Title	制限されたグラフに対する独立集合遷移問題と関連問題
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Citation	
Issue Date	2018-06
Type	Thesis or Dissertation
Text version	ETD
URL	http://hdl.handle.net/10119/15431
Rights	
Description	Supervisor:上原　隆平，情報科学研究科，博士

Independent Set Reconfiguration and Related Problems for Some Restricted Graphs

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June 2018

Abstract

For the last decade, a collection of combinatorial problems called *reconfiguration problems* has been studied extensively. Roughly speaking, a reconfiguration problem is specified in terms of a given collection of *configurations* and a *reconfiguration rule* that describes how to transform one configuration into another. Several real-world situations involving movement and change can be modeled as reconfiguration problems. As an example, consider a network that delivers electricity from suppliers to consumers. In such a network, it may happen that some devices at a power station S is broken and one need to temporarily shut down S for replacing these broken devices with the new ones. Before shutting down S, one may need to reroute the transmission lines that go through S to some other power station in order to maintain the availability of the network. A technician may wonder which station he/she needs to pick for replacing S such that the network remains active and, for saving resources when S becomes active again, the chosen station should be as near S as possible. Such a situation can be modeled as a PATH RECONFIGURATION problem, where each configuration is a path (transmission line) from the main supplier to customers, and the rule is to change a node (power station) such that the network remains connected (active). Another real-world situation where reconfiguration problems arise is the motion planning of moving objects. For instance, in a 3D printer model where multiple printing heads have been used, one need to plan the printing paths (which the heads will follow) to avoid collisions and other unwanted interactions, as well as making the distance traveled by each head as small as possible. Another situation involves multiple robots moving in an environment and one need to plan their movements such that they can avoid obstacles and each other. Such problems can be modeled as different variants of the TOKEN RECONFIGURATION problem on graphs. A classic variant of TOKEN RECONFIGURATION is the so-called 15-PUZZLE – a research topic since 1879. A configuration of 15-PUZZLE consists of 15 tokens labeled $1, 2, \dots, 15$, placed on a 4×4 grid. The rule is that a token can only be slid to an unoccupied adjacent vertex. The 15-PUZZLE problem asks whether one can transform one configuration into another. 15-PUZZLE and its generalized versions can be used as models for the MULTI-ROBOT PATH PLANNING problem. For an overview on both theoretical and practical perspectives of reconfiguration problems, the readers are referred to the surveys by van den Heuvel [*Surveys in Combinatorics 2013*, 127–160, 2013] and Nishimura [*Algorithms*, 11:4, 52, 2018].

Among several reconfiguration problems, the reconfiguration variants of Independent Set are of particular interest. In such variants, an independent set (a set of pairwise non-adjacent vertices of a graph) is often viewed as a set of tokens placing on vertices of the input graph. In this viewpoint, Independent Set Reconfiguration can be seen as a restricted version of the Token Reconfiguration problem where distinct unlabeled tokens are placed on the vertices of a graph and no two tokens are adjacent. Among different reconfiguration rules, the following three models have attracted the attention of many theoretical computer scientists: *Token Sliding* (TS), *Token Jumping* (TJ), and *Token Addition and Removal* (TAR). A TS-step involves moving a token to one of its adjacent vertices. A TJ-step involves moving a token to any other vertex (not necessarily in its neighbors) of the graph. A TAR-step involves either adding or removing a token such that the number of remaining tokens is at least some threshold k. Typically, we are interested in determining whether one can transform an independent set I into another independent set J using TS/TJ/TAR rule such that each intermediate result is also an independent set. For all three rules, the problem is PSPACE-complete even for planar graphs of maximum degree 3 and bounded bandwidth/treewidth/pathwidth/cliquewidth. This raises an open question on whether there exist efficient algorithms for solving the problem (under TS/TJ/TAR) when the bandwidth/treewidth/pathwidth/cliquewidth of the input graph is bounded by some practical (small) constant. Interestingly, when comparing the three rules, TJ and TAR are equivalent, in the sense that for any sequence of p TJ-steps between two independent sets I and J of size k, there is also a sequence of $2p$ TAR-steps between them whose number of tokens in each member is either k or $k-1$, and vice versa. The TS model seems to be more "restricted", in the sense that any sequence of TS-steps can be seen as a sequence of TJ-steps. However, the reverse direction does not hold. This motivates our study for the TS rule. As a result, in this thesis, we made a significant contribution to the computational complexity of Independent Set Reconfiguration under TS rule via designing polynomial-time algorithms for solving the problem for different restricted graphs, namely trees, and cactus graphs (whose treewidth is at most 2). As consequences of our algorithms, we show that one can construct an actual sequence of TS-steps (if exists) between two given independent sets using a polynomial number of token-slides.

Key Words: **polynomial-time algorithm, computational complexity, combinatorial reconfiguration, sliding token, independent set, tree, cactus graph**