotes the cardinality of a set.

Therefore, for each product O_m, a probability distribution function $p : \mathbb{V} \rightarrow [0, 1]$ is defined for each product $O_m(m = 1, \ldots, M)$ on each Kansei attribute $X_n(n = 1, \ldots, N)$, denoted as

$$p_{mn} = [p_{mn}(V_1), p_{mn}(V_2), \ldots, p_{mn}(V_G)]\,, \tag{3}$$

where $m = 1, \ldots, M, n = 1, \ldots, N$. Here, p_{mn} is considered as an uncertain judgment (referred to as *Kansei profile*) of product O_m with respect to Kansei attribute X_n, which can reflect the variation of the Kansei assessments. Moreover, the qualitative scale is treated as categorial or linguistic data, the vagueness of Kansei assessments can also be modeled. Similarly, uncertain Kansei profiles of product $O_m \in \mathbb{O}$ on all Kansei attributes can be generated from $\mathbf{DB}[O_m]$, as shown in Table 1.

3.2 Derivation of stochastic dominance degrees from uncertain Kansei profiles

Given a consumer's Kansei preferences $\mathbf{KW} = \{\mathrm{kw}^*_{\mathbf{1}}, \mathrm{kw}^*_{\mathbf{2}}, \ldots, \mathrm{kw}^*_{\mathbf{N}}\}$ toward the subset of Kansei attributes $\mathbf{X} = \{X_{\mathbf{1}}, X_{\mathbf{2}}, \ldots, X_{\mathbf{N}}\}$, intuitively, if a consumer expresses his preference on Kansei attribute X_n with *left* Kansei preference kw^-_n, he might implicitly assume a preference order on the Kansei data toward V_1, where the left Kansei word is placed. Conversely, if the consumer's preference for that Kansei attribute is *right* Kansei preference kw^+_n, the preference order on the Kansei data corresponding to the Kansei attribute X_n should be determined adaptively according to the particular consumer's preference toward the end V_G, where right Kansei word is placed. Formally, the preference order relation $\succeq_n$ on Kansei attribute X_n can be formally expressed as (Huynh et al, 2010; Yan et al, 2008, 2012)

$$\succeq_n \Leftrightarrow \begin{cases} V_1 \succeq_n \cdots \succeq_n V_G, \text{ if } \mathrm{kw}^*_n = \mathrm{kw}^-_n; \\ V_1 \preceq_n \cdots \preceq_n V_G, \text{ if } \mathrm{kw}^*_n = \mathrm{kw}^+_n. \end{cases} \tag{4}$$

where $n = \mathbf{1}, \ldots, \mathbf{N}$.

In our context, the uncertain profile of each product on each Kansei attribute is expressed in terms of a probability distribution over the qualitative scale $\mathbb{V}$. Therefore, each Kansei attribute can be viewed as a random variable represented by a probability distribution over the qualitative scale $\mathbb{V}$. Quite importantly, in any linguistic decision analysis, the procedure of asking each expert to provide his/her absolute linguistic evaluations for a set of alternatives is based on the mutual independence among the set of alternatives (Bordogna et al, 1997). In our research context, the subjects can be viewed as "experts" and the qualitative scale may be view as "linguistic scale" (Martínez, 2007; Yan et al, 2012). Moreover, in the Kansei experiment, the subjects are asked to provide their absolute Kansei judgements of the products on Kansei attributes independently. Therefore, mutual independence among the subjects, among the products, and among Kansei attributes is naturally assumed here.

By accepting such mutual independence, we are now able to define an evaluation function corresponding to the Kansei preferences $\mathbf{KW} = \{\mathrm{kw}^*_{\mathbf{1}}, \mathrm{kw}^*_{\mathbf{2}}, \ldots, \mathrm{kw}^*_{\mathbf{N}}\}$ based on the concept of *stochastic dominance*, as introduced in the appendix part. In particular, with the preference order relations defined in Eq. (4), a stochastic dominance degree can be derived as follows.

Definition 1 Let O_m, O_l be two products, given Kansei attribute X_n, the stochastic dominance degree of product O_m over product O_l with respect to Kansei attribute X_n is defined as

$$V^n_{ml} = \begin{cases} \Pr(O_m \leq O_l | X_n) - 0.5\Pr(O_m = O_l | X_n), \text{ if } \mathrm{kw}^*_n = \mathrm{kw}^-_n; \\ \Pr(O_m \geq O_l | X_n) - 0.5\Pr(O_m = O_l | X_n), \text{ if } \mathrm{kw}^*_n = \mathrm{kw}^+_n. \end{cases} \tag{5}$$

where $n = \mathbf{1}, \ldots, \mathbf{N}, m, l = 1, \ldots, M$, $\Pr(O_m \leq O_l | X_n)$ and $\Pr(O_m = O_l | X_n)$ can be obtained by the calculation procedures in the appendix part.

Extending two products to M products, for each Kansei attribute X_n, we can derive a matrix of stochastic dominance degrees of different products as

$$\mathbf{V}^n = \begin{array}{c|cccc} & O_1 & O_2 & \ldots & O_M \\ \hline O_1 & V^n_{11} & V^n_{12} & \ldots & V^n_{1M} \\ O_2 & V^n_{21} & V^n_{22} & \ldots & V^n_{2M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_M & V^n_{M1} & V^n_{M2} & \ldots & V^n_{MM} \end{array} \tag{6}$$

where V^n_{ml} represents the stochastic dominance degree of product O_m over product O_l with respect to Kansei attribute X_n and $n = \mathbf{1}, \ldots, \mathbf{N}$. According to the properties of stochastic dominance introduced in the appendix part, it is easy to obtain that the matrix $\mathbf{V}^n$ satisfies the fuzzy reciprocal property such that

$$V_{ml}^n + V_{lm}^n = 1, \tag{7}$$

where $m, l = 1, \ldots, M, n = \mathbf{1}, \ldots, \mathbf{N}$. In this sense, the matrix $\mathbf{V}^n$ is in fact a matrix of fuzzy preference relations satisfying the fuzzy reciprocal property (Sotirov and Krasteva, 1994).

Obviously, there will be $\mathbf{N}$ matrices of stochastic dominance degrees of the products. Each matrix $\mathbf{V}^n$ satisfies the following properties of fuzzy preference relations with respect to Kansei attribute X_n:

- when $V_{ml}^n = 1$, it indicates that product O_m is absolutely preferred to product O_l, i.e., product O_m absolutely dominates product O_l;
- when $0.5 < V_{ml}^n < 1$, it indicates that O_m is slightly preferred to O_l;
- when $V_{ml}^n = 0.5$, there is no preference (i.e., indifference) between O_m and O_l;
- when $0 < V_{ml}^n < 0.5$, it indicates that O_l is slightly preferred to O_m;
- when $V_{ml}^n = 0$, it indicates that O_l is absolutely preferred to O_m, i.e., product O_l absolutely dominates product O_m.

With the matrix $\mathbf{V}^n = [V_{ml}^n]_{M\times M}$ of fuzzy preference relations of different products on Kansei attribute X_n obtained, the stochastic dominance degree of product O_m on Kansei attribute X_n can be obtained as follows;

$$V_m^n = \frac{\sum_{l=1, l\neq m}^{M} V_{ml}^n}{M-1}, \tag{8}$$

where $m, l = 1, \ldots M$. Obviously, for each product O_m, a vector of stochastic dominance degrees $(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}})$ can be obtained.

3.3 Aggregation of individual stochastic dominance degrees

In the sequel, we shall perform the synthesization of the individual degrees into an overall one by taking the priority weights of selected Kansei attributes and the concept of fuzzy majority into consideration.

3.3.1 Priority weighted aggregation

In order to synthesize the individual stochastic dominance degrees into an overall representative one, one commonly used way is to apply the weighted average (WA) method expressed by the following value function

$$\begin{aligned} V_m &= \mathcal{F}_{\text{WA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}; P_{\mathbf{1}}, P_{\mathbf{2}}, \ldots, P_{\mathbf{N}}\right) \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} V_m^n \cdot P_n \end{aligned} \tag{9}$$

where $\sum_{n=\mathbf{1}}^{\mathbf{N}} P_n = 1$ and $m = 1, 2, \ldots, M$.

In Eq. (9), it is important for consumers to consider different importance degrees for the selected Kansei attributes as some Kansei attributes are more important than others. In this case, the consumers may associate different importance weights with different Kansei attributes. However, since the consumer-oriented Kansei evaluation focuses mainly on personalized recommendation in the era of e-commerce and a consumer may choose different Kansei attributes in a dynamic environment, it may be quite difficult and time-consuming for him to assign exact importance weights for the selected Kansei attributes.

In this work, we consider the situation in which the information regarding the importance of the selected Kansei attributes is captured by a prioritization of these attributes. Simply speaking, by saying Kansei attribute X_i has a higher priority than Kansei attribute X_j, we are meaning to indicate that we are not willing to tradeoff satisfaction to attribute X_i for a gain in attribute X_j until perhaps we attain some minimal level of satisfaction to X_i. The priority information may be directly specified by the consumers or defined according to the sequence of selecting Kansei attributes by the consumers. For example, if a consumer chooses Kansei attribute X_i first, and then he selects X_j; then we assume that attribute X_i has a higher priority than attribute X_j.

Assume the set of selected Kansei attributes $\mathbf{X} = \{X_\mathbf{1}, X_\mathbf{2}, \ldots, X_\mathbf{N}\}$ is partitioned into I distinct priority levels, $\mathbb{H} = \{H_1, H_2, \cdots, H_I\}$ such that $H_i = \{X_{i1}, X_{i2}, \cdots, X_{iN_i}\}$, where N_i is the attribute number in priority level H_i, and X_{ij} is the j-th attribute in priority level H_i, as shown in Table 2. We also assume a prioritization of these priority levels is $H_1 \succ H_2 \succ \cdots > H_I$. Then the stochastic dominance degree V_m^n of product O_m with respect to Kansei attribute X_n will be denoted as V_m^{ij}, where $i = 1, \ldots, I, j = 1, \ldots, N_i$.

Table 2 Priority hierarchy of the selected Kansei attributes

Priority level	Kansei attributes
H_1	$X_{11}, X_{12}, \cdots, X_{1N_1}$
H_2	$X_{21}, X_{22}, \cdots, X_{2N_2}$
$\vdots$	$\vdots$
H_I	$X_{I1}, X_{I2}, \cdots, X_{IN_I}$

With the priority hierarchy of the selected Kansei attributes shown in Table 2, a linguistically quantified statement for our consumer-oriented Kansei evaluation problem can be stated generally in form of the following statement:

"I like aesthetic products meeting my *important* Kansei preferences $\mathbf{KW} = \{\mathrm{kw}_\mathbf{1}^*, \mathrm{kw}_\mathbf{2}^*, \ldots, \mathrm{kw}_\mathbf{N}^*\}$ characterized by a priority hierarchy $\mathbb{H}$." ($\bigstar_1$)

Such a linguistic statement can be solved by an aggregation function. Fortunately enough, Yager (2004, 2008)'s prioritized aggregation operator is able to model the priority relationship among attributes, defined as follows.

Definition 2 Let $\left(V_m^\mathbf{1}, V_m^\mathbf{2}, \ldots, V_m^\mathbf{N}\right)$ be the vector of stochastic dominance degrees of product O_m with respect to the consumer's Kansei preferences $\{\mathrm{kw}_\mathbf{1}^*, \mathrm{kw}_\mathbf{2}^*, \ldots, \mathrm{kw}_\mathbf{N}^*\}$. Let $\mathbb{H} = \{H_1, H_2, \cdots, H_I\}$ with $H_i = \{X_{i1}, X_{i2}, \cdots, X_{iN_i}\}$ be the priority hierarchy of Kansei preferences. A prioritized average aggregation operator of dimension $\mathbf{N}$ is a mapping $\mathcal{F}_{\mathrm{PRIA}} : \mathbb{V}^\mathbf{N} \to \mathbb{V}$ such that

$$\begin{aligned} V_m &= \mathcal{F}_{\mathrm{PRIA}}\left(V_m^\mathbf{1}, V_m^\mathbf{2}, \ldots, V_m^\mathbf{N}; \mathbb{H}\right) \\ &= \sum_{i=1}^{I}\left(\sum_{j=1}^{N_i} V_m^{ij} \cdot P_{ij}\right), \end{aligned} \tag{10}$$

where P_{ij} is the priority weight of the jth attribute in category H_i and V_m^{ij} is the stochastic dominance degrees of product O_m with respect to the jth Kansei attribute in priority level H_i.

The priority weight for each Kansei attribute is then derived as follows.

– 1) For each priority category H_i, we calculate a satisfaction degree via $\mathrm{Sat}_i = \min_j[V_m^{ij}]$.

– 2) Associate with the attributes in category H_i a value U_i called their pre-weight as $U_i = \prod_{j=1}^{i} \mathrm{Sat}_{j-1}$, where $U_1 = \mathrm{Sat}_0 = 1$.
– 3) Since the attributes in the same category should have equivalent priority weight, we can obtain for each X_{ij} a priority weight P_{ij} such that

$$P_{ij} = \frac{U_i}{\sum_{i=1}^{I} N_i \cdot U_i}, j = 1, \ldots, N_i. \tag{11}$$

Substituting P_{ij} into Eq. (10), we can obtain the collective stochastic dominance degree of product O_m under the priority hierarchy $\mathbb{H}$.

Remark 1 Note that the satisfaction degree of each priority category depends on an aggregation function ranging from the "min" operator to the "max" operator (Yager, 2004, 2008; Yan et al, 2011). The consumer-oriented Kansei evaluation focuses mainly on personalized recommendation in the era of e-commerce (Huynh et al, 2010). It may be thus difficult and time-consuming for a consumer to specify exact parameter for the aggregation function toward each priority category. In addition, the priority weighted aggregation (Yager, 2004, 2008) assumes that people are not willing to tradeoff satisfaction to attributes with a higher priority for a gain in attributes with a lower priority until perhaps attaining some minimal level of satisfaction to attributes with a higher priority. Therefore, only the "min" operator is considered to obtain the satisfaction degree for each priority category.

3.3.2 OWA aggregation

In addition to the priority/importance weights of Kansei attributes, we also want to incorporate the concept of *majority*, which is a basic element underlying decision-making. The term "majority" indicates that a solution is satisfied by most of its attributes, since in practice it is quite difficult for the solution to be satisfied by all. The concept of "*fuzzy majority*" is used to make the strict concept of majority more vague so as to make it closer to its real human perception (Kacprzyk et al, 2008). A natural manifestation of such a "soft" majority is the so-called linguistic quantifiers as, e.g., *most*, *at least half*, *as many as possible*. With the linguistic quantifier LQ, a "fuzzy majority" quantified statement for our consumer-oriented Kansei evaluation problem can be stated generally by the following statement:

"I like aesthetic products meeting LQ (of) my Kansei preferences $\mathbf{KW} = \{\mathrm{kw}^*_{\mathbf{1}}, \ldots, \mathrm{kw}^*_{\mathbf{N}}\}$." ($\bigstar_2$)

which is equivalent to the one in Huynh et al (2010).

Such a linguistic statement can be solved by an aggregation function. Fortunately enough, Yager (1988) has proposed a special class of aggregation operators, called ordered weighted averaging (OWA for short) operator, which seems to provide an even better and general aggregation in the sense of being able to simply and uniformly model a large class of fuzzy linguistic quantifiers (Chang, 2016), defined as follows.

Definition 3 Let $\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right)$ be the vector of stochastic dominance degrees of product m with respect to the consumer's Kansei preferences, an OWA operator of dimension $\mathbf{N}$ is a mapping $\mathcal{F}_{\mathrm{OWA}} : \mathbb{V}^{\mathbf{N}} \to \mathbb{V}$ if $\mathcal{F}$ is associated with an OWA weighting vector $\mathbf{W} = (W_{\mathbf{1}}, W_{\mathbf{2}}, \ldots, W_{\mathbf{N}})$ such that: $W_n \in [0, 1]$, $\sum_{n=\mathbf{1}}^{\mathbf{N}} W_n = 1$, and

$$\begin{aligned} V_m &= \mathcal{F}_{\mathrm{OWA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right) \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} V_m^{\sigma(n)} \cdot W_n, \end{aligned} \tag{12}$$

where $\left(V_m^{\sigma(\mathbf{1})}, V_m^{\sigma(\mathbf{2})}, \ldots, V_m^{\sigma(\mathbf{N})}\right)$ is a permutation of $\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right)$ such that $V_m^{\sigma(n-1)} \geq V_m^{\sigma(n)}$ for all $n = \mathbf{2}, \ldots, \mathbf{N}$.

The fuzzy majority in terms of linguistic quantifiers can be represented by means of fuzzy sets (Zadeh, 1983), i.e., any relative linguistic quantifier LQ can be expressed as a fuzzy subset Q of the unit interval $[0,1]$, where $Q(x)$ indicates the degree to which x satisfies the concept conveyed by the term LQ. Yager (1996) has further defined a Regular Increasing Monotone (RIM) quantifier to represent the linguistic quantifier, defined as follows.

Definition 4 A fuzzy subset Q of the universe domain $[0,1]$ is called an RIM quantifier function if

$$Q(0) = 0, Q(1) = 1, \text{ and } Q(x) \geq Q(y) \text{ for } x \geq y.$$

With the quantifier function defined, Yager (1996) has proposed a method for obtaining the OWA weighting vector $\mathbf{W}$ via linguistic quantifiers, especially the RIM quantifiers, which can provide information aggregation procedures guided by verbally expressed concepts and a dimension independent description of the desired aggregation. By using the OWA operator and an RIM function Q, the overall stochastic dominance V_m is derived as

$$\begin{aligned} V_m &= \mathcal{F}_{\text{OWA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right) \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} \left[Q\left(\frac{n}{\mathbf{N}}\right) - Q\left(\frac{n-1}{\mathbf{N}}\right)\right] \cdot V_m^{\sigma(n)} \end{aligned} \tag{13}$$

where $m, l = 1, \ldots, M$, $\left(V_m^{\sigma(\mathbf{1})}, V_m^{\sigma(\mathbf{2})}, \ldots, V_m^{\sigma(\mathbf{N})}\right)$ is the permutation of $\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right)$ such that $V_m^{\sigma(n-1)} \geq V_m^{\sigma(n)}$ for all $n = \mathbf{2}, \ldots, \mathbf{N}$.

Note that the OWA operator provides a type of aggregation operator between the "AND" and the "OR" aggregations. Given a linguistic quantifier LQ, the so-called measure "orness" of OWA operator corresponds to

$$orness(Q) = \int_0^1 Q(x)dx \tag{14}$$

This measure of "orness" indicates to which degree the operator $\mathcal{F}_{\text{OWA}}$ behaves like an "OR" aggregation. Also, the measure of "andness" associated with $\mathcal{F}_{\text{OWA}}$ is defined as the complement of its "orness" such that $andness(\mathcal{F}_{\text{OWA}}) = 1 - orness(\mathcal{F}_{\text{OWA}})$. Table 3 provides typical examples of linguistic quantifiers associated with their membership functions and the orness values.

3.3.3 Priority weighted OWA aggregation

Essentially, we want to synthesize the individual degrees of different products into an overall representative one by taking both the weights of Kansei attributes in Eq. (10) and the concept of *fuzzy majority* in Eq. (13) into account. In this case, a linguistically quantified statement may be generally written as

"I like aesthetic products meeting LQ (of) my *important* Kansei preferences $\mathbf{KW} = \{\text{kw}_{\mathbf{1}}^*, \ldots, \text{kw}_{\mathbf{N}}^*\}$ with a priority hierarchy $\mathbb{H}$." ($\bigstar_3$)

Such a linguistically quantified statement can be, fortunately enough, dealt with by the weighted ordered weighted averaging (WOWA for short) operator (Torra, 1997), defined as follows.

Definition 5 Let $\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right)$ be the vector of stochastic dominance degrees of product O_m with respect to the consumer's Kansei preferences $\mathbf{KW}$. Let $\mathbb{H} = \{H_1, H_2, \cdots, H_I\}$ with $H_i = \{X_{i1}, X_{i2}, \cdots, X_{iN_i}\}$ be the priority hierarchy of Kansei preferences. Let the priority weights be re-denoted as $\mathbf{P}$ (priority weights of Kansei attributes) and $\mathbf{W}$ (OWA weights) be weighting vectors of dimension $\mathbf{N}$ such that:

Table 3 Linguistic quantifiers

ID	Linguistic quantifier LQ	Membership function Q	orness
TE	*there exists*	$Q(x)=\begin{cases}0 & \text{if } x=0\\ 1 & \text{if } x>0\end{cases}$	1
ALH	*at least half*	$Q(x)=\begin{cases}2x & \text{if } 0\le x\le 0.5\\ 1 & \text{if } 0.5<x\le 1\end{cases}$	0.8
I	*identity*	$Q(x)=x$	0.5
M	*most*	$Q(x)=\begin{cases}0 & \text{if } 0\le x\le 0.3\\ 2x-0.6 & \text{if } 0.3<x\le 0.8\\ 1 & \text{if } 0.8<x\le 1\end{cases}$	0.45
AMAP	*as many as possible*	$Q(x)=\begin{cases}0 & \text{if } 0\le x\le 0.5\\ 2x-1 & \text{if } 0.5<x\le 1\end{cases}$	0.2
FA	*for all*	$Q(x)=\begin{cases}1 & \text{if } x=1\\ 0 & \text{if } x\ne 1\end{cases}$	0

– $\mathbf{P}=(P_{\mathbf{1}},P_{\mathbf{2}},\ldots,P_{\mathbf{N}}), P_n\in[0,1]$ and $\sum_{n=\mathbf{1}}^{\mathbf{N}} P_n=1$.
– $\mathbf{W}=(W_{\mathbf{1}},W_{\mathbf{2}},\ldots,W_{\mathbf{N}}), W_n\in[0,1]$ and $\sum_{n=\mathbf{1}}^{\mathbf{N}} W_n=1$.

In this case, a mapping $\mathcal{F}_{\text{WOWA}}:\mathbb{V}^{\mathbf{N}}\to\mathbb{V}$ is a WOWA operator of dimension $\mathbf{N}$ if

$$\begin{aligned} V_m &= \mathcal{F}_{\text{WOWA}}\left(V_m^{\mathbf{1}},V_m^{\mathbf{2}},\ldots,V_m^{\mathbf{N}};P_{\mathbf{1}},P_{\mathbf{2}},\ldots,P_{\mathbf{N}}\right)\\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}}\omega_n\cdot V_m^{\sigma(n)} \end{aligned} \tag{15}$$

where $\left(V_m^{\sigma(\mathbf{1})},V_m^{\sigma(\mathbf{2})},\ldots,V_m^{\sigma(\mathbf{N})}\right)$ is the permutation of $\left(V_m^{\mathbf{1}},V_m^{\mathbf{2}},\ldots,V_m^{\mathbf{N}}\right)$ such that $V_m^{\sigma(n-1)}\ge V_m^{\sigma(n)}$ for all $n=\mathbf{2},\ldots,\mathbf{N}$, and the weight ω_n is defined as

$$\omega_n = W^*\left(\sum_{l\le n}P_{\sigma(l)}\right)-W^*\left(\sum_{l<n}P_{\sigma(l)}\right) \tag{16}$$

with W^* a monotone increasing function that interpolates the points $\left(\frac{n}{\mathbf{N}},\sum_{l\le n}P_{\sigma(n)}\right)$ together with the point $(0,0)$. The value $P_{\sigma(l)}(l=\mathbf{1},\ldots,\mathbf{N})$ means the permutation according to $\left(V_m^{\sigma(\mathbf{1})},V_m^{\sigma(\mathbf{2})},\ldots,V_m^{\sigma(\mathbf{N})}\right)$.

When W^* is replaced with a RIM linguistic quantifier introduced in Definition 4, then

$$\omega_n = Q\left(\sum_{l\le n}P_{\sigma(l)}\right)-Q\left(\sum_{l<n}P_{\sigma(l)}\right), n=\mathbf{1},\ldots,\mathbf{N}, \tag{17}$$

which indicates that the WOWA operator becomes the importance weighted quantifier guided aggregation method (Yager, 1996). Using the WOWA operator and RIM linguistic quantifier, the overall stochastic dominance degree of product O_m is derived by

$$\begin{aligned} V_m &= \mathcal{F}_{\text{WOWA}}\left(V_m^{\mathbf{1}},V_m^{\mathbf{2}},\ldots,V_m^{\mathbf{N}};P_{\mathbf{1}},P_{\mathbf{2}},\ldots,P_{\mathbf{N}}\right)\\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}}\left[Q\left(\sum_{l\le n}P_{\sigma(l)}\right)-Q\left(\sum_{l<n}P_{\sigma(l)}\right)\right]\cdot V_m^{\sigma(n)} \end{aligned} \tag{18}$$

where $\left(V_m^{\sigma(\mathbf{1})},V_m^{\sigma(\mathbf{2})},\ldots,V_m^{\sigma(\mathbf{N})}\right)$ is the permutation of $\left(V_m^{\mathbf{1}},V_m^{\mathbf{2}},\ldots,V_m^{\mathbf{N}}\right)$ such that $V_m^{\sigma(n-1)}\ge V_m^{\sigma(n)}$ for all $n=\mathbf{2},\ldots,\mathbf{N}$.

Interestingly enough, our priority weighted OWA aggregation function in Eq. (18) generalizes the WA method in Eq. (10) and OWA method in Eq. (13) as follows.

– If $Q(x) = x$, then

$$\begin{aligned} V_m &= \mathcal{F}_{\text{WOWA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}; P_{\mathbf{1}}, P_{\mathbf{2}}, \ldots, P_{\mathbf{N}}\right) \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} \left[Q\left(\sum_{l \leq n} P_{\sigma(l)}\right) - Q\left(\sum_{l < n} P_{\sigma(l)}\right)\right] \cdot V_m^{\sigma(n)} \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} V_m^n \cdot P_n \\ &\triangleq \mathcal{F}_{\text{WA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}; P_{\mathbf{1}}, P_{\mathbf{2}}, \ldots, P_{\mathbf{N}}\right) \end{aligned} \tag{19}$$

which indicates when the linguistic quantifier "identity" is used, our priority weighted OWA aggregation function reduces to the priority weighted aggregation function.

– If all Kansei attributes are in the same priority level, i.e., all Kansei attribute are equivalently prioritized, then $P_n = \frac{1}{\mathbf{N}}, n = \mathbf{1}, \mathbf{2} \ldots, \mathbf{N}$, then our priority weighted OWA aggregation transforms to the OWA aggregation function such that

$$\begin{aligned} V_m &= \mathcal{F}_{\text{WOWA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}; P_{\mathbf{1}}, P_{\mathbf{2}}, \ldots, P_{\mathbf{N}}\right) \\ &= \sum_{n=\mathbf{1}}^{\mathbf{N}} \left[Q\left(\sum_{l \leq n} P_{\sigma(l)}\right) - Q\left(\sum_{l < n} P_{\sigma(l)}\right)\right] \cdot V_m^{\sigma(n)} \\ &= \sum_{n=1}^{\mathbf{N}} \left[Q\left(\frac{n}{\mathbf{N}}\right) - Q\left(\frac{n-1}{\mathbf{N}}\right)\right] \cdot V_m^{\sigma(n)} \\ &\triangleq \mathcal{F}_{\text{OWA}}\left(V_m^{\mathbf{1}}, V_m^{\mathbf{2}}, \ldots, V_m^{\mathbf{N}}\right) \end{aligned} \tag{20}$$

4 A comparative application case study

In this section, the proposed approach will be applied to a case study introduced in Yan et al (2012). In Japan, there are a large number of traditional craft products which are closely related to Japanese traditional culture. As explained on the Web site of The Association for the Promotion of Traditional Craft Industries, each of traditional craft products is"unique fostered through regional differences and loving dedication, and provides a continual wealth of pleasure." The artistic and aesthetic aspects play a crucial role in perception of traditional crafts. In this paper, our main concern is about the consumer-oriented Kansei evaluation of Japanese traditional products for the personalized recommendation problem.

4.1 Experiment data: Japanese hand-painted Kutani cups

In this paper, a particular emphasis is laid on the hand-painted Kutani cups in Ishikawa Prefecture, Japan. The Kutani cup is a traditional craft product with a long history of over 400 years. Within the framework of a research project supported by the local government, a total of 35 representative hand-painted Kutani cups were first selected for Kansei experiment, as shown in Fig. 1. Before gathering Kansei assessment data of these Kutani cups for evaluation, preliminary research was carried out to select Kansei attributes by consultation with local manufacturers and shops. Finally, 26 bipolar pairs of Kansei words were selected and refined through a brainstorming process by consulting with local manufacturers and selling shops in Ishikawa, Japan. The 26 refined bipolar pairs of Kansei words were first used in Japanese and then approximately translated into English, as shown in Table 4.

Fig. 1 The 35 hand-painted Kutani cups to be evaluated

Table 4 26 Kansei attributes with bipolar pairs of Kansei words

X_n	$< \mathrm{kw}_n^-,\ \mathrm{kw}_n^+ >$	X_n	$< \mathrm{kw}_n^-,\ \mathrm{kw}_n^+ >$
X_1	<conventional,unconventional>	X_{14}	<delicate,large-hearted>
X_2	<simple,compound>	X_{15}	<luxurious,frugal>
X_3	<solemn,funny>	X_{16}	<gentle,pithy>
X_4	<formal,casual>	X_{17}	<bright,dark>
X_5	<serene,forceful>	X_{18}	<reserved,imperious>
X_6	<still,moving>	X_{19}	<free,regular>
X_7	<pretty,austere>	X_{20}	<level,indented>
X_8	<friendly,unfriendly>	X_{21}	<lustrous,matte>
X_9	<soft,hard>	X_{22}	<transpicuous,dim>
X_{10}	<blase,attractive>	X_{23}	<warm,cool>
X_{11}	<flowery,quiet>	X_{24}	<moist,arid>
X_{12}	<happy,normal>	X_{25}	<colorful,sober>
X_{13}	<elegant,loose>	X_{26}	<plain,gaudy-loud>

A 7-point qualitative scale was then used to put a value for each Kutani cup with respect to 26 Kansei attributes such that $\mathbb{V} = \{1, 2, 3, 4, 5, 6, 7\}$. Finally, a total of 60 people from Ishikawa Prefecture including relevant researchers of Kansei engineering, elder residents in Ishikawa, and certified masters of traditional crafts, were chosen as subjects. The 60 subjects were then asked to provide their Kansei assessments simultaneously for the 35 Kutani cups on the 26 Kansei attributes in several sessions. The Kansei assessment data are then used to generate Kansei profiles for the Kutani cups via Eq.(2). For example, the uncertain Kansei profile of Kutani cup O_1 on Kansei attribute X_2 is derived as a probability distribution over $\mathbb{V} = \{1, 2, 3, 4, 5, 6, 7\}$ such that $p_{12} = [0.0, 0.05, 0.05, 0.0, 0.1, 0.3833, 0.4167]$. Such Kansei profiles are considered as uncertain feeling assessments of Kutani cups, serving as knowledge for the following consumer-oriented Kansei evaluation problem.

Table 5 Preferred 5 Kansei attributes with Kansei request

Attribute	Bipolar Kansei words	Kansei preference	Priority
X_2	<simple,compound>	*compound*	2
X_9	<soft,hard>	*soft*	2
X_{11}	<flowery,quiet>	*flowery*	1
X_{17}	<bright,dark>	*dark*	1
X_{23}	<warm,cool>	*warm*	2
Fuzzy majority	As many as possible		

4.2 Consumer-oriented Kansei evaluation

To illustrate how our approach proposed in the preceding section works, let us now consider the following example. Assume a consumer prefers the following 5 Kansei attributes $\{X_2, X_9, X_{11}, X_{17}, X_{23}\}$ and specifies his Kansei request as

$$\begin{cases} \mathbf{KW} & = \{\mathrm{kw}_2^+, \mathrm{kw}_9^-, \mathrm{kw}_{11}^-, \mathrm{kw}_{17}^+, \mathrm{kw}_{23}^-\} \\ \mathbb{H} & = \{2, 2, 1, 1, 2\} \\ LQ & = \text{As many as possible} \end{cases} \tag{21}$$

as explained in Table 5, here $\mathbf{1} = 2, \mathbf{2} = 9, \mathbf{3} = 11, \mathbf{4} = 17, \mathbf{5} = 23$. Verbally, the consumer prefers Kutani cups that ***as many as possible*** firstly meeting his preferences of ***flowery*** and ***dark***, and secondly meeting his preferences of ***compound***, ***soft***, and ***warm***.

According to Table 5, the consumer has *left* Kansei preferences on Kansei attributes X_9, X_{11}, X_{23}, and right Kansei preferences toward Kansei attributes X_2, X_{17}, respectively. We then determine preference order relations on $\mathbb{V} = \{1, 2, 3, 4, 5, 6, 7\}$ for $X_2, X_9, X_{11}, X_{17}, X_{23}$ via Eq. (4) as follows:

$$\begin{cases} \mathrm{kw}_2^+ : V_1 \preceq V_2 \preceq V_3 \preceq V_4 \preceq V_5 \preceq V_6 \preceq V_7 \\ \mathrm{kw}_9^- : V_1 \succeq V_2 \succeq V_3 \succeq V_4 \succeq V_5 \succeq V_6 \succeq V_7 \\ \mathrm{kw}_{11}^- : V_1 \succeq V_2 \succeq V_3 \succeq V_4 \succeq V_5 \succeq V_6 \succeq V_7 \\ \mathrm{kw}_{17}^+ : V_1 \preceq V_2 \preceq V_3 \preceq V_4 \preceq V_5 \preceq V_6 \preceq V_7 \\ \mathrm{kw}_{23}^- : V_1 \succeq V_2 \succeq V_3 \succeq V_4 \succeq V_5 \succeq V_6 \succeq V_7 \end{cases}$$

We are now ready to derive 5 matrices $\mathbf{V}^n = [V_{ml}^n]_{35\times 35}$ $(n = 2, 9, 11, 17, 23)$ of stochastic dominance degrees of the 35 products via Eq. (5), respectively. For example, the stochastic dominance degree of product O_1 over O_2 on Kansei attribute X_2 is derived as $V_{12}^2 = 0.66$, which indicates that O_1 is slightly preferred to O_2 regarding the consumer's Kansei preference *compound*. According to Eq. (8), the stochastic dominance degrees of each Kutani cup over all the others with respect to the five selected Kansei attributes can be obtained.

We now consider the priority weighted OWA aggregation. To do so, we first consider priority information $\{2, 2, 1, 1, 2\}$, which indicates that $H_1 = \{X_{11}, X_{17}\}$ and $H_2 = \{X_2, X_9, X_{23}\}$. Taking Kutani cup O_{18} as an example, we know

$$\left(V_{18}^2 = 0.802, V_{18}^9 = 0.2317, V_{18}^{11} = 0.8228, V_{18}^{17} = 0.4344, V_{18}^{23} = 0.3525\right).$$

Then, we first calculate

$$\mathrm{Sat}_1 = \min\left[V_{18}^{11}, V_{18}^{17}\right] = 0.4344$$
$$\mathrm{Sat}_2 = \min\left[V_{18}^{2}, V_{18}^{9}, V_{18}^{23}\right] = 0.2317$$

Table 6 Top 3 Kutani cups under priority information $\{2, 2, 1, 1, 2\}$ and different linguistic quantifiers.

Linguistic quantifier LQ	Top 3 Kutani cups
There exist (TE)	$O_{23} \succ O_{18} \succ O_5$
At least half (ALH)	$O_{18} \succ O_{23} \succ O_5$
Identity (I)	$O_{18} \succ O_{33} \succ O_{31}$
Most (M)	$O_{31} \succ O_{33} \succ O_{32}$
As many as possible (AMAP)	$O_{35} \succ O_3 \succ O_{14}$
For all (FA)	$O_{35} \succ O_3 \succ O_{26}$

Using this, we can get

$$U_1 = 1$$
$$U_2 = \mathrm{Sat}_1 \cdot U_1 = 0.4344$$

Finally, the priority weights with respect to Kansei attributes $\{X_2, X_9, X_{11}, X_{17}, X_{23}\}$ under product O_{18} are derived via Eq. (11) as follows

$$\mathbf{P} = (0.1315, 0.1315, 0.30275, 0.30275, 0.1315)$$

Moreover, according to the fuzzy set-based semantics of linguistic quantifiers listed in Table 3, by substituting stochastic dominance degrees and the priority weighting vector $\mathbf{P}$ of Kutani cup O_{18} into the priority weighted OWA aggregation in Eq. (18), we have

$$V_{18} = \mathcal{F}_{\mathrm{WOWA}}\,(0.802, 0.2317, 0.8228, 0.4344, 0.3525; 0.1315, 0.1315, 0.30275, 0.30275, 0.1315)$$

Finally, a ranking of Kutani cups can be easily obtained. Table 6 shows the top three Kutani cups that would best meet the Kansei preferences of "***compound***, ***soft***, ***flowery***, ***dark***, and ***warm***" with different linguistic quantifiers and the priority information $\{2, 2, 1, 1, 2\}$.

4.3 Analysis of the results obtained

For the sake of facilitating the analysis of the results obtained, all the recommended Kutani cups with their uncertain Kansei profiles on the 5 selected Kansei attributes $\{X_2, X_9, X_{11} X_{17}, X_{23}\}$ are graphically illustrated in Fig 2. Accordingly, the stochastic dominance degrees, the priority weights, and their aggregate values corresponding to the different quantifiers, are shown in Table 7, respectively.

We first consider the priority hierarchy of the 5 selected Kansei attributes $\{X_2, X_9, X_{11}, X_{17}, X_{23}\}$. According to Table 5, the consumer has the first priority level toward Kansei attributes X_{11}, X_{17} and the second priority level toward Kansei attributes X_2, X_9, X_{23}, respectively. From Table 7, it is easily seen that for each Kutani cup, the priority weights of Kansei attributes X_{11}, X_{17} are higher than those of Kansei attributes X_2, X_9, X_{23}; the attributes in the same priority level have the same priority weight. Such phenomena reflect the fact that the weights of attributes in lower priority levels depend on the stochastic dominance degrees of attributes in higher priority levels. In other words, for each Kutani cup O_m, the value $\min[V_m^{11}, V_m^{17}]$ determines the priority weights of attributes in the second priority level $\{X_2, X_9, X_{23}\}$, i.e., the greater the value $\min[V_m^{11}, V_m^{17}]$ is, the greater the priority weights of Kansei attributes $\{X_2, X_9, X_{23}\}$ are. Seen from Table 7, it is obvious that $\min[V_5^{11}, V_5^{17}]$ has the smallest value as 0.1571; therefore Kansei attributes $\{X_2, X_9, X_{23}\}$ of Kutani cup O_5 have the smallest priority weights as 0.0636.

Fig. 2 Kansei profiles of all the top 3 recommended Kutani cups.

Now let us consider the linguistic quantifiers used in our consumer-oriented Kansei evaluation. If the linguistic quantifier "there exists" is used, the priority information has no effect on the aggregation results. As shown in Table 6, the ranking of the top 3 Kutani cups is $O_{23} \succ O_{18} \succ O_5$, and the three Kutani cups

Table 7 Stochastic dominance degrees and priority weights of the selected 5 Kansei attributes.

Cups	Stochastic dominance degrees					Priority weights					Aggregated results					
	X_2	X_9	X_{11}	X_{17}	X_{23}	X_2	X_9	X_{11}	X_{17}	X_{23}	TE	ALH	I	M	AMAP	FA
O_3	0.6121	0.4363	0.5743	0.4491	0.6223	0.1342	0.1342	0.2987	0.2987	0.1342	0.6223	0.5974	0.5299	0.5160	**0.4625**	**0.4363**
O_5	0.2257	0.4317	0.1571	0.8133	0.3517	0.0636	0.0636	0.4046	0.4046	0.0636	**0.8133**	**0.7355**	0.4568	0.3628	0.1782	0.1571
O_{14}	0.4775	0.6378	0.5986	0.4078	0.6489	0.1265	0.1265	0.31025	0.31025	0.1265	0.6489	0.6212	0.5354	0.5259	**0.4495**	0.4078
O_{18}	0.8020	0.2317	0.8228	0.4344	0.3525	0.1315	0.1315	0.30275	0.30275	0.1315	**0.8228**	**0.7663**	**0.5629**	0.5229	0.3596	0.2317
O_{23}	0.4801	0.2845	0.1736	0.8459	0.3920	0.0689	0.0689	0.39675	0.39675	0.0689	**0.8459**	**0.7642**	0.4841	0.3912	0.2039	0.1736
O_{26}	0.4444	0.4640	0.5607	0.4185	0.4826	0.1285	0.1285	0.30725	0.30725	0.1285	0.5607	0.5282	0.4796	0.4554	0.4310	**0.4185**
O_{31}	0.6950	0.5714	0.7053	0.3374	0.6096	0.1120	0.1120	0.3320	0.3320	0.1120	0.7053	0.6923	**0.5563**	**0.5544**	0.4203	0.3374
O_{32}	0.6881	0.3488	0.5518	0.6238	0.3755	0.1510	0.1510	0.27365	0.27365	0.1510	0.6881	0.6323	0.5348	**0.5338**	0.4373	0.3488
O_{33}	0.7145	0.5104	0.7354	0.3198	0.6319	0.1081	0.1081	0.33785	0.33785	0.1081	0.7354	0.7197	**0.5572**	**0.5452**	0.3947	0.3198
O_{35}	0.6908	0.5322	0.6329	0.4693	0.4579	0.1377	0.1377	0.29345	0.29345	0.1377	0.6908	0.6350	0.5549	0.5295	**0.4748**	**0.4579**

have strong dominance degrees on Kansei attributes X_{11} and X_{17}. In the case where quantifier "at least half" is used, we still obtain the same top 3 Kutani cups but with a different ranking. The main reasons are twofold. (1) Our priority weighted OWA function corresponding to the linguistic quantifier "there exists" is a pure "OR" operator with the orness value as 1; whereas the one corresponding to quantifier "at least half" still behaves like an "OR"-type aggregation as well with the orness value as 0.8. (2) The priority information plays a role in the priority weighted OWA function corresponding to quantifier "at least half".

Looking at the case of using linguistic quantifier "identity", the priority weighted OWA aggregation reduces to the priority weighted aggregation. As shown in Table 6, the ranking of the top 3 Kutani cups is $O_{18} \succ O_{33} \succ O_{31}$. Seen from Table 7, the Kutani cups O_{18}, O_{33}, O_{31} have greater priority weights and higher stochastic dominance degrees over the others with respect to Kansei attributes X_{11}, X_{17}. Therefore, O_{18}, O_{33}, O_{31} are recommended as the top 3 Kutani cups to the consumer. In the case where quantifier "most" is used, it is found that O_{31} is ranked the first one, followed by O_{33}, O_{32}. The main reason for such a difference may come from the fact that the priority weighted OWA function corresponding to the quantifier "identity" has the orness value as 0.5, whereas the one corresponding to the quantifier "most" has the orness value as 0.45.

Finally, let us consider the results with the quantifiers "for all" and "as many as possible". If the quantifier "for all" is used, the priority information has no effect on the aggregation results. As shown in Table 6, the ranking of the top 3 Kutani cups is $O_{35} \succ O_3 \succ O_{26}$. In the case where quantifier "as many as possible" is used, the top 3 Kutani cups are ranked as $O_{35} \succ O_3 \succ O_{14}$, which is different from the one corresponding to quantifier "for all". The main reasons are twofold. (1) Our priority weighted OWA function corresponding to quantifier "for all" is a pure "AND" operator with the andness value as 1; whereas the one corresponding to quantifier "as many as possible" still behaves like an "AND"-type aggregation with the andness value as 0.8. (2) The priority information plays a role in the priority weighted OWA function corresponding to quantifier "as many as possible".

4.4 A comparative study

In order to clarify the efficiency of our proposed approach, we will conduct in this section a comparative study with the target-based approach to consumer-oriented Kansei evaluation (Huynh et al, 2010), which is summarized as follows.

- Step 1): With the Kansei database obtained, the uncertain Kansei profiles of the Kutani cups on Kansei attributes are derived via Eq. (2), as $p_{mn} = [p_{mn}(V_1), p_{mn}(V_2), \ldots, p_{mn}(V_G)]$, where $g = 1, \ldots, G, m = 1, \ldots, M, n = 1, \ldots, N$.
- Step 2): Given a consumer's Kansei words preferences $\mathbf{KW} = \{\text{kw}^*_{\mathbf{1}}, \text{kw}^*_{\mathbf{2}}, \ldots, \text{kw}^*_{\mathbf{N}}\}$, preference order relations $\succeq_n$ are obtained according to Eq. (4).

  Due to the vagueness inherent in the consumer's Kansei preferences in terms of Kansei words, fuzzy targets are defined for the Kansei preferences such that $(T_{\mathbf{1}}, \ldots, T_{\mathbf{N}})$, expressed as follows:

$$\pi_{T_n}(V_g) = \begin{cases} \left(\frac{g-1}{G-1}\right)^{\lambda} & \text{if } \text{kw}^*_n = \text{kw}^-_n \\ \left(\frac{G-g}{G-1}\right)^{\lambda} & \text{if } \text{kw}^*_n = \text{kw}^+_n \end{cases} \tag{22}$$

  where $n = \mathbf{1}, \ldots, \mathbf{N}$ and $\lambda \geq 0$ expresses the degree of intensity of the consumer's feelings about the target, which is in fact represented as a possibility variable whose possibility distribution is denoted by π.

  By making use of the possibility-probability conversion method (Yager, 2002), the possibility distribution of Kansei target T_n can be transformed into an associated probability distribution as

$$p_{T_n}(V_g) = \frac{\pi_{T_n}(V_g)}{\sum_{g=1}^{G} \pi_{T_n}(V_g)}, g = 1, \ldots, G, \tag{23}$$

  where $n = \mathbf{1}, \ldots, \mathbf{N}$.
- Step 3): By accepting the assumption that Kansei target T_n is stochastically independent of Kansei performance on X_n of any product O_m, the probability of product O_m meeting the Kansei target T_n in terms of the preference order $\succeq_n$ can be calculated as

$$\begin{aligned} \Pr_{mn} &\triangleq \Pr(p_{mn} \succeq_n p_{T_n}) \\ &= \sum_{g=1}^{G} p_{mn}(V_g) \cdot \Pr(V_g \succeq_n p_{T_n}) \end{aligned} \tag{24}$$

  where $n = \mathbf{1}, \ldots, \mathbf{N}$.
- Step 4): The OWA aggregation operator is used to take the linguistic quantifier LQ into account such that

$$V_m = \mathcal{F}_{\text{OWA}}\left(\Pr_{m\mathbf{1}}, \ldots, \Pr_{m\mathbf{N}}\right) \tag{25}$$

  The aggregated values V_m $(m = 1, \ldots, M)$ are used to rank the products and provide the consumer recommendations.

  It is obvious that our proposed approach differs from the target-based approach from two aspects.

- **Evaluation function**. The target-based approach firstly quantifies a consumer's Kansei preferences in terms of fuzzy targets; secondly it transforms the fuzzy targets into probabilistic targets and then computes the probabilities of meeting the consumer's Kansei targets. Instead of quantifying the Kansei preferences in terms of fuzzy targets, our approach uses the concept of stochastic dominance to build the evaluation function solely based on the order relations of Kansei preferences.
- **Aggregation function**. The target-based approach performs aggregation by the OWA operator, in which the concept of fuzzy majority can be considered. Our aggregation function incorporates both the concept of fuzzy majority and the weights of Kansei attributes, especially the priority hierarchy of Kansei attributes. In this sense, our approach generalizes the aggregation function in Huynh et al (2010).

Table 8 Top 3 Kutani cups using our approach and the target-based approach.

LQ	Top 3 Kutani cups			
	Our approach	Target-based approach		
		Kansei targets in Eq. (26)	Kansei targets in Eq. (27)	Kansei targets in Eq. (28)
TE	$O_{23} \succ O_{18} \succ O_5$	$O_{18} \succ O_{13} \succ O_1$	$O_{18} \succ O_{13} \succ O_1$	$O_{18} \succ O_{33} \succ O_{12}$
ALH	$O_{18} \succ O_{13} \succ O_{33}$	$O_{18} \succ O_{33} \succ O_{13}$	$O_{18} \succ O_{33} \succ O_{13}$	$O_{18} \succ O_{20} \succ O_{34}$
I	$O_{31} \succ O_{33} \succ O_{24}$	$O_{33} \succ O_{24} \succ O_{31}$	$O_{33} \succ O_{31} \succ O_{24}$	$O_{18} \succ O_{33} \succ O_{24}$
M	$O_{31} \succ O_{24} \succ O_{33}$	$O_{24} \succ O_{31} \succ O_{33}$	$O_{31} \succ O_{33} \succ O_{24}$	$O_{24} \succ O_6 \succ O_{10}$
AMAP	$O_{31} \succ O_{35} \succ O_{14}$	$O_{31} \succ O_{24} \succ O_{35}$	$O_{31} \succ O_{35} \succ O_{33}$	$O_{24} \succ O_{35} \succ O_{14}$
FA	$O_{35} \succ O_3 \succ O_{26}$	$O_{32} \succ O_{17} \succ O_{28}$	$O_{32} \succ O_{28} \succ O_{17}$	$O_{35} \succ O_3 \succ O_{22}$

To compare our approach with evaluation function of the target-based approach, let us return to the consumer's preferences as considered before with $\mathbf{KW} = \{\mathbf{compound}, \mathbf{soft}, \mathbf{flowery}, \mathbf{dark}, \mathbf{warm}\}$. Here, we assume that all the selected Kansei attributes are equivalently prioritized, i.e., the priority information is $\{1, 1, 1, 1, 1\}$. With the preference orders on the selected Kansei attributes $\{X_2, X_9, X_{11}, X_{17}, X_{23}\}$ obtained via Eq. (4), a set of fuzzy targets $(T_2, T_9, T_{11}, T_{17}, T_{23})$ are defined for the Kansei preferences **KW** as

$$\pi_{T_n}(V_g) = \begin{cases} (g-1)/6, & \text{if } n = 2, 17 \\ (7-g)/6, & \text{if } n = 9, 11, 23 \end{cases} \tag{26}$$

where $g = 1, \ldots, 7$. Then by means of possibility-probability transformation method and the computation method in Eq. (24), the probabilities of meeting the Kansei targets can be obtained. Finally, the results of the top 3 recommended Kutani cups with different linguistic quantifiers can be obtained, as shown in Table 8, indexed by Kansei targets in Eq. (26).

As we have seen from Table 8, the results yielded by the target-based approach is somewhat similar with but different from those obtained by our proposed approach. When the linguistic quantifier "at least half", "identity", or "most" is specified by a consumer, the recommended top 3 Kutani cups by our approach are the same those by the target-based approach. For example, corresponding to "at least half", our approach yields the ranking as $O_{18} \succ O_{13} \succ O_{33}$; whereas, the target-based approach derives a ranking as $O_{18} \succ O_{33} \succ O_{13}$. Corresponding to quantifier "as many as possible", two common Kutani cups $\{O_{31}, O_{35}\}$ are recommended by our approach and the target-based approach; corresponding to quantifier "there exists", one common Kutani cup O_{18} is recommended by our approach and the target-based approach; if quantifier "for all" is used, no common Kutani cup is recommended by our approach and the target-based approach.

The main reason for such phenomena may come from the quantitative definition of fuzzy targets. Theoretically, the consumer's Kansei targets may also be defined as

$$\pi_{T_n}(V_g) = 1, n = 2, 9, 11, 17, 23 \tag{27}$$

or

$$\begin{aligned} \text{For } n = 2, 17 \quad & \pi_{T_n}(V_g) = (g-4)/3, \text{ if } g \geq 4; 0, \text{ otherwise.} \\ \text{For } n = 9, 11, 23 \quad & \pi_{T_n}(V_g) = (4-g)/3, \text{ if } g \leq 4; 0, \text{ otherwise.} \end{aligned} \tag{28}$$

Substituting these two types of fuzzy targets into the target-based approach, the final top 3 Kutani cups can be obtained, as shown in Table 8, indexed by Kansei targets in Eq. (27) and Eq. (28), respectively. It is seen that different results have been yielded corresponding to these two types of targets.

In essence, the target-based approach needs a procedure of quantitatively defining the possibility distributions of the consumer's targets based on the preference orders derived from the consumer's Kansei preferences. The definition in Eq. (22) is assumed for the sake of simplicity. However, the consumer may not be intelligent enough to define their fuzzy targets. Moreover, the consumer's Kansei preference is in fact a qualitative concepts, the quantification of which in terms of fuzzy sets is in fact the process of transforming the ordinal information into a cardinal scale that represents an “arbitrary passage”. Even if the quantification is rational, the semantics of fuzzy targets is often defined subjectively and context dependently, which have sensitively influenced the final recommendation results, as shown in Table 8. In addition, the target-based approach needs to transform a possibility distribution into a probability distribution. The normalization based transformation method is used in the target-based approach. However, there is another method called the least prejudiced probability. In fact, in the literature of fuzzy target-based decision-making, the results by those two methods are quite different, see Huynh et al (2008). Quite differently, our approach performs pairwise comparisons of the Kutani cups based on the preference orders derived from the consumer's Kansei preferences, which is solely based on order-based semantics of Kansei preferences and thus can reduce the cognitive burden of quantifying consumer's Kansei preferences, which gives a practical convenience and is easy of use in the process of consumer-oriented Kansei evaluation.

5 Concluding remarks

It has become more and more important and quite difficult for consumers to choose their preferred aesthetic products, since decisions on which product(s) to purchase or use are heavily influenced by personal aesthetic feelings/characteristics. Taking Kansei as one aspect of quality of products, consumer-oriented Kansei evaluation focuses on evaluation of existing commercial products based on consumers' Kansei preferences. This paper proposed a stochastic dominance based approach to consumer-oriented Kansei evaluation with multiple priorities. To do so, uncertain Kansei profiles in terms of probability distributions over qualitative scale were first derived. Secondly, the preference orders on Kansei scale were defined according to a consumer's Kansei preferences toward the selected Kansei attributes. Then, the concept of stochastic dominance was utilized to build an evaluation function, which derived a fuzzy preference matrix of all the products for each selected Kansei attribute. Finally, the importance weights of Kansei attributes, captured by a prioritization of Kansei attributes, together with the concept of fuzzy majority, were incorporated into an aggregation function to obtain the global dominance degrees for the products. An application to the hand-painted Kutani cups in Ishikawa, Japan, was conducted to illustrate the effectiveness and efficiency of the proposed approach. It is shown that, on one hand the proposed approach can reduce the cognitive burden of quantifying Kansei preferences, which is easy of use in practice; on the other hand, it incorporates both the fuzzy majority and priority information of Kansei attributes to help the consumers find their preferred products.

Essentially, our proposed approach aims at providing highly individualized recommendations to consumers based on their Kansei preferences, which are expressed in terms of Kansei words, priority information, and linguistic quantifiers. In fact, it simulates an artificial salesperson (recommender system) to recommend options based on consumers' Kansei preferences. Due to the high vagueness of the aesthetics, the consumer may not find their preferred aesthetic products at one time. Moreover, a typical consumer has many constraints and preferences that are not stated up front, i.e., the consumer becomes aware of these latent preferences only when proposed solutions violate them. For a consumer to finally reach his ideal aesthetic product, a number of such evaluation cycles are often required. Indeed, consumers are likely to construct their preferences in a context-dependent and adaptive fashion during the decision process (Tversky and Simonson, 1993). Critiquing-based recommender systems (Chen and

Pu, 2012) elicit consumers' feedback, called critiques, which they made on the recommended products. This conversational style of interaction is in contrast to the standard model where consumers receive recommendations in a single interaction. Therefore, one potential extension is to explore the use of our approach to critiquing-based recommender system in greater detail.

Appendix: Deriving stochastic dominance degrees from uncertain profiles

In this section, we will introduce an approach to derive stochastic dominance degrees from uncertain profiles, which is based on our previous work (Yan et al, 2013b).

Let Z_1 and Z_2 be two independent discrete random variables with respective probability distributions p_1 and p_2 defined over a finite set of qualitative scale $\mathbb{V} = \{V_1, V_2, \ldots, V_G\}$ with $V_1 < V_2 < \cdots < V_G$, where

$$\sum_{z\in\mathbb{V}} p_1(z) = 1 \text{ and } \sum_{z\in\mathbb{V}} p_2(z) = 1. \tag{29}$$

The probability distribution over the qualitative scale is referred to as *uncertain profile* in this paper.

Let z_1 and z_2 be possible outcomes of Z_1 and Z_2, respectively. Let $\Pr(z_1 \geq z_2)$, $\Pr(z_1 = z_2)$, and $\Pr(z_1 \leq z_2)$ denote the probabilities of $z_1 \geq z_2$, $z_1 = z_2$, and $z_1 \leq z_2$, respectively. Since the two random variables Z_1 and Z_2 are stochastically independent, we have

$$\begin{aligned}
\Pr(z_1 \geq z_2) &= \sum_{z_1=V_1}^{V_G} \sum_{z_2=V_1}^{z_1} p_1(z_1)\cdot p_2(z_2) \\
\Pr(z_1 = z_2) &= \sum_{z_1=V_1}^{V_G} p_1(z_1)\cdot p_2(z_1) \\
\Pr(z_1 \leq z_2) &= \sum_{z_1=V_1}^{V_G} \sum_{z_2=z_1}^{V_G} p_1(z_1)\cdot p_2(z_2)
\end{aligned} \tag{30}$$

Accordingly, we have

$$\begin{aligned}
\Pr(z_1 > z_2) &= \Pr(z_1 \geq z_2) - \Pr(z_1 = z_2) \\
\Pr(z_1 < z_2) &= \Pr(z_1 \leq z_2) - \Pr(z_1 = z_2)
\end{aligned} \tag{31}$$

Due to the above analysis, we are able to give the definition of stochastic dominance degree of two random variables with discrete probability distributions defined over a qualitative scale as follows.

Definition 6 Let Z_1 and Z_2 be two independent discrete random variables with (discrete) probability distributions p_1 and p_2 over a qualitative scale $\mathbb{V} = \{V_1, V_2, \ldots, V_G\}$ with $V_1 < V_2 < \cdots < V_G$, where $\sum_{z\in\mathbb{V}} p_1(z) = 1$ and $\sum_{z\in\mathbb{V}} p_2(z) = 1$. Then the stochastic dominance degree of p_1 over p_2 (noted as $R_{p_1 \succ p_2}$) is defined as

$$\begin{aligned}
R_{12} &= R_{p_1 \succ p_2} \\
&= \Pr(z_1 \geq z_2) - 0.5\Pr(z_1 = z_2)
\end{aligned} \tag{32}$$

where $\Pr(z_1 \geq z_2)$ is the probability of $z_1 \geq z_2$ and $0.5\Pr(z_1 = z_2)$ in Eq. (32) may be regarded as the probability of $z_1 > z_2$ when event $z_1 = z_2$ occurs (Fan et al, 2010).

Accordingly, the stochastic dominance degree of p_2 over p_1 (noted as $R_{p_2 \succ p_1}$) is defined as

$$\begin{aligned}
R_{21} &= R_{p_2 \succ p_1} \\
&= \Pr(z_2 \geq z_1) - 0.5\Pr(z_1 = z_2)
\end{aligned} \tag{33}$$

where $\Pr(z_2 \geq z_1)$ is the probability of $z_2 \geq z_1$ and $0.5\Pr(z_1 = z_2)$ in Eq. (33) may be viewed as the probability of $z_2 > z_1$ when event $z_1 = z_2$ occurs (Fan et al, 2010).

Extending two random variables to a vector of N random variables $\mathbb{Z} = (Z_1, Z_2, \ldots, Z_N)$, we are able to derive a matrix $\mathbf{R}$ of stochastic dominance degrees of the N discrete random variables as

$$\mathbf{R} = \begin{array}{c|cccc}
 & Z_1 & Z_2 & \ldots & Z_N \\
\hline
Z_1 & R_{11} & R_{12} & \ldots & R_{1N} \\
Z_2 & R_{21} & R_{22} & \ldots & R_{2N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
Z_N & R_{N1} & R_{N2} & \ldots & R_{NN}
\end{array} \tag{34}$$

Such a matrix of stochastic dominance degrees has the following interesting corollaries (Yan et al, 2013b).

Corollary 1 *Let* $\mathbf{R} = [R_{nl}]_{N\times N}$ *be a matrix of stochastic dominance degrees of the* N *discrete random variables, then we have* $R_{nl} + R_{ln} = 1, \forall n, l = 1, 2, \ldots, N$.

Corollary 2 *Let* $\mathbf{R} = [R_{nl}]_{N\times N}$ *be a matrix of stochastic dominance degrees, the stochastic dominance degree of one discrete random variable over itself is*

$$R_{nn} = 0.5, n = 1, 2, \ldots, N.$$

Corollary 3 *Let* $\mathbf{R} = [R_{nl}]_{N\times N}$ *be a matrix of stochastic dominance degrees, then the sum of all the elements of* $\mathbf{R}$ *is* $N^2/2$*, that is*

$$\sum_{n=1}^{N}\sum_{l=1}^{N} R_{nl} = \frac{N^2}{2}.$$

Corollary 4 *Let* $\mathbf{R} = [R_{nl}]_{N\times N}$ *be a matrix of stochastic dominance degrees, then we have* $0 \le R_{nl} \le 1, \forall n, l = 1, \ldots, N$.

Interestingly, the matrix $\mathbf{R}$ of stochastic dominance degrees with respect to a vector of N random variables $\mathbb{Z} = (Z_1, Z_2, \ldots, Z_N)$ satisfies the following properties of fuzzy preference relations.

Property 1 When $R_{nl} = 1$, it indicates that Z_n is absolutely preferred to Z_l, i.e., indicates the maximum degree of preference of Z_n over Z_l.

Property 2 When $0.5 < R_{nl} < 1$, it indicates that Z_n is slightly preferred to Z_l.

Property 3 When $R_{nl} = 0.5$, there is no preference (i.e., indifference) between Z_n and Z_l.

Property 4 When $0 < R_{nl} < 0.5$, it indicates that Z_l is slightly preferred to Z_n.

Property 5 When $R_{nl} = 0$, it indicates that Z_l is absolutely preferred to Z_n.

Therefore, the matrix of stochastic dominance degrees of the random variable set $\mathbb{Z}$ is in fact a matrix of fuzzy preference relations formulated as $\mu_{\mathbf{R}} : (Z_n, Z_l) \in \mathbb{Z} \times \mathbb{Z} \longrightarrow R_{nl} \in [0, 1]$, where $n, l = 1, \ldots, N$, and R_{nl} reflects the degree of fuzzy preference of Z_n over Z_l. Moreover, it is obvious that the matrix of fuzzy preference relations satisfies the condition of *fuzzy reciprocity* such that $R_{nl} + R_{ln} = 1, \forall n, l = 1, \ldots, N$.

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