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On Equitability for Housemate Problem

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The problem of fair division is to find a division of some objects and an allocation of each portion to players which is considered to be fair for every players. The objects to be divided can be either a set of divisible items, a set of indivisible items, or a set of both divisible and indivisible items. Moreover, each of such items can be "*good*" or "*bad*". Such problem have been studied for long time.

In this thesis, a fair division problem in which indivisible goods and divisible bads are allocated among players is concerned. The problem is formulated and analyzed in terms of *housemate problem*. An informal description of housemate problem can be presented as follows: Suppose there is a house with n rooms to be rented by n housemates, and each housemate bids a nonnegative value for each room in such a way that the total bids given by each housemate is equal to the house rent. The problem is to assign to each housemate exactly one room and its room rent, i.e., a solution to this problem is a pair of room allocation and room rents. Observe that, in this problem, rooms are indivisible goods, and the house rent is divisible bad.

To decide whether a solution of the problem is fair, we need concepts of fairness. For housemate problem, concepts of fairness is defined by surpluses (i.e., difference of bids and room rents) that housemates may get. In the literature, there are several concepts of fairness are proposed, such as proportionality, envy-freeness and exactness. A solution is called proportional if each housemate gets surplus with nonnegative value. A solution is called envy-free if each housemate is assigned a room such that he or she gets surplus with the largest value. A solution is called exact if different the room assignment make no change in the value of surplus that each housemate may get.

Brams and Kilgour proposed a simple procedure, called Gap procedure, for finding a proportional solution such that the sum of surpluses of housemates is maximized. They also showed that an envy-free solution may not exist if $n \geq 4$, and gave a sketch of proof that an envy-free solution always exists if $n \leq 3$. Later, Sung and Vlach showed that room assignment of an envy-free solution must be a maxsum assignment, i.e., an assignment which maximizes the sum of corresponding bids. They proposed a procedure to decide whether an envy-free solution exists or not, and when an envy-free solution exists the procedure returns an envy-free solution. They also showed that in general there are uncountably many envy-free solutions.

In this thesis, we study the housemate problem with equitability as its fairness. First, we show that an equitable solution always exists. Since the value of surplus that each housemate may get changes depending on solutions, we consider the problem to decide whether an equitable and proportional solution exists. We show that the problem is NP-complete. As a consequence, the problem to find an equitable solution in which surpluses of housemates are maximized is NP-hard. Furthermore, as an extension of the algorithm proposed by Sung and Vlach, we propose an efficient algorithm for finding an envy-free solution in which the differences of surplus among housemates are minimized, i.e., the algorithm finds an envy-free solution which is most equitable.